

## DAMAGE QUANTIFICATION OF BEAMS USING FREQUENCY SIGNATURE

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**ABSTRACT:** A rapid method for determining the damage severity sustained by a beam proved to be challenging due to either limited studies conducted on the subject or alternative methods require highly sophisticated and costly equipment to perform. In this research, the unique frequency signature emitted by a beam when excited by an external force was utilized in order to determine the changes in the properties of the beam. Experiments were performed using a roving accelerometer hammer impact test on a beam with a grounded configuration to test the changes occurring as the controlled damage sustained by the beam increases. The acceleration response of the beam obtained from the experiment is then processed using software incorporating Kalman Filter and structural dynamics. Results show that the dominant frequency obtained in both the Fast Fourier Transform and Power Spectral Density of the acceleration response of the beam decreases as the damage incurred by the beam increases. The results also show that regardless of the position of the accelerometer, dominant frequencies tend to converge to a value depending on the damage sustained in the beam. Damping ratio of the beam also decreased as the damage sustained by the beam increased. Inversely, the increase in damage of the beam corresponds to an increase in the dissipation rate of the beam. The study was able to achieve its goal of quantifying damage in a beam through the use of frequency signature by identifying the changes in its dominant frequencies and the damping ratio and dissipation rate.

*Keywords: Damage Quantification, Impact Hammer Test, Frequency Signature, Damping Ratio*

### 1. INTRODUCTION

In a modern community, engineering structures play a vital role and therefore they are usually designed to have long life spans. Human lives and property may be affected or disrupted by the failure or unsatisfactory performance of these structures and is therefore necessary to ensure the safety and reliability of structural members.

Structures are prone to damage due to loading from continuous use or due to stress caused by external loads such as earthquakes. With this, structural damage detection is a vital factor in maintaining the integrity of the structure and reduces the likelihood of structural failures which can have grave consequences [1]. At present, the use of non-destructive examination is being used by engineers in determining the damages of a structure without the need to demolish the structural element of the building being analyzed [2].

The method of detecting damage in a structure should also consider the quickness of identifying the problem in the structure. Vital structures including hospital, bridges and fire stations require rapid identification of damage in the structure in order to prevent secondary damage [3]. A recent study shows that there are five levels of damage detection to be considered in monitoring the condition of a structure. These are mainly the identification of existing

damage, localization of damage, identification of damage type, quantification of damage severity and prediction of the remaining life service of the structure [2].

Several studies have been made in order to determine the damage sustained by beams. A study using the Artificial Neural Networks that makes use of global and local vibration-based analysis data as input were conducted to determine the location and quantified depth of damage in beam like structures were done in previous years [4]. Another recent study on the other hand, makes use of guided waves and Bayesian statistical framework for the characterization of the damage in beams [5]. Other studies conducted made use of modal power flow generated by structural elements subjected to vibrations in determining the crack length and depth [6], [7]. The most recent studies made for damage identification made use of the acoustic emission techniques which make use of state-of-the-art equipment to determine the location and severity of the damage [8], [9].

This research therefore aims to provide a means of detecting the level of damage severity a structural element has sustained. With this method, damage can be quantified and the damages incurred by the structure can be prioritized according to the degree of severity. The need for the prioritizing the damage sustained by the beams would be useful to further

understand the situation and be able to quickly remediate the problem and avoid serious damage. The repairs necessary for the structure can be determined based on the output of the study.

Damage severity is one of the five key levels in damage detection of a structure [2]. Damage severity is a necessary procedure when conducting damage detection in a structural element of a building. However, the lack of a quick and convenient method for determining the severity of a structural damage is currently not present. On the other hand, methods for determining the damage severity of the structure require serious amounts of data to provide accurate results. There is a need to study a method for determining and quantifying the severity of damage in a structural element of a building.

As seen in several stated literatures, the main focus of other studies is on damage identification and localization. Damage quantification alone was not dwelled in most literature. On the other hand, damage quantification that is determined along with the other levels of damage quantification requires state-of-the art machinery or tools to determine the unknown parameter.

Damage severity detection is crucial in determining the status of a structure. It should be determined along with the other levels of damage detection such as existence detection, localization of damage, damage type identification and prediction of remaining service life [2]. Fewer studies have been conducted to determine the severity of the damages as compared to studies which determine the location and existence of damage. This gives all the more reason to conduct the study. Also, most important structures are found in urbanized and densely populated areas, the study is vital to ensure the safety of both the structure and the people occupying it.

