International Journal of GEOMATE, Nov., 2018 Vol.15, Issue 51, pp.188-194
Geotec., Const. Mat. & Env., DOI: https://doi.org/10.21660/2018.51.7332
ISSN: 2186-2982 (Print), 2186-2990 (Online), Japan

STRESS-STRAIN-DILATANCY RELATIONSHIPS OF NORMALLY CONSOLIDATED DHAKA CLAY

Muhammad Abdur Rahman [1], *Hossain Md Shahin [2] and Teruo Nakai [3]

[1]Lecturer, Department of Civil Engineering, MIST, Mirpur, Dhaka-1216, Bangladesh.
[2]Professor, Department of Civil Engineering, IUT, Board Bazar, Gazipur, Bangladesh.
[3]Technical Adviser, Geo-Research Institute; Emeritus Professor at NIT, Nagoya, Japan.

*Corresponding Author, Received: 12 June 2017, Revised: 22 Jan. 2018, Accepted: 12 June 2018

ABSTRACT: In this paper, stress-strain-dilatancy relationships of normally consolidated Dhaka clay is presented. Two constitutive models Cam-clay model (both original and modified) and sub loading t_{ij} model are used in this paper. To obtain the parameters, drained triaxial tests of saturated cylindrical specimens under compression is conducted. One dimensional consolidation test is also conducted to obtain other model parameters namely compression index and swelling index. Total four sets of the undisturbed specimen are prepared where each set contains two specimens. Consolidated drained triaxial compression tests are conducted on first three test sets and thereby stress-dilatancy relation in compression condition is evaluated. To understand the behavior in extension one-dimensional consolidation test and drained triaxial test are performed on the test set four to obtain the model parameters. It is observed from the stress-dilatancy relationship that, sub loading t_{ij} model can well describe the stress-dilatancy behavior of normally consolidated Dhaka clay than that of the Cam-clay model both in compression and extension conditions. In compression condition, sub loading t_{ij} model presents almost a unique stress-dilatancy relationship for all three sets of specimens. Also in extension condition, a unique relationship between stress and dilatancy is observed where Cam-clay model failed to describe it uniquely. Therefore, sub loading t_{ij} model can be used with better accuracy for the Dhaka clay.

Keywords: Stress-Dilatancy Relationship, Constitutive Modelling, Stress-Strain Relation, Drained Tri-axial Test, Consolidation Test.

1. INTRODUCTION

With the development of modern geotechnical engineering, application of numerical techniques like finite element method becomes essential to solve the complex geotechnical engineering problems. These methods depend largely on the stress-strain behavior of geomaterials to solve any problem.

Again, most of the cases geotechnical engineering problems are solved by considering soil as an elastic or rigid plastic material where stress-dilatancy characteristics are not taken into considerations. As soil is an elastoplastic material, most of the designs are either over designed or under designed for not taking proper considerations of constitutive model. There is the necessity of a constitutive model which can simulate the soil behavior accurately. On the contrary, most of the constitutive models are not able to simulate the behavior of the soil accurately and does not fit with all the soil types as well.

The first simple model which considered the soil as an elastoplastic material is the Cam-clay model. In the Cam-clay model [12] positive dilatancy during strain hardening is not taken into consideration along with some other limitations [6] which are as follows:

(i) Influence of intermediate principal stress on the deformation and strength of soil (ii) Stress path dependency on the direction of plastic strain increments (iii) Positive dilatancy during strain hardening (iv) Behavior of soil under cyclic loading (v) Soil anisotropy (vi) Influence of density and/or confining pressure on the deformation and strength (vii) Behavior of structured soil (viii) Soil anisotropy and non-coaxially (ix) Time effect and age effect (x) Unsaturated Soils.

To overcome the limitations of Cam-clay model, Subloading t_{ij} model [6] had been developed which provides a better performance in loose to dense sand and soft to stiff clay. Some of the features of the model are as follows:

(i) The influence of intermediate principal stress on the deformation and strength is considered [8].
(ii) The stress path dependency of plastic flow is considered with the introduction of the plastic strain increment dividing into two components: a plastic strain increment which satisfies an associated flow rule in the t_{ip} space and an isotropic plastic strain increment due to increasing mean stress.
(iii) Positive dilatancy during strain hardening is considered in sub loading t_{ij}, model [9].
(iv) Influence of density and/or confining pressure on the deformation and strength are taken into consideration by introducing and

revising the sub loading surface concept [2]-[3].

