International Journal of GEOMATE, Nov., 2018 Vol.15, Issue 51, pp.252-257
Geotec., Const. Mat. & Env., DOI: https://doi.org/10.21660/2018.51.36469
ISSN: 2186-2982 (Print), 2186-2990 (Online), Japan

STABILITY ANALYSIS OF SLENDER REINFORCED CONCRETE RECTANGULAR COLUMNS UNDER AXIAL COMPRESSION

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ABSTRACT: The objective of this paper is to produce new design and verification charts for reinforced concrete rectangular columns subjected to axial compression in accordance with the Eurocode 2. The reason behind such a research is that stability analysis is a complex problem that includes not only first order effects but also second order ones. The solution depends on a number of parameters and comes from non-polynomial equations which makes the resolution very tedious and needs the use of computer programs. This article considers the case of rectangular beams and provides solutions to it. The presented solutions of the problem are in form of design charts. The design charts presented in this paper are an important tool, quick and easy to use, which will enable engineers to design slender columns or verify their stability without the use of a computer. It will also help them design columns in a economic way as they will see the influence of each parameter on the stability of the element.

Keywords: Slender columns; Buckling; Design charts; Axial loading; Reinforced concrete

1. INTRODUCTION

For buildings, the designer opts generally for a load-bearing system such as a strong column-weak beam. Therefore, due to the importance of the columns for structural stability, the design codes require that the second order effects must be considered.

When these columns are slender they are more subjected to the buckling phenomenon and it is necessary to perform a complete nonlinear analysis which requires the use of numerical methods.

It is proposed in this paper to make charts that allows verifying stability of reinforced concrete rectangular columns under axial compressive loads without computers.

2. PROBLEM PRESENTATION

Buckling is characterized by a sudden sideways deflection of axially loaded structural members when they are slender. This phenomenon has been highlighted by Euler [1] who determined the expression of the critical load $P_c = \frac{\pi^2 . E . I}{l^2}$, at which an ideal element will buckle.

In reinforced concrete, stability analysis consists of proving that there is a deflection of the element that equilibrates the design solicitations while taking into account second-order effects [2], [3], [4].

The BAEL code and the Eurocode 2 admit that for usual structures, regardless of the end conditions, the study of a compressed column under an axial load $\mathrm{N_u}$ can be brought to the case of a double articulated column of length $\mathrm{l_f}$ well known as model column [4], [5], [7], [8].

The advantage of the model column is to rally the buckling problem into the study of one cross-section at the ultimate limit state. It is sufficient to verify, in the middle cross-section, that there is equilibrium between internal and external loads.

The fundamental assumption of the model column is that the deformation is sinusoidal. The maximum deflection f and the curvature $\frac{1}{\mathrm{r}}$ are therefore tied by the following equation:

$$f = \frac{1}{\mathrm{r}} \times \frac{\mathrm{l_f^2}}{\pi^2}$$

The external eccentricity or the eccentricity of the axial load $\mathrm{N_u}$ in the middle cross section is therefore:

$$e_{ext} = e_1 + \frac{1}{r} \times \frac{l_f^2}{\pi^2} \quad (1)$$

$e_1 = e_c + e_a$ where e_c isthe structural eccentricity and e_a is the accidental eccentricity due to execution imperfections.

Furthermore, in the middle cross-section, each state of deformation defined by its curvature $\frac{1}{r}$ and the strain $\mathcal{E}$ in a particular point of the cross section ($\mathcal{E}_{bc}$ for the most compressed fiber for example) leads to the equilibrium equations.

