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TWO-DIMENSIONAL NUMERICAL MODELLING OF MODULAR-BLOCK SOIL RETAINING WALLS COLLAPSE USING MESHFREE METHOD

C.T. Nguyen[1,3], Ha H. Bui[2], and R. Fukagawa[3]

[1]Institute of Mechanics, Vietnam Academy of Science and Technology, Vietnam
[2]Department of Civil Engineering, Monash University, Australia
[3]Department of Civil Engineering, Ritsumeikan University, Japan

ABSTRACT: Modular-block retaining wall system has been extensively used to stabilize natural and cut-slope. Despite this fact, numerical development for prediction of the large deformation and flexible behavior of modular retaining wall bocks is still not advanced. To overcome this limitation, this paper presents a new numerical approach which can be used to simulate large deformation and flexible behavior of the modular-block retaining wall system. Herein, soil is modelled using an elasto-plastic constitutive model, while wall blocks are assumed rigidity with full degree of motion. A linear contact model is proposed to simulate interaction between soil and wall block. Experiments were also conducted to validate the proposed numerical framework. It is showed that the proposed numerical framework can simulate well behavior of the modular-block retaining wall system.

Keywords: Modular-Block Retaining Wall, Large Deformation, Elasto-Plastic, SPH

1. INTRODUCTION

Modular-block retaining walls (MRW) have been used as an effective method to stabilize cuts and fills adjacent to highways, driver-ways, embankment, etc. Because they are flexible structures, modular-block retaining walls can tolerate movement and settlement without causing crack and damage, particularly under seismic loading conditions. Despite of its advantage, very few numerical studies of large deformation of the MRW systems were found in the literature. This is because it is very difficult to simulate large deformation and flexible behaviour of wall blocks (i.e. full rotational and translational motions) in the MRWs system using traditional continuum based numerical methods such as finite element method (FEM) which is suffered from grid distortions. The Discrete Element Method (DEM) proposed by Cundall & Strack [1] which is another popular numerical method in geotechnical applications may be applied to simulate dynamic behaviour of the modular-block retaining wall blocks in the MRW system. However, the DEM suffers from low accuracy in predicting soil behaviour due to the difficulty in selecting parameters for contact laws. In addition, the DEM cannot make use of advanced soil constitutive models which have been extensively developed in the literature. The discontinuous deformation analysis (DDA) method proposed by Shi et al. [2] has also been applied to geotechnical applications, but is mainly used for rock engineering, etc. In order to overcome the above limitations of traditional numerical methods, continuum based mesh-free methods such as the mesh-free Galerkin element method (EFG) [3], material point method (MPM) [4], particle in cell method (PIC) [5], etc., could be also applied to simulate large deformation of soil. However, these methods are quite time consuming and complicated to implement into a computer code as they consist of both interpolation points and the background mesh. On the other hand, the smoothed particle hydrodynamics (SPH) method, originally proposed by Gingold & Monaghan [6], has been recently developed for solving large deformation and post-failure behaviour of geomaterials [7-12] and represents a powerful way to understand and quantify the failure mechanisms of soil in such challenging problems.

In this paper, taking into consideration the unique advantage of the SPH method, it is further extended to simulate large deformation and post-failure of the MRW systems. Herein, soil is modelled using the elasto-plastic Drucker-Prager constitutive model [8] and wall blocks are assumed rigidity with complete degree of rotation. A linear contact model which is similar to the penalty contact law is proposed and is implemented in the SPH code to simulate interaction between soil and wall blocks, and between wall blocks in the MRW systems. The developed model is then applied to simulate large deformation of the MRW system and comparing to a two-dimensional experiment. Results showed good agreement with the experiment, suggesting that the proposed method is a promising approach for further design of modular-block retaining wall systems subjected to earthquake.