(v) The behavior of structured soil such as naturally deposited clay can be described [7], [10].

In this paper comparison between this two simple constitutive models in terms of stress-dilatancy relationship is made to choose a better constitutive model for Dhaka soils.

2. DESCRIPTION OF DHAKA CLAY

Geologically Dhaka is situated in the southern half of the Madhupur Tract which is the oldest sediment exposed. The tropical clay soils of Dhaka are mainly composed of illite, kaolinite, chlorite and some non-clay minerals [4]. The Dhaka clay varies from light yellowish gray to brick red in color. In general, the soil is normally consolidated to slightly overconsolidated and of intermediate to high plasticity [5].

In the present study, the sample is collected from the Mirpur area (23°49'49.8"N, 90°22'34.2"E) where the reddish clay layer extends to about 10 m depth.

3. STRESS-DILATANCY EQUATIONS

In this paper, comparison of stress-dilatancy relationship is made among the original Cam-clay model, Modified Cam-clay model, and sub loading t_{ij} model.

3.1 Cam-Clay Model

Like most of the constitutive models for soils, Cam-clay model is formulated using the stress invariants, e.g. mean stress p and deviatoric stress q and the strain increment invariants, e.g. volumetric strain increment $d\varepsilon_v$ and deviatoric strain increment $d\varepsilon_d$ [6],[12].

Yield function of the model is defined by the following equation:

$$\ln p + \zeta(\eta) - \ln p_1 = \ln \frac{p}{p_0} + \zeta(\eta) - \ln \frac{p1}{p_0} = 0 \tag{1}$$

Where, $\zeta(\eta)$ is an increasing function of η ($\eta = q/p$) and satisfies the condition $\zeta(0) = 0$, p_0 is the value of the initial yield surface at p-axis, and p_1 determines the size of the current yield surface (the value of p at $\eta = 0$).

The stress ratio function $\zeta(\eta)$ is expressed as follows in Cam-clay model:

Original:

$$\zeta(\eta) = \frac{1}{\mathrm{M}}\,\eta \tag{2}$$

Modified:

$$\zeta(\eta) = \frac{\mathrm{M}^2+\eta^2}{\mathrm{M}^2}\,\eta \tag{3}$$

Here, M is the stress ratio, η at a critical state.

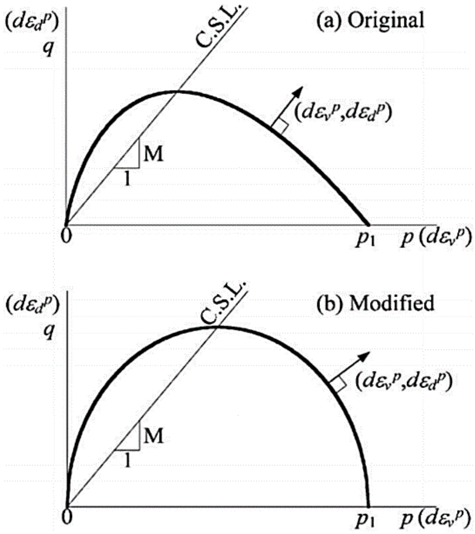


Fig. 1 Yield surface of Cam-clay model in p-q plane.

The plastic strain increment assuming associated flow rule than can be expressed as follows [6]:

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = \frac{\partial f/\partial p}{\partial f/\partial q} = \frac{1-\zeta'(\eta).\eta}{\zeta'(\eta)} \tag{4}$$

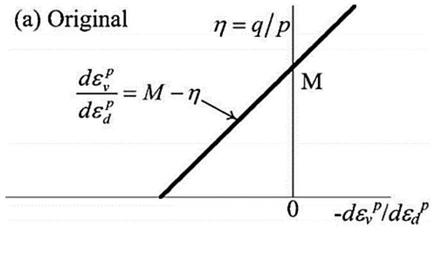
Using Eq. (2), Eq. (3) and Eq. (4), the stress-dilatancy relation can be derived as follows:

Original:

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = \mathrm{M} - \eta \tag{5}$$

Modified:

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = \frac{\mathrm{M}^2-\eta^2}{2\eta} \tag{6}$$



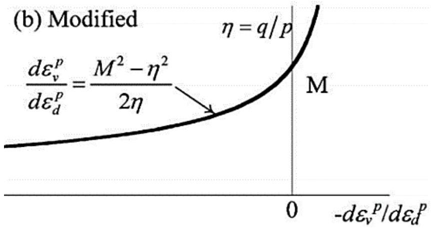


Fig. 2 Stress-dilatancy relations of (a) Original (b) Modified Cam-clay model.