The stresses are functions of the strains, thus they depend on the curvature $\frac{1}{r}$ according to the compatibility relations. So, by setting the state of deformation of the cross section by the couple $(\varepsilon_{bc}\ ,\ \frac{1}{r})$ the internal loads $N_i(\varepsilon_{bc}\ ,\ \frac{1}{r})$ and

$M_i(\varepsilon_{bc}, \frac{1}{r})$ can be calculated, and the internal eccentricity deduced:

$$e_{int} = \frac{M_i}{N_i} \quad (2)$$

In the $(\frac{1}{r}, e)$ plane(Fig.1), the geometrical equation (1) is represented by a straight line and the mechanical equation (2) is represented by network curves parameterized by N_i = constant.
The critical load N_{uc} corresponds to the curve N_i that is tangent to the straight line:

$$e_{ext} = e_1 + \frac{1}{r} \times \frac{l_f^2}{\pi^2}$$

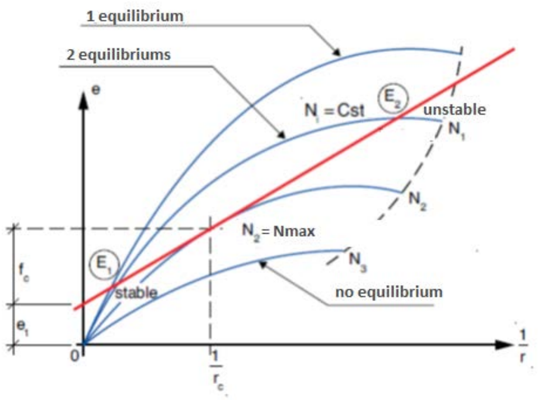


Fig.1:Network curves for several values of N_i

3. NONLINEAR BUCKLING ANALYSIS METHOD

To assess most precisely the buckling resistance of columns, the general method analysis has been used. It is based on a nonlinear analysis including:

- Geometric nonlinearity (second order effects),
- Nonlinear mechanical behavior of materials.

The equilibrium state is proved if there is a point P which coordinates are $(\frac{1}{r}, e_{int})$, localized in the colored area in Fig.2. This point verifies the following inequalities:

and
$$\begin{cases} N_i(\varepsilon, \frac{1}{r}) \geq N_u \\ e_{int}\left(\varepsilon, \frac{1}{r}\right) \geq e_{ext} \end{cases}$$

If the equilibrium is largely insured, the colored zone is very large which makes easier the search for the point P.
In the case where the cross-section is close to the ultimate state of stability, this method becomes tedious. So, it is proposed in this paper to facilitate the task of the columns stability analysis by using charts.

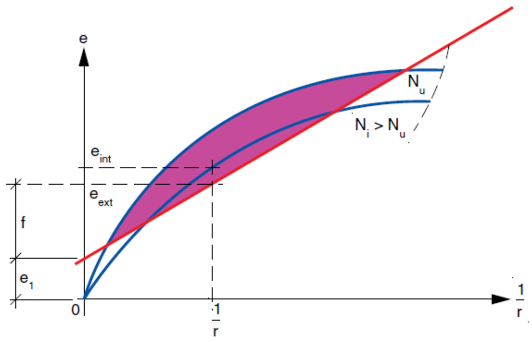


Fig.2: Area of stability

Hence charts will be established for rectangular cross-section columns which are the most common.

4. GEOMETRIC IMPERFECTIONS EFFECTS

According to Eurocode 2, the perfectly centered loading does not exist [6], there will always be an eccentricity due to geometric defects. For isolated columns, the geometrical imperfections may be taken into account as an additional eccentricity e_a given by:

$$e_a = \frac{a_h a_m l_f}{400}$$

With
$$\begin{cases} a_h = \frac{2}{\sqrt{l}} \text{ with } \frac{2}{3} \leq a_h \leq 1 \\ a_m = \sqrt{0.5(1 + \frac{1}{m})} \\ \text{m=1 for an isolated column} \\ \text{l= column's length} \end{cases}$$

Note: For cross sections loaded by the compression force with symmetrical reinforcements, the Eurocode2 recommends to assume the minimum eccentricity $e_0 = \max\left(\frac{h}{30}, 20\text{mm}\right)$ where his the depth of the cross section.