2. SIMULATION APPROACHES

2.1 Simulation of Soil in SPH Framework

In the SPH method, motion of a continuum is modeled using a set of moving particles (interpolation points); each assigned a constant mass and "carries" field variables at the corresponding location. The continuous fields and their spatial derivatives are taken to be interpolated from the surrounding particles by a weighted summation, in which the weights diminish with distance according to an assumed kernel function. Details of the interpolation procedure and its application to soil can be found in Bui et al. [8]. The motion of a continuum can be described through the following equation,

$$\rho \frac{d\mathbf{v}}{dt} = \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g} + \mathbf{f}_{ext} \tag{1}$$

where $\mathbf{v}$ is the velocity; ρ is the density; $\boldsymbol{\sigma}$ is the total stress tensor, where negative is assumed for compression; $\mathbf{g}$ is the acceleration due to gravity; and $\mathbf{f}_{ext}$ is the additional external force(s). The total stress tensor of soil is normally composed of the effective stress ($\boldsymbol{\sigma}'$) and the pore-water pressure (p_w), and follows Terzaghi's concept of effective stress. Because the pore-water pressure is not considered, the total stress tensor and the effective stress are identical throughout this paper, and can be computed using a constitutive model.

In the SPH framework, Eq. (1) is often discretized using the following form [8, 11],

$$\frac{dv_a^\alpha}{dt} = \sum_{b=1}^{N} m_b \left(\frac{\sigma_a^{\alpha\beta}}{\rho_a^2} + \frac{\sigma_b^{\alpha\beta}}{\rho_b^2} + C_{ab}^{\alpha\beta} \right) \nabla_a^\beta W_{ab} + g_a^\alpha + f_{ext \to a}^\alpha \tag{2}$$

where α and β denote Cartesian components x, y, z with the Einstein convention applied to repeated indices; a is the particle under consideration; ρ_a and ρ_b are the densities of particles a and b respectively; N is the number of "neighbouring particles", *i.e.* those in the support domain of particle a; m_b is the mass of particle b; C is the stabilization term [10]; W is the smoothing kernel function which is taken to be the cubic Spline function [13]; and $f_{ext \to a}$ is the unit external force acting on particle a.

The stress tensor of soil particles in Eq. (2) can be computed using any soil constitutive model developed in the literature. For the purpose of soil-structure interaction, the Drucker-Prager model has been chosen with non-associated flow rule, which was implemented in the SPH framework by Bui et al. [8] and shown to be a useful soil model for simulating large deformation and post-failure behaviour of aluminum rods used in the current paper as model ground. The stress-strain relation of this soil model is driven from the assumption of additive decomposition of the total strain rate tensor,

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p \tag{3}$$

where a raised dot denotes the time derivative; $\dot{\boldsymbol{\varepsilon}}$ is the total strain rate tensor; $\dot{\boldsymbol{\varepsilon}}^e$ is its elastic component; and $\dot{\boldsymbol{\varepsilon}}^p$ is its plastic component. The elastic component is computed using the well-known Hooke's law; while the plastic component can be calculated using the plastic flow rule [8],

$$\dot{\boldsymbol{\sigma}} = \mathbf{D}\dot{\boldsymbol{\varepsilon}}^{\mathbf{e}} \tag{4}$$

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda} \frac{\partial g_p}{\partial \boldsymbol{\sigma}} \tag{5}$$

where $\mathbf{D}$ is the elastic stiffness matrix, $\dot{\lambda}$ is the rate of change of plastic multiplier, and g_p is the plastic potential function.

According to the plasticity theory, the plastic deformation occurs only if the stress state reaches the yield surface. Therefore, the plastic deformation will occur only if the following yield criterion is satisfied,

$$f = \alpha_\phi I_1 + \sqrt{J_2} - k_c = 0 \tag{6}$$

where I_1 and J_2 are the first and second invariants of the stress tensor; and α_ϕ and k_c are Drucker-Prager constants that are calculated from the Coulomb material constants c (cohesion) and ϕ (internal friction). In the plane strain, the Drucker-Prager constants are computed by,

$$\alpha_\phi = \frac{\tan\varphi}{\sqrt{9 + 12\tan^2\varphi}} \tag{7}$$

$$k_c = \frac{3c}{\sqrt{9 + 12\tan^2\phi}} \tag{8}$$

The non-associated plastic flow rule specifies the plastic potential function by [9-10],

