

## A MESOSCALE INVESTIGATION ON THE SIZE EFFECT OF THE FRACTURE CHARACTERISTICS IN CONCRETE

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**ABSTRACT:** The size effect or scaling phenomenon is a fundamental problem when dealing with fracture of geomaterials. Concrete is considered as one of world's most widely used construction materials. The design in civil engineering of structural concrete should take into account the size effect when extrapolating the experimental tests at laboratory scale to structures with large dimensions. The size effect is generally investigated on the nominal strength which is an important parameter for design in civil engineering. However, the fracture characteristics are also considered in structural concrete codes. The limitation of cracking (crack width, crack length, etc.) are important aspects to consider in order to guarantee a safe conception. This paper deals with a numerical analysis of size effect on the nominal strength and on the cracking characteristics in concrete beams. Numerical simulations of geometrically similar notched beams of various sizes are performed using a mesoscopic approach with an isotropic damage model. In order to properly describe the cracking process, a post processing method based on the fracture energy regularization is adopted. The size effect on crack-opening and crack length evolution is investigated by comparing the numerical results with experimental data obtained by digital image correlation technique. The numerical results prove the ability of the mesoscopic approach to describe the size effect on the global behavior and the local behavior of concrete. The numerical investigation reveals the existing of a size effect on the cracking process evolution.

*Keywords: Meso-scale, Size effect, Concrete, Cracking process.*

### 1. INTRODUCTION

The behaviors of quasi-brittle geomaterials are characterized by a complex failure. For concrete, due to its inhomogeneous microstructure, the cracking process is due to the growth and coalescence of microcracks leading to the formation of a macroscopic crack and structural failure.

According to the plasticity theory (or elasticity theory), the failure load or the nominal strength of a concrete structure is independent of the structure size. However, experimental results show that the specimen size has an effect on the nominal strength and the fracture properties (the fracture toughness, the fracture energy, etc.). The size effect is mostly investigated on the nominal strength. For small sizes, the size effect disappears and plasticity theory (Limit Analysis) could be applied. For large structures, the failure is brittle and therefore the behavior could be approximated by Linear Elastic Fracture Mechanic (LEFM). The experimental tests are mostly performed on small and medium sizes; however practical size-range of real structures is different (Dams, bridge, etc.). The design in civil engineering should take into account the size effect (scaling phenomenon) when extrapolating the experimental tests at laboratory scale to structures with large dimensions in order

to assess the failure loads and the bearing capacities [1] [2].

For quasi-brittle behaviors, different approaches have been proposed to explain the size effect either statistically [2] or deterministically [3]. One of the most used criterion is the Bazant's SEL based on the Griffith energetic criteria. For concrete, it is now widely accepted that the size effect is mostly due to the existence of a non-linear zone of micro-cracking around the crack tip called the Fracture Process Zone (FPZ) (Fig. 1). Concrete is known to have a relatively large FPZ length compared to other quasi-brittle geomaterials. Due to the existence of the FPZ, during the fracture process, the material behavior becomes soft, and neither the LEFM nor the plasticity theory could represent correctly the stress field within this zone. A size effect law is therefore needed to bridge the gap between plasticity and LEFM. Other geomaterials (rock, ice, etc.) exhibits tension-softening and are characterized by a quasi-brittleness leading to a fracture process zone different from plastic zone in metallic engineering materials. Therefore, the tension-softening (or the fracture energy characteristic) should be taken into account to describe the failure of such materials.

As it has been outlined above, the size effect is generally investigated on the nominal strength which is an important parameter for design in civil

engineering. However, the fracture characteristics are also considered in structural concrete codes. Concrete structures are usually designed to allow cracking under service loading. For serviceability limit state design, obtain an adequate strength does not guarantee a safe behavior and other requirements are needed. The limitation of crack opening (crack width) is an important aspect to consider in order to ensure a safe conception. Therefore, the question whether or not a size effect on crack opening exists arises !