5. CONCRETE STRESS STRAIN RELATION FOR NONLINEAR STRUCTURAL ANALYSIS

The relation between σ_c and ε_c, for short-term uniaxial loading, as represented in Fig.3, is described by the following expression:

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1+(k-2)\eta} \quad \text{for } 0 < |\varepsilon_c| < |\varepsilon_{cu1}|$$

where:

$$\eta = \frac{\varepsilon_c}{\varepsilon_{c1}}$$

$$k = 1.05E_{cm} \times \frac{|\varepsilon_{c1}|}{f_{cm}},$$

E_{cm}: Secant modulus of elasticity of concrete,
ε_{c1}: Compressive strain at the peak stress,
f_{cm}: Mean value of concrete cylinder compressive strength,
ε_{cu1} : Nominal value of ultimate compressive strain in the concrete,
To compute a design value of the ultimate load, the Eurocode2 recommends replacing f_{cm} by f_{cd} and E_{cm} by $E_{cd} = \frac{E_{cm}}{\gamma_{CE}}$ where $\gamma_{CE} = 1.2$ and f_{cd} is the design value of concrete compressive strength. Therefore :

$$k = 1.05\frac{E_{cm}}{1.2} \times \frac{|\varepsilon_{c1}|}{f_{cd}},$$

$$\mathbf{f_{cd}} = \mathbf{f_{ck}}/\gamma_{\mathbf{c}} \qquad \text{with} \qquad \gamma_{\mathbf{c}} = \mathbf{1.5}$$

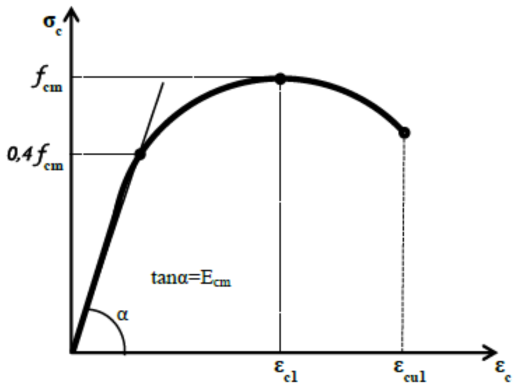


Fig.3:Stress-strain relation for concrete

6. EFFECT OF THE LOADS DURATION

The creep may be taken into account by multiplying all strain values in the concrete stress-strain diagram with a factor$(1 + \varphi_{ef})$, where $\varphi_{ef}$is the effective creep ratio given by:

$$\varphi_{ef} = \varphi(\infty, t_0)\frac{M_{0Eqp}}{M_{0Ed}}$$

$\varphi(\infty, t_0)$is the final value of creep coefficient,
$M_{0Eqp}$is the first order bending moment in quasi-permanent load combination (SLS),
$M_{0Ed}$is the first order bending moment in design load combination (ULS).

7. STRESS-STRAIN RELATIONSHIP FOR STEEL REINFORCEMENT

The design diagram is represented Fig.4. Either of the following assumptions may be made:

- An inclined top branch with a strain limit of ε_{ud} and a maximum stress of Kf_{yd} at ε_{uk}, where $K = (f_t/f_y)_k$.
- A horizontal top branch without the need to check the strain limit.

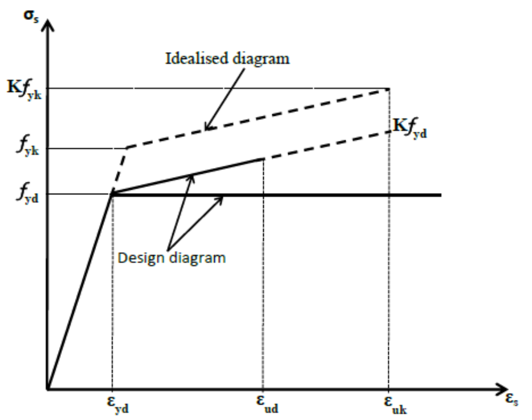


Fig.4:stress-strain diagrams for reinforcing steel

The equation of the stress in reinforcement bars will then be:

$$\sigma_s = \frac{f_{yd}}{\varepsilon_{yd}}\varepsilon_s \ \text{ if } \ \varepsilon_s \le \varepsilon_{yd}$$

$$\sigma_s = f_{yd} + k.f_{yd}(\varepsilon_s - \varepsilon_{yd}) \ \text{ if } \ \varepsilon_s > \varepsilon_{yd}$$

$$\mathbf{f_{yd}} = \mathbf{f_{yk}}/\gamma_{\mathbf{s}} \qquad \text{with} \qquad \gamma_{\mathbf{s}} = \mathbf{1.1}$$

The advised value for $\varepsilon_{ud}$is $\varepsilon_{ud} = 0.9\varepsilon_{uk}$ and $\varepsilon_{uk}$is the elongation of reinforcement at maximum force,
f_{yk} is the characteristic yield strength of reinforcement,
$f_{yd}$is the design yield strength of reinforcement.

8. NUMERICAL BUCKLING ANALYSIS

The buckling critical load problem was solved by numerical simulation with Matlab-Simulink. In this program the case considered was the one of a double articulated column with rectangular cross-section b. h , reinforced by symmetrical steel bars and subjected to centered load N_u.

Taking into account geometric imperfections, the total eccentricity e_t of the load after deformation is :

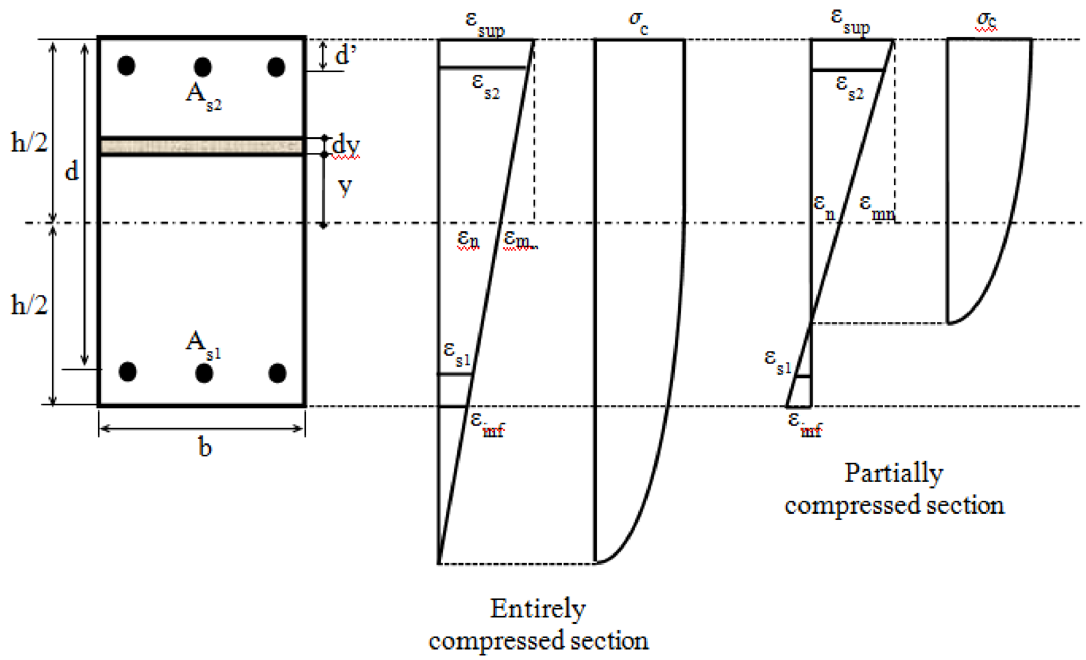


Fig.5: strains and stresses diagrams

$$e_t = \frac{a_h.l_f}{400} + \frac{1}{r} \times \frac{l_f^2}{\pi^2} = e_{ext}$$

8.1 Analysis of the middle cross section

According to the position of the neutral axis, there are two cases (Fig.5):

- The cross-section is entirely compressed,
- The cross-section is partially compressed.