## 2. CONCEPTUAL FRAMEWORK

Structural elements gave off a frequency signature when the element experienced an external load. These frequency signatures were captured through the use of a sensor to gather data and store on to a laptop. Each beam gave off a unique frequency signature due to the minute differences caused by the imperfections found in the beams. Given that the structural element was set at a constant properties and imperfections are set to a minimum, the frequency signature produced was also presumed to be at a constant.

In order to test and determine the degree of severity that a structure has sustained, the frequency of the beam was tested at different depth of damage. The cracks or fractures of the beam was represented by a chipped off portion and these damages are varying in depth to record the change in frequency signature. The recorded frequency signatures of the

damaged and undamaged beam were then compiled and were then used in the Kalman filter program. The data is then analyzed to obtain the final output in terms of a frequency of the fast Fourier transform and power spectral density as well as the damping ratio of the element.

## 3. THEORETICAL FRAMEWORK

The study aims to identify the changes to the dynamic properties of the box beam using the different theories and techniques presented in previous researches and published material. The approach to the study involves the use of three theories, i.e., Kalman Filter, direct integration method and the approximation of the damping ratio.

## 4. KALMAN FILTER

The Kalman Filter is an optimal solution to filtering problems which have systems and observation models that are both linear and have Gaussian probability density functions. With the following assumption considered, the equations taken from the optimal Bayesian equations would be reduced to the following.

$$\mathbf{x}_k = f\mathbf{x}_{k-1} + \mathbf{w}_k \quad (1)$$

$$z_k = h\mathbf{x}_k + \mathbf{v}_k \quad (2)$$

And  $K_k$  is the Kalman gain given as

$$K_k = f_{k+1/k} P_{n/n-1} h_k^T (h_k P_{n/n-1} h_k^T + R_k)^{-1}, \quad (3)$$

And  $P_k$  is the Variance-covariance matrix which is given as

$$P_k = P_{n/n-1} - f_{k/k+1} K_k h_k P_{n/n-1} \quad (4)$$

### 4.1 Equation of motion for a discretized beam

Considering a finite element beam model wherein the properties are distributed to each element, a motion of equation can be setup for each element present in the beam. Figure 1 shows a typical beam with an n-th number of elements. Each element is considered for analysis and that the motion of the entire beam can be determined with respect to the movement of each element. Similar to a single oscillating object, the elements of the beam have properties such as mass, damping coefficient, stiffness and external force acting on the element which are independent from one element to the other and values such as acceleration, velocity and displacement which are the basis for the motion of the beam.

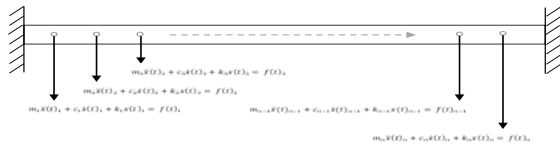


Fig. 1 Typical Beam with n number of elements

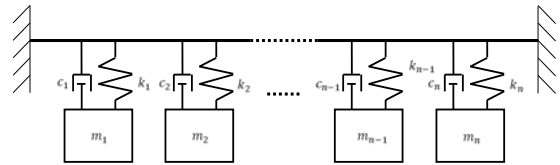


Fig. 2 Free Body Diagram of a beam with an n-th number of elements

#### 4.2 State of the system equation and Observation equation

As one of the methods for direct integration of second order derivatives, Newmark's Constant Average Acceleration method make use of the fact that the acceleration of a given time interval is equal to a constant value. The algorithm numerically updates the response acceleration, velocity and displacement of an object from  $t_i$  to  $t_{i+1}$ . Following the previous stated assumption and isolating  $\ddot{x}_i$ , the equation of motion becomes

$$\ddot{x}_i = \frac{f_i - c \left( \dot{x}_{i-1} + \frac{\Delta t}{2} \ddot{x}_{i-1} \right) - k \left( x_{i-1} + \Delta t \dot{x}_{i-1} + \frac{\Delta t^2}{4} \ddot{x}_{i-1} \right)}{\left( m + \frac{\Delta t}{2} c + \frac{\Delta t^2}{4} k \right)} \quad (5)$$

Using the truncated Taylor's series expansion, the following equation that would complement the previous equation and complete the algorithm are

$$\dot{x}_i = \dot{x} + \frac{\Delta t}{2} (\ddot{x}_{i-1} + \ddot{x}_i) \quad (6)$$

$$x_i = x_{i-1} + \Delta t \dot{x}_{i-1} + \frac{\Delta t^2}{4} (\ddot{x}_{i-1} + \ddot{x}_i) \quad (7)$$

Incorporating previous equations and the new equation becomes the state equation of the Kalman filter updates the acceleration based on the numerical integration of the equation of motion and would be used to compared the measured acceleration obtain from the experimental portion of the study.