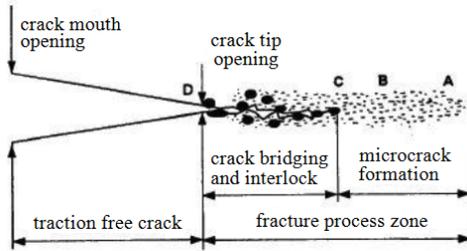


Fig. 1 The Fracture Process Zone

The present paper discusses the existing of size effect during the cracking process for concrete member. A numerical investigation at a mesoscale level is performed. The objective is to study the presence of a size effect on the cracking process (crack opening, crack length, etc.). The cracking process is assessed using a post processing method developed by Matallah et al [4]. Based on an energetic regularization method; it permits the study of the fracture process using damage and/or plastic models. Numerical modeling is conducted at a mesoscale level using a mesoscopic approach which permits an explicit representation of concrete constituents. We consider concrete as a biphasic material where the cement paste and the aggregates are described by their own mechanical characteristics.

Firstly, the energetic regularization method is explained. Furthermore, the fracture process of three geometrically similar beam specimens is studied numerically at a mesoscopic level. Finally the existing of a size effect on the evolution of the cracking process (crack opening, crack length, etc.) is deeply investigated.

## 2. THEORETICAL ASPECTS OF THE POST-PROCESSING METHOD

### 2.1 Energetic Regularization

Within a non-linear behavior formulation, the area under the stress-strain curve represents the dissipated energy density per unit volume

$$W = \int_0^\infty \sigma_{ij} d\varepsilon_{ij} \quad (1)$$

and the product of this quantity by the size of the localization area " $h$ " gives the energy of cracking  $G_f$  needed to create a cracking unit surface (energy dissipated per unit area)

$$G_f = \int_0^\infty \sigma du = hW \quad (2)$$

To represent the cracking process under mode I, Bazant and Oh [5] consider that the crack is spread over a band of width  $h$  which allows to calculate the displacement of the jump as the product of the fracture strain by the band width

$$G_f = \int_0^\infty \sigma du = h \int_0^\infty \sigma d\varepsilon^f \quad (3)$$

Using the energetic approach, the cracking process in concrete is thus governed by the fracture energy. The value of the fracture energy controls the dissipation process during the degradation of the material via a parameter of the nonlinear behavior law that controls the tension-softening. In the following analysis, a simple isotropic damage model is used [6][7]. The stress-strain relationships is written as

$$\sigma_{ij} = (1-d)\tilde{\sigma}_{ij} = (1-d)C_{ijkl}^0 \varepsilon_{kl} \quad (4)$$

The damage evolution is described by an exponential evolution function of the equivalent strain. For the equivalent strain, we use the Mazars's definition.

$$d = 1 - \frac{\varepsilon_{d0}}{\tilde{\varepsilon}_e} \exp(B(\varepsilon_{d0} - \tilde{\varepsilon}_e)) \quad (5)$$

$$\tilde{\varepsilon}_e = \sqrt{\langle \varepsilon_e^1 \rangle^2 + \langle \varepsilon_e^2 \rangle^2 + \langle \varepsilon_e^3 \rangle^2} \quad (6)$$

For a one-dimensional case with a mode I of crack propagation, we obtain [8]

$$\begin{aligned} G_f &= h \int_0^\infty E(1-d)\varepsilon d\varepsilon \\ &= h \int_0^\infty E \left( \frac{\varepsilon_{d0}}{\varepsilon} \exp[B(\varepsilon_{d0} - \varepsilon)] \right) \varepsilon d\varepsilon \\ &= h \frac{E\varepsilon_{d0}^2}{2} + h \frac{E\varepsilon_{d0}}{B} \end{aligned} \quad (7)$$

With  $h$  the size of the finite element and  $B$  the parameter controlling the softening. Under a uniaxial stress state, using the formulation given in Eq. (7) ensures that the model dissipates the same fracture energy injected in the computation.

## 2.2 Crack Opening Evaluation

A practical method to estimate crack opening from a finite element computation based on damage and/or plastic model has been developed by Matallah & al in [4]. This method is proposed in the Finite Element code Cast3M (OUVFISS Procedure). The method is based on the fracture energy regularization. The fundamental basic equations are recalled here.