$$g_p = \alpha_\psi I_1 + \sqrt{J_2} - \textbf{constant} \tag{9}$$

where α_ψ is a dilatancy factor that can be related to the dilatancy angle ψ in a fashion similar to that between α_ϕ and friction angle ϕ.
Substituting Eq. (9) into Eq. (5) in association with the consistency condition, that is the stress state must be always located on the yield surface f during the plastic loading, the stress-strain relation of the current soil model can be written as [10],

$$\frac{d\sigma_a^{\alpha\beta}}{dt} = 2G_a\dot{e}_a^{\alpha\beta} + K_a\dot{\varepsilon}_a^{\gamma\gamma}\delta_a^{\alpha\beta} - \dot{\lambda}_a\left[3K_a\alpha_{\psi a}\delta^{\alpha\beta} + (G_a/\sqrt{J_{2a}})s_a^{\alpha\beta}\right] \tag{10}$$

where $e^{\alpha\beta}$ is the deviatoric strain-rate tensor; $s^{\alpha\beta}$ is the deviatoric shear stress tensor; and $\dot{\lambda}$ is the rate of change of plastic multiplier of particle a [10],

$$\dot{\lambda}_a = \frac{3\alpha_{\phi a}K_a\dot{\varepsilon}_a^{\gamma\gamma} + (G_a/\sqrt{J_{2a}})s_a^{\alpha\beta}\dot{\varepsilon}_a^{\alpha\beta}}{9\alpha_{\phi a}K_a\alpha_{\psi a} + G_a} \tag{11}$$

where the strain-rate tensor is computed by

$$\dot{\varepsilon}_a^{\alpha\beta} = \frac{1}{2}\left(\nabla^\beta\dot{u}^\alpha + \nabla^\alpha\dot{u}^\beta\right)_a \tag{12}$$

When considering a large deformation problem, a stress rate that is invariant with respect to rigid-body rotation must be employed for the constitutive relations. In the current study, the Jaumann stress rate is adopted:

$$\dot{\hat{\sigma}}_a^{\alpha\beta} = \dot{\sigma}_a^{\alpha\beta} - \sigma_a^{\alpha\gamma}\dot{\omega}_a^{\beta\gamma} - \sigma_a^{\gamma\beta}\dot{\omega}_a^{\alpha\gamma} \tag{13}$$

where $\dot{\omega}$ is the spin-rate tensor computed by

$$\dot{\omega}_a^{\alpha\beta} = \frac{1}{2}\left(\nabla^\beta\dot{u}^\alpha - \nabla^\alpha\dot{u}^\beta\right)_a \tag{14}$$

As a result, the final form of the stress-strain relationship for the current soil model is,

$$\frac{d\sigma_a^{\alpha\beta}}{dt} = \sigma_a^{\alpha\gamma}\dot{\omega}_a^{\beta\gamma} + \sigma_a^{\gamma\beta}\dot{\omega}_a^{\alpha\gamma} + 2G_a\dot{e}_a^{\alpha\beta} + K_a\dot{\varepsilon}_a^{\gamma\gamma}\delta_a^{\alpha\beta} - \dot{\lambda}_a 3K_a\alpha_{\psi a}\delta^{\alpha\beta} + \dot{\lambda}_a(G/\sqrt{J_2})_a s_a^{\alpha\beta} \tag{15}$$

Validation of the elasto-plastic Drucker-Prager soil model with SPH has been extensively documented in the literature [8-10], and readers can refer to these references for further details on the validation process.