Let's consider a linear deformation represented by the couple of parameters $(\varepsilon_{sup}, \varepsilon_{inf})$. For each values of this couple parameters can be evaluated:

- The resisting solicitations N_i, M_i,
- The corresponding deformation of the section at the middle span.

The curvature of the middle cross section is $\frac{1}{r} = \frac{2\varepsilon_m}{h}$. Therefore :

$$e_{ext} = \frac{a_h.l_f}{400} + \frac{2\varepsilon_m}{h} \times \frac{l_f^2}{\pi^2} \qquad (3)$$

The concrete resistant solicitations can be computed by the formula:

$N_c = \int dN_c$ with $dN_c = \mathrm{b}.\sigma.dy$
$M_c = \int dM_c$ with $dM_c = \mathrm{b}.\sigma.y.dy$

The internal solicitations are, then, given by:

$$N_i = N_c + A_{s1}\sigma_{s1} + A_{s2}\sigma_{s2}$$

$$M_i = M_c + A_{s1}\sigma_{s1}\left(\frac{h}{2} - d'\right) + A_{s2}\sigma_{s2}\left(d - \frac{h}{2}\right)$$

Hence:

$$e_{int} = \frac{M_i}{N_i} \qquad (4)$$

If the equation $e_{int} = e_{ext}$ has a solution then the column will be stable, else it will buckle. To determine the critical normal load, the initialization of the normal effort should be at a suitable N_0 value, then the stability of the column is checked. Next the value of N_u is incremented until the column is no longer stable. The maximum normal load supported without instability is the sought critical normal load, named N_{max}.

8.2 Drawing of design charts

Hypothesis:

- Properties of steel bars:
$E_s = 200$ GPa and $f_{yk} = 500$ MPa.
The stress-strain diagram chosen is the perfect elastoplastic one, i.e. k=0
The section contains bars put symmetrically in the top (named s1) and the bottom (named s2)
A : total area of the steel reinforcements $A_{s1} = A_{s2} = \frac{A}{2}$,