We consider a second order strain tensor  $\varepsilon^{uco}$  (the Unitary Crack Opening strain which is equivalent to the fracture strain  $\varepsilon^f$ ). The fracture energy per unit width is calculated as the area under the complete stress-strain diagram. The total strain is decomposed into two parts: an elastic part  $\varepsilon^e$  and a cracking part represented by the Unitary Crack Opening strain variable  $\varepsilon^{uco}$ . So, the effective stress is given by

$$\tilde{\sigma}_{ij} = C_{ijkl} \varepsilon_{kl} = C_{ijkl} \varepsilon_{kl}^e + C_{ijkl} \varepsilon_{kl}^{uco} = \sigma_{ij} + \sigma_{ij}^{in} \quad (8)$$

From a finite element computation, we get a nominal (total) stress  $\sigma$ , the effective stress is computed using the total strain. The inelastic stress tensor is therefore given by:

$$\sigma_{ij}^{in} = \tilde{\sigma}_{ij} - \sigma_{ij} \quad (9)$$

The tensor of the crack openings strain is given by:

$$\varepsilon_{kl}^{uco} = C_{ijkl}^{-1} \sigma_{kl}^{in} \quad (10)$$

Eq. (10) gives the Unitary Crack Opening strain tensor. The normal crack-opening displacement value is given by:

$$\delta_n = n_i \delta_{ij} n_j = n_i h \varepsilon_{ij}^{uco} n_j \quad (11)$$

So for each finite element, the crack opening displacement is given by

$$\delta_n = \int_{element} \varepsilon_n^{uco} dn = \int_{element} n_i \varepsilon_{ij}^{uco} n_j dn \quad (12)$$

Where  $n$  is the normal crack direction.

## 3. NUMERICAL INVESTIGATION

### 3.1 Experimental Vs Numerical results

The experimental program has been carried out by Yassir Alam [9]. Three geometrically similar beam specimens were tested with a span to depth ratio  $l/d = 3:1$  for all the specimens. The cross sectional

heights ( $d$ ) of the specimens were 100, 200 and 400 mm respectively. The thickness ( $b$ ) for all the sizes is 100mm. The beams were notched at midspan with a notch “a” constant  $a/d = 0.2$  for all the beams. The notch is about 3 mm of thickness

The beams are classified into three classes depending upon their dimensions small (D1), medium (D2) and large sizes respectively (D3). Two types of concrete mixes (M1 and M2) were used, in which size of aggregates is varied. M1 with  $d_{max} = 20\text{mm}$  and M2 with  $d_{max} = 12\text{mm}$ .

The global behavior (Load-CMOD) and the local one (crack opening) have been assessed experimentally. The objective of the present study is to reproduce the experimental results at a mesoscale level and to investigate the size-dependency of the fracture characteristics (crack opening and crack length).

### 3.2 Meso-scale description of concrete

Numerical modelling is conducted at a meso scale which permits an explicit representation of concrete constituents. At mesoscale level, concrete can be represented by lattice models where a system of discrete particles is used to represent the mesostructure or by using the finite element method. When using the FE approach, we consider concrete as a multiphase material where the cement paste and aggregates are described by their own mechanical characteristics. Beyond the matrix phase, only the large aggregates are represented explicitly. The small aggregates and other components are assumed to be mixed up with the mortar phase establishing the matrix.

Numerical modeling is performed using the finite element code Cast3M [10]. The mesh used for simulation is presented in Fig. 2. The mesoscopic mesh is adopted in the central part of the beam where the crack appears. A macroscopic mesh is considered for the other parts in order to reduce the computation time. The size effect on the nominal strength is firstly investigated. Three random draws are used to generate the mesoscopic structures.

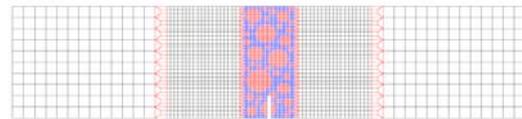


Fig. 2 Meso-Macro Mesh of the beam (ExempleD1)

### 3.3 Size Effect on the global behavior

The comparison between the experimental results and the numerical ones are shown in Fig. 3 for M1

and in Fig. 4 for M2. The curves Load -Crack Mouth Opening Displacements (P-CMOD) are compared. Experimentally, for the mix M2, only two geometrically similar beams specimens have been tested.