2.2 Motion of Rigid Wall Blocks

An arbitrary motion of a rigid body can be represented as a superposition of translational motion in which all points of the body, including the centre of mass, move with the same speed along parallel trajectories, and rotation around the centre of mass. Accordingly, the motion of a rigid wall block in the MRW system can be determined by specifying the translational motion of the centre of mass and the rotational motion about its mass central. The equation of motion of the central mass is given as follows,

$$M\frac{d\mathbf{V}}{dt} = \mathbf{F} \tag{16}$$

where M is the central mass, $\mathbf{V}$ is the velocity vector of the central mass, $\mathbf{F}$ is total force vector acting on the body.
The equation of rotation about the central mass is,

$$I\frac{d\mathbf{\Omega}}{dt} = \mathbf{T} \tag{17}$$

where I is the inertial moment, $\mathbf{\Omega}$ is the angular velocity which is perpendicular to the plane of the motion, and $\mathbf{T}$ is the total torque about the central mass.
In the computation, the rectangular block is represented by the set of boundary particles that are equi-spaced around the boundary. Denoting the force vector acting on each boundary particle i located on the moving block is f_i, Eq. (16) and Eq. (17) can be rewritten, respectively, as follows,

$$M\frac{d\mathbf{V}}{dt} = \sum_i \mathbf{f}_i \tag{18}$$

$$I\frac{d\mathbf{\Omega}}{dt} = \sum_k (\mathbf{r}_i - \mathbf{R})\times\mathbf{f}_i \tag{19}$$

where $\mathbf{r}_i$ and $\mathbf{R}$ are vector coordinates of boundary particle and central mass, respectively. The rigid body boundary particles move as a part of the rigid body, thus the change on position of boundary particle i is given by,

$$\frac{d\mathbf{r}_i}{dt} = \mathbf{V} + \mathbf{\Omega}\times(\mathbf{r}_i - \mathbf{R}) \tag{20}$$

The force $\mathbf{f}_i$ acting on a boundary particle on the rigid body is due to the surrounding soil particles or boundary particles that belong to different rigid bodies. This force can be calculated using any suitable contact model.

2.3 Contact Force Model

A linear soft contact model based on a concept of the spring and dash-pot system is proposed to model the interaction between soil and retaining wall blocks and between blocks. Accordingly, the radial force acting between two particles can be calculated using the following equation,

$$\mathbf{f}^{n}_{a\to i} = \begin{cases} -K_{ai}\boldsymbol{\delta}_{n} - c_{n}\mathbf{v}^{n}_{ak} & h_{ai} > 2d_{ai} \\ 0 & h_{ai} \le 2d_{ai} \end{cases} \tag{21}$$

where K is the radial stiffness; $\boldsymbol{\delta}_n$ is the allowable overlapping distance between two particles; c_n is the radial damping coefficient; $\mathbf{v}^n$ is the relative radial velocity vector between particle a and particle i; h_a and h_{ai} are the sum of the initial separation between soil particles and between boundary particles, respectively; and d_{ai} is the distance between two particles. The overlapping distance and radial damping coefficient can be calculated using the following relationships,

$$\delta_n = d_{ai} - h_{ai}/2 \tag{22}$$

$$c_n = 2\sqrt{m_{ai}K_{ai}} \tag{23}$$

The contact force in the shear direction which is perpendicular to the radial direction can be calculated in the same manner,

$$\mathbf{f}^{s}_{a\to i} = \begin{cases} -k_{ai}\boldsymbol{\delta}_{s} - c_{s}\mathbf{v}^{s}_{ai} & h_{ai} > 2d_{ai} \\ 0 & h_{ai} \le 2d_{ai} \end{cases} \tag{24}$$

where k_{ai} is the shear stiffness which is taken similar to K; $\boldsymbol{\delta}_s$ is the relative shear displacement between the two particles; c_s is the shear damping coefficient; $\mathbf{v}^s$ is the relative shear velocity vector between particle a and particle i. The relative shear displacement and shear damping coefficient are,

$$\boldsymbol{\delta}_s = \oint \mathbf{v}^{s}_{ai}\,dt \tag{25}$$

$$c_s = 2\sqrt{m_{ai}k_{ai}} \tag{26}$$

The current shear force must satisfy Coulomb's friction law which implies that the shear force must not exceed the maximum resisting force,

$$\mathbf{f}^{s}_{a\to i} \le \mu \frac{\boldsymbol{\delta}_s}{\left|\boldsymbol{\delta}_s\right|}\left|\mathbf{f}^{n}_{a\to i}\right| \tag{27}$$

where μ is the friction coefficient. These forces are finally added to Eq. (2), Eq. (18) and Eq. (19) to progress the motion of soil and wall block.