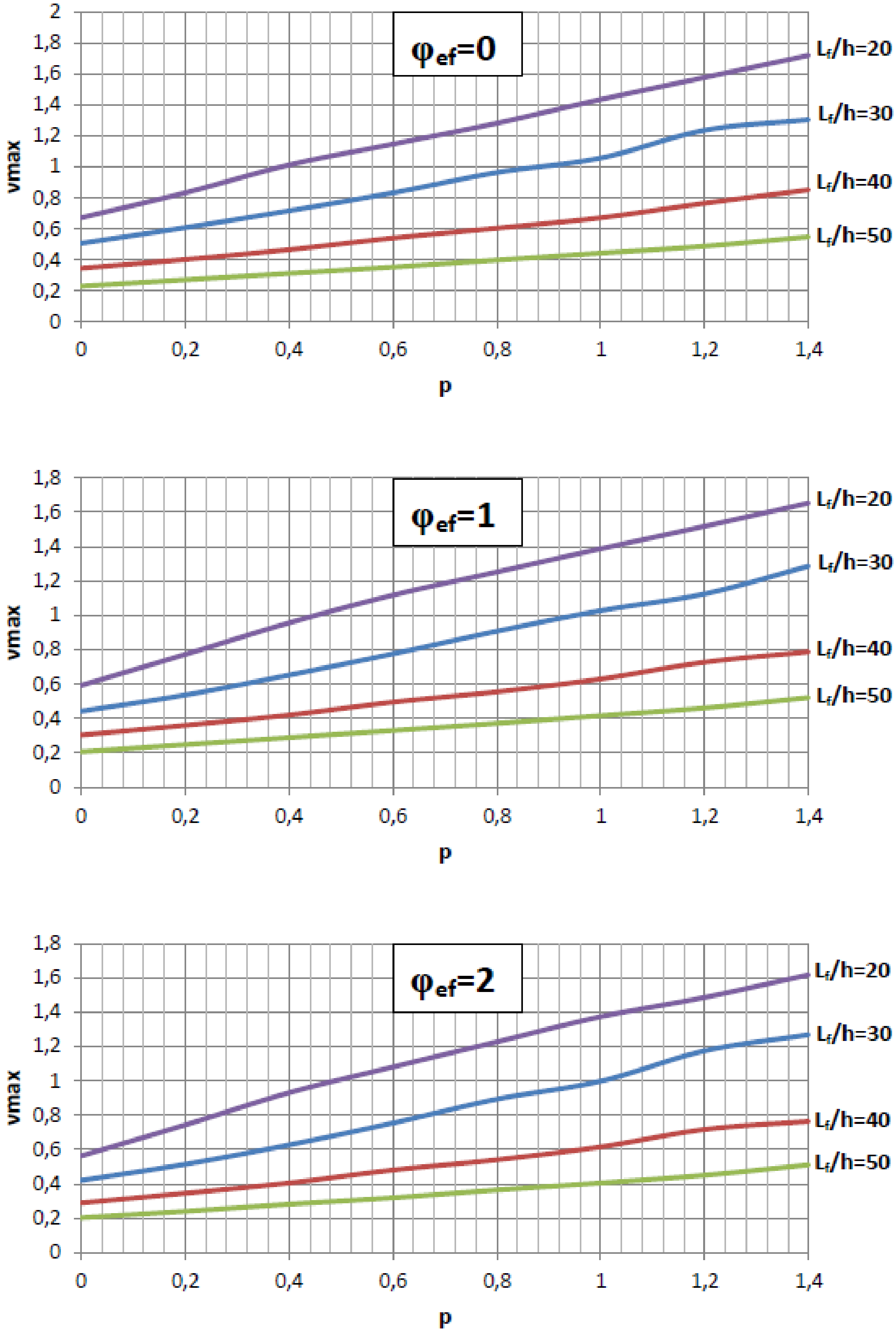
- Properties of concrete:
$f_{ck} = 25\ MPa$

$$E_{cm} = 22000 \times \left(\frac{f_{ck} + 8}{10}\right)^{0.3}$$

- Geometrical properties:

The cross-section of the columns is rectangular.
b: cross section width,

Figure 6: Stability Design Charts



h : cross-section height in the buckling plane, in most cases $h < b$,
l_f: effective length of the column,

- Notations:
$p = \frac{A.f_{yd}}{b.h.f_{cd}}$ mechanical percentage of reinforcement,

$\nu = \frac{N_u}{b.h.f_{cd}}$ relative normal load,

ν_{max} is the critical relative normal load. It is the maximum relative normal load that can be applied on the columns without failure due to instability. ν_{max} will be used to calculate N_{umax} which is the maximum normal load that can be applied on the columns without failure due to instability.

The Fig.6 is a plot of ν_{max} versus p for $\varphi_{ef} = 0, 1$ and 2 and for different values of the slenderness l_f/h.
It can be noted note that the curves ν_{max} versus p are practically linear.

Note: For intermediate values of slenderness (l_f/h) and creep coefficient (φ_{ef}) a linear interpolation will be done.

9. CONCLUSION

In this article have been produced new design charts for the determination of buckling resistance of rectangular columns. They are congruent to the prescriptions of Eurocode 2 and apply to all rectangular columns under centered axial compression.

These charts can be used to determine the buckling resistance of the column and verify its stability without having to use computer programs. They also can be used to identify the column reinforcements.

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