$$x_k = (\dot{x}_i) = \begin{bmatrix} A_1 & 0 & \dots & 0 & 0 \\ 0 & A_2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & A_{n-1} & 0 \\ 0 & 0 & \dots & 0 & A_n \end{bmatrix} \begin{bmatrix} \ddot{x}_{t-1} \\ \ddot{x}_{t-1.2} \\ \vdots \\ \ddot{x}_{t-1-n} \end{bmatrix} + \begin{bmatrix} B_1 & 0 & \dots & 0 & 0 \\ 0 & B_2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & B_{n-1} & 0 \\ 0 & 0 & \dots & 0 & B_n \end{bmatrix} \begin{bmatrix} \dot{x}_{t-1} \\ \dot{x}_{t-1.2} \\ \vdots \\ \dot{x}_{t-1-n} \end{bmatrix} + \begin{bmatrix} C_1 & 0 & \dots & 0 & 0 \\ 0 & C_2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & C_{n-1} & 0 \\ 0 & 0 & \dots & 0 & C_n \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-1.2} \\ \vdots \\ x_{t-1-n} \end{bmatrix} + \begin{bmatrix} D_1 & 0 & \dots & 0 & 0 \\ 0 & D_2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & D_{n-1} & 0 \\ 0 & 0 & \dots & 0 & D_n \end{bmatrix} \begin{bmatrix} f_n(t)_1 \\ f_n(t)_2 \\ \vdots \\ f_n(t)_{n-1} \\ f_n(t)_n \end{bmatrix} \quad (8)$$

Wherein

$$A_n = \frac{\left( -\frac{\Delta t}{2} c - \frac{\Delta t^2}{4} k_n \right)}{\left( m_n + \frac{\Delta t}{2} c + \frac{\Delta t^2}{4} k_n \right)}; \quad (9)$$

$$B_n = \frac{(-c - k_n \Delta t)}{\left( m_n + \frac{\Delta t}{2} c + \frac{\Delta t^2}{4} k_n \right)};$$

$$C_n = \frac{(-k_n)}{\left( m_n + \frac{\Delta t}{2} c + \frac{\Delta t^2}{4} k_n \right)};$$

$$D_n = 1 / M_n$$

Considering the latter of the two equations, the observation equation is based on the independent variable that has been measured in the experimental portion of the study. Given that the input actuators or the accelerometers each measures only the acceleration response of the material being tested, the observation equation would be a vector matrix and an identity matrix defined as

$$x_k = (\ddot{x}_{meas}) = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{x}_{meas_1} \\ \ddot{x}_{meas_2} \\ \vdots \\ \ddot{x}_{meas_{n-1}} \\ \ddot{x}_{meas_n} \end{bmatrix} \quad (10)$$

#### 4.3 Measuring Damping Ratio

In evaluating the damping ratio of a free vibrating structure, the logarithmic decrement method can be used. The logarithmic decrement method is used to measure damping in time domain. In this method, the free vibration displacement amplitude history of a system to an impulse is measured and recorded. Logarithmic decrement is

the natural logarithmic value of the ratio of two adjacent peak values of displacement in free decay vibration. Given that the damping ratio is small and with the exponential in the ratio  $\frac{x_1}{x_2}$  can be expanded in series retaining only the first two terms, since  $\omega_d \cong \omega_n$  this leads to

$$\varepsilon \cong \frac{x_1 - x_2}{2\pi x_2} \quad (11)$$

For cases where the difference between two amplitude peaks are very small, it is more convenient to choose two non-consecutive peaks and the equation would be

$$\varepsilon \cong \frac{x_i - x_{i+m}}{2\pi m x_{i+m}} \quad (12)$$

## 5. RESEARCH METHODOLOGY

### 5.1 Pre-experimentation phase

As the initial step to the study, the research was required to design the appropriate beam in order to produce result which will represent beams in its true scale and purpose in a structure.

The following were considered in the design of the beam element which would represent the structural element of a building and would provide the necessary data needed for the study.

The beam element was made of a rectangular aluminum box beam, specifically the material was a 6061 T6 3x3" Aluminum Square Tube.

As for the length of the beams, the original piece used for the test were 3 pieces of standard length which amounts to a total length of 21 ft. per piece or approximately equal to 6.4m per piece. The each material was cut into 3 equal part wherein the 2m of the beam was considered as the effective length of the beam. The remaining portions of the beam was buried inside the wooden box frame to dissipate vibration that would reflect back. Figure 3 illustrates the dimensions of the beam from the front view.