3.2 Subloading t_{ij} model

In the subloading t_{ij} model [9] yield function is given by,

$$\ln t_N + \zeta(X) - \ln t_{N1} = \ln \frac{t_N}{t_{N0}} + \zeta(X) - \ln \frac{t_{N1}}{t_{N0}} = 0 \quad (7)$$

Here, t_N and t_S are the stress invariants in t_{ij} concept [9].

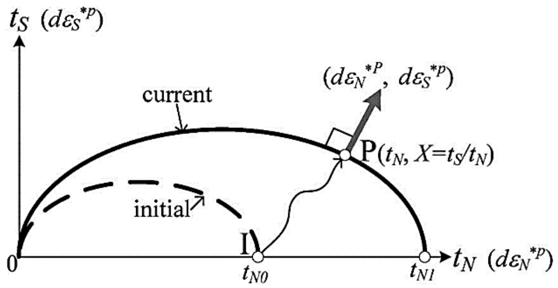


Fig. 3 Yield surface of sub loading t_{ij} model in t_N-t_S plane.

The stress ratio function $\zeta(X)$ is then given by the equation below [1],[9]:

$$\zeta(X) = \frac{1}{\beta}\left(\frac{X}{M^*}\right)^{\beta} \quad (8)$$

Where β (≥1) is the parameter which controls the shape function.

Finally, the stress dilatancy relation can be expressed as,

$$\frac{d\varepsilon_N^{*p}}{d\varepsilon_S^{*p}} = \frac{1-\zeta'(X).X}{\zeta'(X)} = \frac{(M^*)^{\beta}-X^{\beta}}{X^{\beta-1}} \quad (9)$$

Here, $d\varepsilon_N^{*p}$ and $d\varepsilon_S^{*p}$ are the strain increment invariants in t_{ij} concept.

Again, M^* is expressed using X_{CS} and Y_{CS}, which are the stress ratio X and Y at the critical state ($d\varepsilon_v^p = 0$).

$$M^* = (X_{CS}^{\ \beta} + X_{CS}^{\ \beta-1} Y_{CS})^{1/\beta} \quad (10)$$

X_{cs} and Y_{cs} are expressed as follows [8]:

$$X_{CS} = \frac{\sqrt{2}}{3}\left(\sqrt{R_{CS}} - \frac{1}{\sqrt{R_{CS}}}\right) \quad (11)$$

$$Y_{CS} = \left(\frac{1-\sqrt{R_{CS}}}{\sqrt{2}(\sqrt{R_{CS}}+0.5)}\right) \quad (12)$$

Where Rcs is the critical stress ratio and expressed in terms of principal stress ratio as follows:

$$R_{CS} = (\sigma_1/\sigma_3)_{CS(comp)}. \quad (13)$$

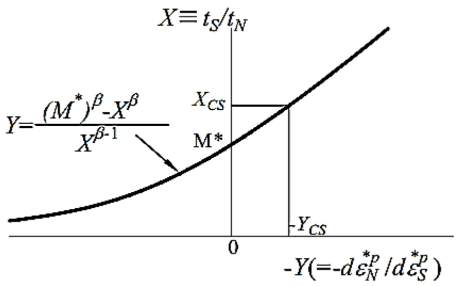


Fig. 4 Stress-dilatancy relation of sub loading t_{ij} model.

4. SAMPLE AND TEST DETAILS

In the present study, the sample is collected from the Mirpur Ceramic Road area, Dhaka (23°49'50.4"N 90°22'30.9"E). Total four tubes are collected from different depths at the location.