3. TWO-DIMENSIONAL EXPERIMENTS

A serial of two-dimensional modular-block retaining wall system collapses were conducted to investigate the failure mechanism of the MRW system and to verify the proposed numerical framework. Fig. 1 outlined the initial setting condition of the MRW system which consists of six rectangular wall blocks. The size of the model ground is 15cm in height and 50cm in width at the base. Soil was modelled using aluminum rods of 1.6mm and 3mm in diameters, 50mm in length, and mixed with the ratio of 3:2 in weights. The wall block is 3.2cm in width, 2.5cm in height, and 5cm in length, which is also manufactured from aluminum. In the experiment, the MRW system was constructed by successively placing one wall block on the top of the other with an overlapping of 1.2cm, followed by filling the model ground at each layer. The MRW system was stabilized by a stopper stand as shown in Fig. 1. To visualize the failure pattern of the model ground after collapse, square grids (2.5×2.5cm) were drawn on the soil specimen. The experiments were initiated by quickly removing the stopper stand and digital photos were taken to record the failure process as well as the final configuration of the MRW system after collapse.

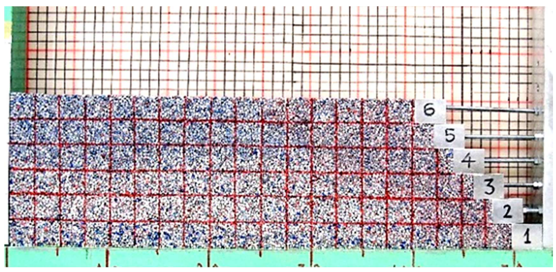

Fig. 1 Initial setup of model ground and the retaining wall blocks in the MRW system.

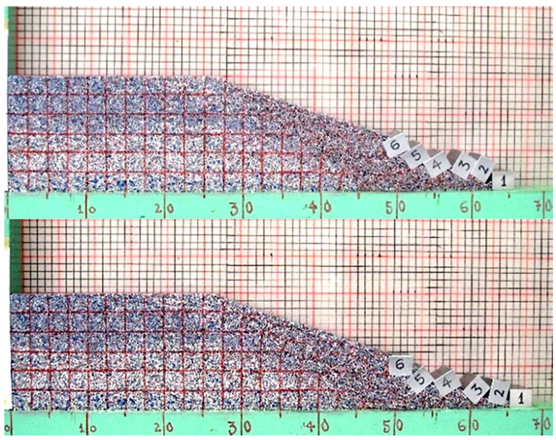

Fig. 2 Typical configurations of the MRW system observed in the experiment after collapse for two different tests.

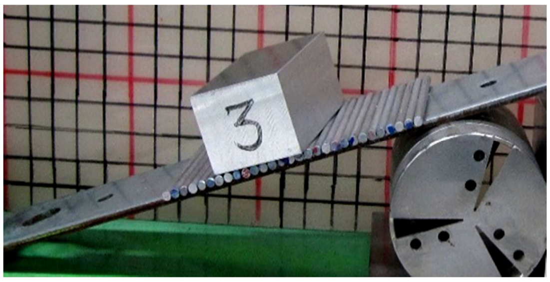

Fig. 3 Outline of the sliding test used to measure friction coefficient between model ground and modular-block.

A series of experiments were conducted starting from one block and then gradually increasing the number of blocks in the BRW system until the retaining wall is collapsed. The experimental evidences consistently showed that the MRW system will collapse when reaching 6 blocks height. Accordingly, a numerical model consisting of six retaining wall blocks will be conducted in the next section to verify the proposed numerical framework. A total of six experiments were conducted to verify the failure mechanism of the MRW system and the final run-out distance of each block in the MRW system. In all experiments, the failure mechanism of the BRW systems was more and less the same as shown in Fig. 2.