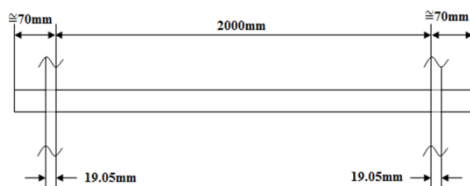


Fig. 3 Box beam dimensions

The beams were classified in to two cases, wherein there will be one undamaged and five damaged beam. The damaged beam were the representation of the deterioration or cracks developed by the beam with each case having a different degree of damage that was predetermined in order to consider only the quantification of damage. The damage ranges from 1 to 4 percent of the total length of the beam. Its design also had a constant depth of damage equal to 50 percent of the total height of the beam. Figure 4 shows the dimensions of the damage.

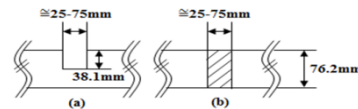


Fig. 4 Dimensions of damage (a) Elevation (b) top All damages on the beam trial were located on the first quarter mark which was equal to 0.5m from origin.

The beams were supported on box filled with sand in order to make use of the boundary effects when vibrations passed through the beam.

Once the pieces were sawed, it is then nailed and wood glue is placed on its corners to avoid breaking from the weight of both the beam and the sand. A hole with the exact size of the beam was then punctured and filled with sand up to the bottom portion of the hole.

The input actuator with negligible mass that was used for recording and measuring the response of the beam was a triaxial accelerometer as seen in Fig. 5.



Fig. 5 Triaxial Accelerometer

The hammer that was for all trial in the experiments was an ordinary rubberized hammer typically used for basic construction and carpentry

### 5.2 Experimentation proper

With all beams cut according to specification, the beam ends were covered using packaging tape to reduce the amount of unnecessary matter such as sand from entering the beam.

Proceeding to the next step of the experiment, five accelerometers were used for the undamaged

beam and six accelerometers for all damaged beams.

Given that the setup is ready and the triaxial accelerometers' capabilities were tested, the vibration test could now proceed. The vibration test used for the experiments was the impact hammer test. The design of the experiment incorporated the use of a rubberized hammer that was dropped at the center of the beam which provided the maximum movement possible for the beam to produce.

In Summary, Fig. 6 shows the complete experimental setup

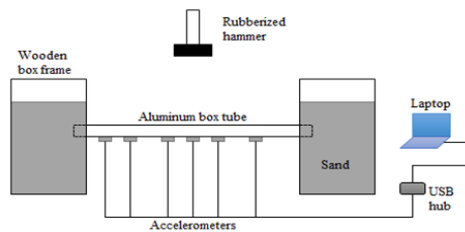


Fig. 6 Complete experimental setup

### 5.3 Post-experimentation phase

Using the obtained acceleration data from the experiments, the Kalman filter processed this data for the observed stated and has been incorporated by the use of the observation equation as defined in previous sections of the study which will be expounded depending on the case being studied.

## 6. DATA PRESENTATION AND ANALYSIS

Sensors were based on the predetermined location set to each sensor; FL2, FL1, L, M, R & FR corresponds to Far Left 2, Far Left 1, Left, Middle, Right and Far Right respectively. From the acceleration data, the time domain of the acceleration is transformed to a frequency domain using the fast Fourier transform and the Welch power spectral density analysis.

With regards to the Fast Fourier Frequency, the estimate acceleration response of the beam which was taken from the Kalman filter code was subjected to the process of determining the magnitude of the Fast Fourier Frequency. Based from Fast Fourier Frequency of all test trials, all frequency exhibit a dominant frequency in the higher frequencies levels. In each case, dominant frequencies of all trial fall on a specific frequency level. The sample data was then subjected to an increase in the sampling rate to reduce the amount of spectral leakage when a signal is being filtered or processed. To further eliminate the possible spectral leakage in the Fourier analysis,

a process called Hanning windowing.

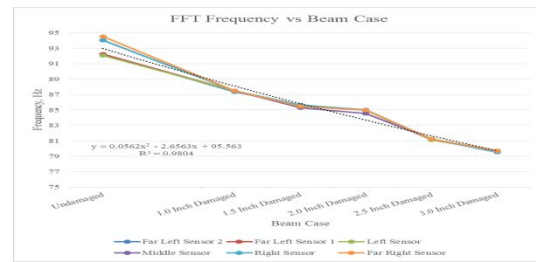


Fig. 7 Resampled FFT Frequency vs beam case

The trend indicates that as the damage sustained by the beam or any structural element increase, the frequency produced by the said beam or element decreases accordingly. Also, it could be pointed out that the drop in frequency is consistent regardless of the position of the accelerometer whether it is position nearest to the sensor or it is place nearest to the point of impact or even positioned at the father point from the damage. Table 1 established the percent difference between the frequencies recorded by each sensors and the mean frequency of each beam case.