To evaluate the physical and index properties of Dhaka clay, grain size analysis, specific gravity test, liquid limit and plastic limit tests were conducted according to the corresponding ASTM standards. Grain size analysis shows that the sample has 30 - 34 % clay, 60 - 66 % silt and 4-8 % sand. The specific gravity ranges from 2.67 to 2.7. The liquid limit ranges from 27.0 % to 29.0 % and plastic limit ranges from 24.0 % to 26.3 %.

The mechanical tests include one-dimensional consolidation test and consolidated drained triaxial compression test.

Table 1 Test and Sample Details

Triaxial compression test					
Test set	Specimen	Diameter (mm)	Height (mm)	B Value	Effective confining pressure (σ_3'), kPa
1	A	38	76	0.97	199
	B	38	76	0.98	397
2	A	38	76	0.95	150
	B	38	76	0.96	390
3	A	38	76	0.95	196
	B	38	76	0.94	398
4	A	38	76	0.96	201

One dimensional consolidation test					
Test set	Specimen	Diameter (mm)	Height (mm)	Initial Degree of Saturation (%)	Final Degree of Saturation (%)
4	B	63.5	20	85	99

5. ANALYSIS AND RESULTS

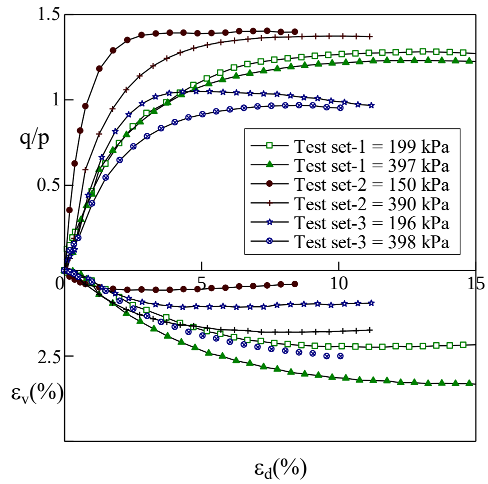


Fig. 5 Observed Stress-strain relation.

Fig. 5 shows the observed stress-strain, volumetric strain – deviatoric strain relationships of the test sets (test set 1-3) under different effective confining pressures ranges from 150 kPa to 398 kPa. Compression loading is applied to observe the stress-strain behavior.

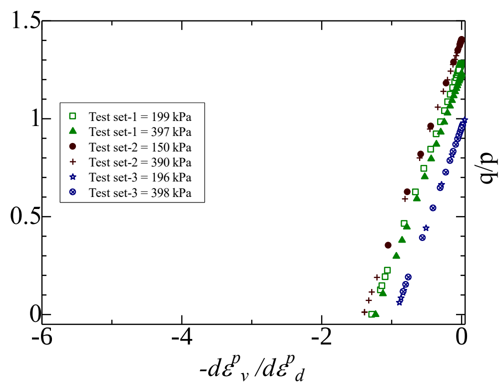


Fig. 6 Observed Stress-dilatancy relation of Cam-clay model.

Figures 6 – 8 show the observed stress – dilatancy relationship of Cam-clay model and sub loading t_{ij} model. In Fig. 6 it is observed that stress – dilatancy relation of original Cam-clay model is linear. From both Fig. 6 and Fig. 7 it is observed that in Cam-clay model there is no unique stress dilatancy relationship established as the shape of yield surface on the p-q plane is dependent on the intermediate principal stress. In case of subloading t_{ij} model as shown in Fig. 8 a unique relation is established amongst the different specimen of the test sets due to its independency from the influence of intermediate principal stress.

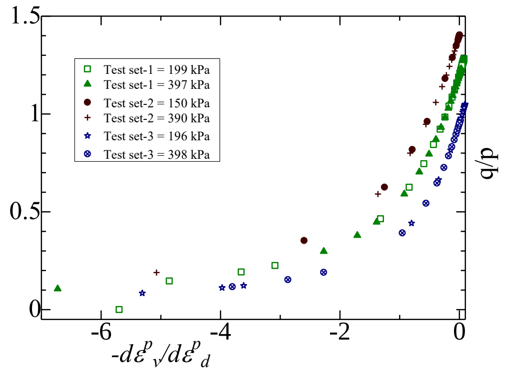


Fig. 7 Observed Stress-dilatancy relation of Modified Cam-clay model.