In addition to the retaining wall collapse experiments, sliding tests were also conducted to measure friction coefficients between the wall blocks, between the soil model and the wall block, and between the block and the bottom solid wall. Outline of these tests are shown in Fig. 2. Based on these experiment, it was found that the friction (μ) between retaining wall blocks is μ ~ 0.31, between the retaining wall block and model ground is μ ~ 0.38, and between wall block and the bottom wall boundary is μ ~ 0.40.

4. SIMULATION OF MODULAR-BLOCK RETAINING WALL COLLAPSE USING SPH

The model test shown in Figure 1 was simulated using 11,304 SPH particles arranged in a rectangular lattice with an initial separation of 0.25cm. Rigid blocks were created by placing boundary particles uniformly around the boundary at a constant distance. In order to simulate the smooth surface, half of particle spacing was chosen for the rigid body boundary particles. Model ground parameters including elastic modulus E = 5.84MPa, Poisson's ratio ν = 0.3, friction angle ϕ = 21.9^o, dilatant angle ψ = 0^o, and cohesion c = 0kPa were taken similar to those measured by Umezaki et al. [15]. The unit weight of the ground model is γ_s = 20.4kN/m^3. In addition to the ground parameters, parameters for the linear contact model needed to be specified. In this paper, the radial and shear stiffness were assumed to be $K = k = 1\times10^9$N/m. The friction coefficients between the ground model and the block, between the rigid blocks, and between the block and the bases of the wall boundary were taken similar to those measured in the sliding tests as explained in the experimental section. The boundary conditions for the model ground are restrained with a roller boundary at the lateral boundaries and fixed in both directions at the base [8].

Fig. 4 shows the comparison between the experiment and the computation for the final configuration of the MRW system after collapse. It can be seen that the computed result could predict fairly well the behavior of all rigid blocks observed in the experiment after the MRW system was collapsed. The good agreement between experiment and simulation can be attributed to the fact that the complete degrees-of-freedom of the rigid wall was taken into consideration in the simulation and large deformation and post-failure behavior of soil could be simulated well in the current SPH framework.

Comparing the final run-out distance of each block, for instance Block No.1, it can be seen that the final position (right edge) of Block No.1 in the simulation is approximately ~68.3cm from the left-most solid boundary. This result is in fairly good agreement with that observed in experiment which was approximately ~66.2cm. This suggests that the proposed numerical framework could be applied to simulate the soil-structure interaction in the MRW system. However, further refinement of the contact model should be considered to provide a more theoretical sound framework to specify the parameters for the contact model.

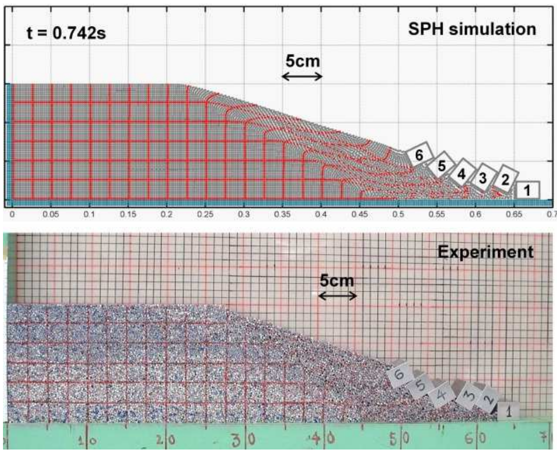

Fig. 4 Comparison between the SPH simulation and the experiment for the final configuration of the MRW system.

5. CONCLUSION

This paper presented a new numerical approach for simulation of large deformation and post-failure of modular-block retaining wall system. It was shown that the proposed method provides good agreement with the experimental results. One of the key advantages of the proposed method is that the complete degrees-of-freedom of the retaining wall blocks, which could not be simulated using traditional numerical approaches such as finite element method, can now be simulated in the proposed numerical framework. Large deformation and post-failure behaviour of geomaterials can also be readily simulated. In order to broaden the application of the proposed numerical approach in geotechnical engineering, generalized contact model is necessary. This work is in progress and will be reported in the near future.

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Corresponding Author: C.T. Nguyen