Table 1 Percent differences of FFT frequencies in beam cases

Sensor/ Beam Case	0"	1"	1.5"	2"	2.5"	3"
FL2	0.00 %	0.18 %	0.18 %	0.00 %	0.00 %	0.20 %
FL1	0.17 %	0.18 %	0.00 %	0.00 %	0.19 %	0.00 %
L	1.98 %	0.18 %	0.37 %	0.55 %	0.00 %	0.20 %
M	1.98 %	0.18 %	0.00 %	0.05 %	0.00 %	0.20 %
R	2.48 %	0.18 %	0.18 %	0.00 %	0.00 %	0.00 %
FR	1.05 %	0.15 %	0.12 %	0.08 %	0.03 %	0.10 %

Using the same acceleration estimate obtained from the impact hammer test which has been refined by the Kalman filter, the PSD or the Power Spectral Density of the beams response was established. Similar to the FFT, a decline in the peak frequency was also determined in the PSD and position of the sensors were also negligible.

Considering the damping ratio as a parameter to quantify and determine the damage severity of a structural element, the results of the analysis show that the damping ratio of the beam decreases as the as the damaged found in the beam increases is an indication that the damage inflicted on the beam was detected using the damping ratio as the parameter.

The severity of the damage was identified due to the fact that a continuous decrease in the damping ratio was observed in all sensors. Although sensors further away from the actual damage are less reliable due to inconsistent decrease in damping ratio was observed, sensors that are nearer the damage could be noted to be more reliable and consistent.

Dissipation time however, increases as the damaged sustained by the beam decreases. The gradual increase in dissipation time go hand in hand with the gradual decrease in damping ratio due to the loss in the ability to remove the external force from the beam. Results show that the sensor furthest from the actual damage tend to have more sporadic data and is therefore less reliable as compared to the result shown by the sensors near the cut.

In order to justify the completion of the study’s objectives a summary of all results is tabulated to show that the damaged was quantified and that several methods can be used to determine the damage severity levels of a beam or any other structural element being analyzed. Table 2 collected all the information from all previous results of the study.

Table 2 Summary of Results

Beam Case	Damage w/L	FFT, Hz	PSD, Hz	Damping Ratio	Dissipation Rate
0"	0.00%	93.19	94.585	0.2884	0.1451
1"	1.27%	87.47 (6.14%)	87.318 (7.68%)	0.1915 (33.60%)	0.1702 (17.30%)
1.5"	1.91%	85.52 (8.23%)	85.144 (9.98%)	0.1911 (33.74%)	0.1778 (22.54%)
2"	2.54%	84.93 (8.86%)	84.778 (10.37%)	0.1531 (46.91%)	0.1982 (36.60%)
2.5"	3.18%	81.22 (12.84%)	80.931 (14.44%)	0.1110 (61.51%)	0.2442 (68.30%)
3"	3.81%	79.61 (14.57%)	79.237 (16.23%)	0.0981 (65.98%)	0.2769 (90.83%)
R-Value		0.9804	0.9761	0.9023	0.9629

## 7. CONCLUSION

The study was able to establish a link between the current condition of the beam to the unique frequency signature it emits using experimental modal analysis such as the impact hammer test and the use of filtering techniques such as the Kalman filter. Moreover, the damage severity of the structural element, which in the study is a beam element, was determined using the parameters such as the dominant frequency of the frequency signature, damping ratio and duration of damping. Based from all the data gathered and analyzed from the study, the following observations and conclusion were identified:

For determining the severity of the damage sustained by the beam, sensor placement may be negligible as seen in several analyses wherein position of the sensor did not affect the results of the study. This could prove useful in monitoring the condition of structural elements which are not easily accessible for placement of measuring devices. This however may not be the case for all parameters such as the damping ratio wherein position was key to having improved results.

Properties and parameter which include the damping ratio, dissipation rate and dominant frequencies found in both the fast Fourier transform and the power spectral density remain constant in a structural elements until its condition deteriorates through damage. Besides the dissipation rate, these parameters decrease depending on the damage sustained by the said element. Larger decrease indicate a severer damage whereas smaller changes can be caused by smaller damages.as for the dissipation rate, damage is directly proportional to the dissipation of energy in a structural element. By evaluating any of these parameters, the damage was quantified to determine its overall condition.

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