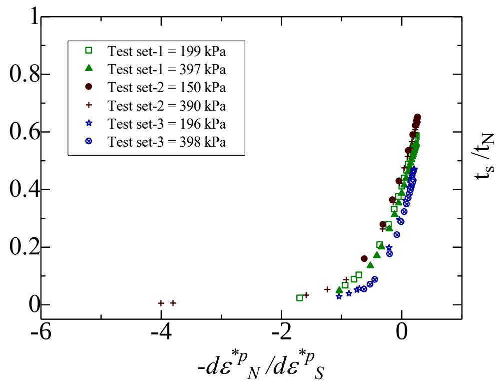


Fig. 8 Observed Stress-dilatancy relation of subloading t_{ij} model.

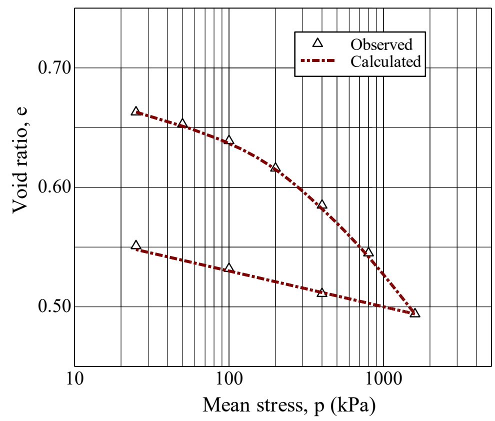


Fig. 9 Observed and calculated e-log curve.

In Fig. 9 observed results of void ratio, e and mean stress, p of the specimen B of the test set 4 is plotted. Calculated results are obtained using the observed parameters from the test following the sub loading t_{ij} model. Hence, density and bonding parameters of the soil is obtained as presented in Table 2.

Table 2 Material parameters of Dhaka clay

Parameter	Notations	Value
Compression index	λ	0.07
Swelling index	κ	0.013
Void ratio at atmospheric pressure (98 kPa)	N	0.68
Critical state stress ratio	R_{CS}	2.8
Poisson's ratio	ν_e	0.2
The shape of the yield surface	β	2.0
Influence of density	a	980
Influence of bonding	b	85

In Fig. 10 material parameters obtained from Table 2 is verified by plotting calculated results of stress ratio *(q/p)* and deviatoric strain (ε_d) using the sub loading t_{ij} model with the observed results of specimen A of the test set 4. Hence with the verified parameters, stress-strain relation under extension is calculated and plotted using the sub loading t_{ij} model. It is observed that for the same specimen there is no unique relationship between stress-strain under two different loading conditions.

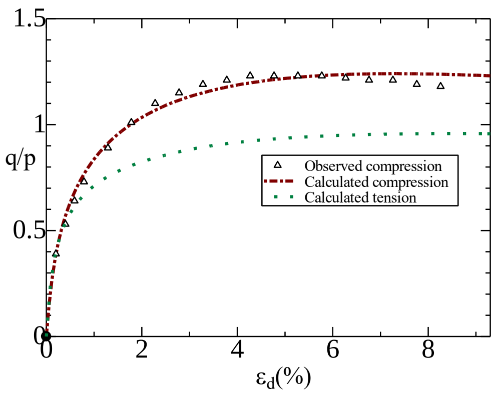
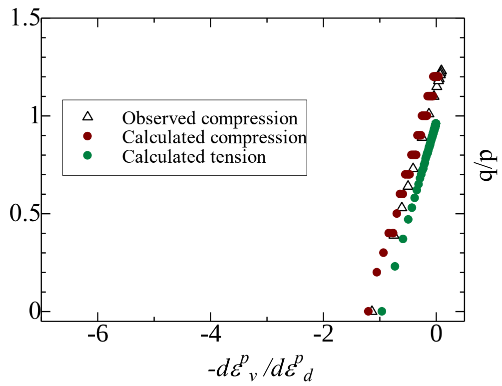
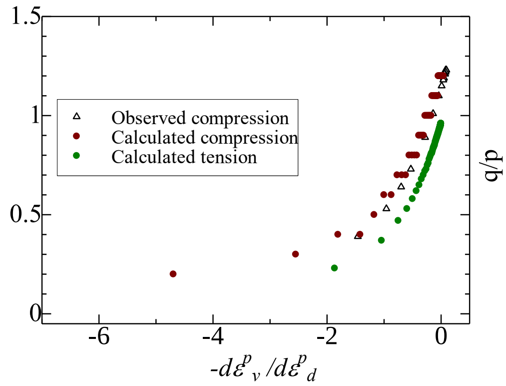


Fig. 10 Observed and calculated Stress-strain relation under triaxial compression and extension.

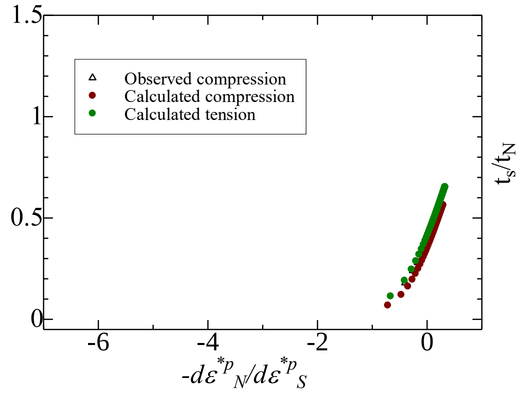
Fig. 11 shows the observed and calculated stress – dilatancy relationship under triaxial compression and extension. In Fig. 11(a) and Fig. 11(b) it is observed that in Cam-clay model there is no unique stress dilatancy relation established. This is because of the influence of intermediate principal stress on the relation between q/p and $d\varepsilon_v$ / $d\varepsilon_d$. Thus the models formulated using these stress invariants cannot provide a unique stress-dilatancy relationship. On the contrary Fig. 11(c) shows that in case of Subloading t_{ij} model a unique stress-dilatancy relation is established both under compression and extension. In t_{ij} concept the, stress-dilatancy relationship is independent of intermediate principal stress.



(a)



(b)



(c)

Fig. 11 Observed and Calculated Stress-dilatancy relation of (a) Cam-clay model (b) Modified Cam-clay model (c) Subloading t_{ij} model under compression and extension.

6. CONCLUSION

From the analyses presented in the paper, it can be concluded that,

1) Stress-dilatancy relations of both original and modified Cam-clay model is not unique for all three sets of the specimen under triaxial compression tests. On the contrary, Subloading t_{ij} model can describe it almost uniquely.
2) For the same specimen, the relation between stress ratio (*q/p*) and deviatoric strain **(ε_d)** of triaxial compression and extension tests are not unique when expressed using these stress invariants.
3) Stress-dilatancy relations derived from compression and extension test of the test set 4 shows that Subloading t_{ij} model provides unique relationship whereas Cam-clay model failed.

7. ACKNOWLEDGMENTS

All the tests were conducted in the Terzaghi Laboratory, Military Institute of Science and Technology (MIST), Dhaka-1216, Bangladesh. Also, the study was financially aided by Military Institute of Science and Technology (MIST) authority.

8. REFERENCES

[1] Chowdhury, E. Q. and Nakai, T. (1998): Consequence of the t_{ij} concept and a new modeling approach. Computers and Geotechnics 23 (3): 131–164.

[2] Hashiguchi, K. (1980): Constitutive equation of elastoplastic materials with the elastoplastic transition. Journal of Applied Mechanics, ASME 102 (2): 266–272. (2009): Elastoplasticity theory. In Lecture notes in applied and computational mechanics. New York: Springer, 42.

[3] Hashiguchi, K. and Ueno, M. (1977): Elastoplastic constitutive laws for granular materials, constitutive equations for soils. Proceedings of Specialty Session 9, 9th International Conference on Soil Mechanics and Foundation Engineering, Tokyo, 73–82.

[4] Hossain, A.T.M.S, and TOLL D.G. 2006: Geomechanical aspects of some tropical clay soils from Dhaka, Bangladesh. Engineering Geology for Tomorrow's Cities, Special publication 22, Book CD-ROM published by Geological Society of London, U.K. ISBN: 978-1-86239-290-8.

[5] Islam, M. S., Siddique, A. and Muqtadir, A. (2004). "Mechanical properties of soft organic Dhaka clay". Journal of Civil Engineering (IEB), 32(2); 143-161

[6] Nakai, T., Shahin, H. M. Kikumoto, m., Kyokawa, H., Zhang, F. and Farias, M. M. (2011): a simple and unified three-dimensional model to describe various characteristics of soils, Soils, and Foundation, 51(6), 1149-1168.

[7] Nakai, T. (2007): Modeling of soil behavior based on t_{ij} concept, Proc. Of 13th Asian Regional Conf. on Soil Mech. And Geotechnical Eng.m Keynote Paper, 2, 69-89.

[8] Nakai, T. and Mihara, Y. (1984): A new mechanical quantity for soils and its application to elastoplastic constitutive models. Soils and Foundations 24 (2): 82–94.

[9] Nakai, T. and Hinokio, T. (2004): A simple elastoplastic model for normally and overconsolidated soils with unified material parameters. Soils and Foundations 44 (2): 3–70.

[10] Nakai, T., Kyokawa, H., Kikumoto, M. and Zhang, F (2009): Elastoplastic modeling of geomaterials considering the influence and density and bonding. Proceedings of Prediction and Simulation Methods for Geohazard Mitigation, Kyoto, 367–373.

[11] Roscoe, K. H. & Burland, J. B. (1968) - On the generalized stress-strain behavior of wet clay. Heyman and F. A. Leckie (eds.), Engineering Plasticity, Cambridge University Press, pp535-609.

[12] Schofield, A. N., and Wroth, C. P. (1968) - Critical StateSoil Mechanics, McGrow- Hill, London.

[13] Teruo Nakai, Constitutive Modeling of Geomaterials, Principles, and Applications, *ISBN 13:978-0-203-86884-3*, CRC Press.

9. AUTHOR'S BIOGRAPHY

Muhammad Abdur Rahman is a Lecturer of Geotechnical Engineering in the Department of Civil Engineering, MIST. He obtained his B.Sc. in Civil Engineering degree from Khulna University of Engineering and Technology, Khulna in the year 2013. At present, he is pursuing his M.Sc. in Civil Engineering at Military Institute of Science and Technology. His research interests include constitutive modeling of soils, slope stability analysis, foundation system, etc.

Dr. Hossain Md. Shahin is currently working as the Professor in the Department of Civil and Environmental Engineering of Islamic University of Technology (IUT), Dhaka, Bangladesh. He has completed his Ph.D. in Geotechnical Engineering from Nagoya Institute of Technology, Nagoya, Japan in March 2004, Master of Engineering in Geotechnical Engineering from the same institution in March 2001 and Bachelor of Science in Civil Engineering from Bangladesh University of Engineering & Technology, Dhaka, Bangladesh in July 1997. Before joining IUT, Dr. Shahin worked as an Associate Professor from April 2008-February 2015, Geotechnical Division, Graduate School of Engineering, Nagoya Institute of Technology, Nagoya, Japan. His research interest includes, Tunneling, Constitutive model of soils, Bearing capacity of shallow and deep foundations, Liquefaction (earthquake) and Ground improvement problems, Ground settlement problems, Slope stability problems, Retaining wall problems, Finite Element Method, Soil-water interaction problems, River bank protection problems, Low cost road pavement systems, Geotechnical site investigation, Land reclamation and embankment problems, Soil – structure interaction.

Dr. Teruo Nakai has got his bachelor and master's degree in civil engineering in 1972 and 1974 respectively from Kyoto University. He has got the Doctor of Engineering degree in 1981 from Kyoto University. He has become a Professor of Civil Engineering of Nagoya Institute of Technology in 1991. He was in Glasgow University as a visiting research fellow from 1988 to 1989. Dr. Nakai has been an active researcher, he has numerous publications in international journals and conferences and his fields of research. His consistent research subjects are (a) laboratory testing of geomaterials and their constitutive modeling in general stress systems, (b) application of the constitutive model to boundary value problems such as tunneling, braced excavation, bearing capacity of various foundations, reinforced soils, and other soil-structure interaction problems, and the corresponding model tests. He was awarded the prize for active young researchers in 1982, the prize for excellent researchers in 1991 and the prize for excellent papers in 2005, from the Japanese Geotechnical Society.

10. AUTHOR'S CONTRIBUTIONS

Muhammad Abdur Rahman: Testing, analysis and interpretation of data and drafting the article. Prof. Hossain Md. Shahin and Prof. Teruo Nakai: Critical reviewing and final approval of the version to be submitted

11. ETHICS

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.