The comparison shows that the global behavior is well reproduced. The size effect on the peak load is globally well represented at mesoscopic level. The agreement between numerical and experimental results for both the peak and post-peak part is very good since the same set of model parameters were used for obtaining all the load-CMOD curves. For the mix M1, the shape of the post-peak is in good agreement with the experimental results. However, for mix M2, even if the numerical peak-load results are in good agreement with the experimental one, using the same parameter set for the mix M2 leads to a post-peak with a greater dissipated energy compared with the experimental one. The experimental post-peak is more brittle. Regarding the random distribution of the aggregates, the global behavior is globally the same as the dissipation is governed by the fracture energy. The position of the aggregates has an influence on the crack path.

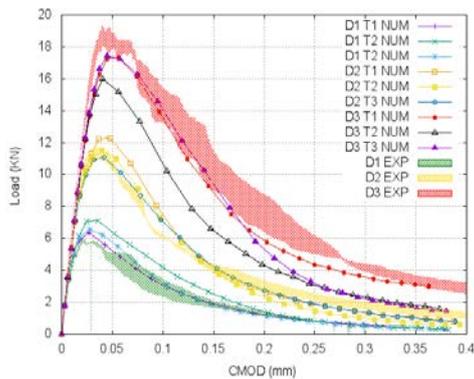


Fig. 3 Comparison between numerical and experimental P-CMOD curves for concrete mix M1

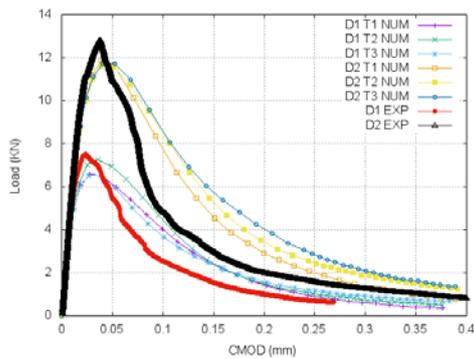


Fig. 4 Comparison between numerical and Experimental P-CMOD curves for concrete mix M2

#### 4. SIZE EFFECT ON THE LOCAL BEHAVIOR –CRACK LENGTH-

The post processing method developed by Matallah et al [4] is used to evaluate the crack path for each beam both for concrete M1 and M2. Fig. 5 gives an example of the crack path.

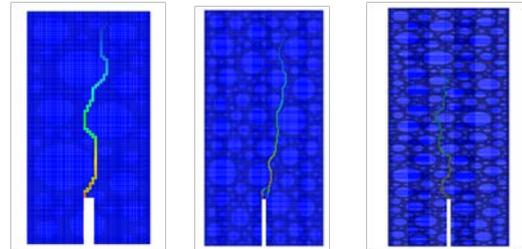


Fig. 5 Crack path in beams D1, D2 and D3 (concrete M1)

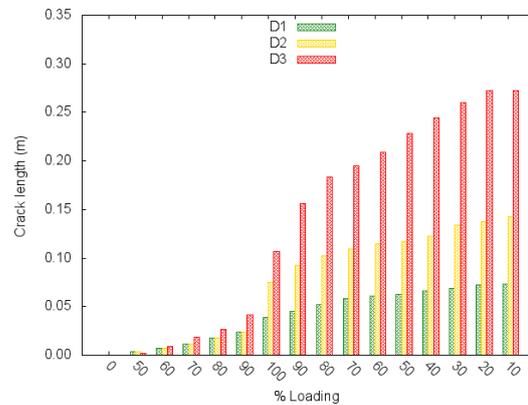


Fig. 6 Crack-length evolution during loading (M1)

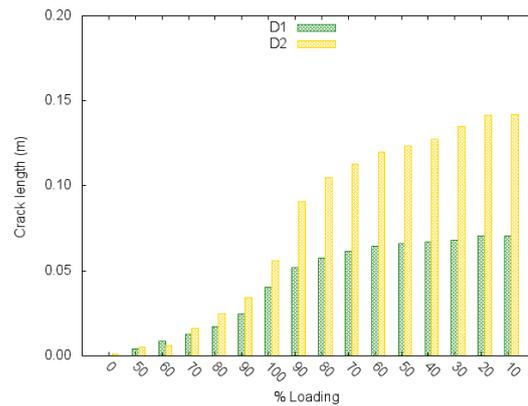


Fig. 7 Crack-length evolution during loading (M2)

Figs. 6 and 7 show the crack length evolution for concrete M1 and M2 plotted at different loading stage (From 0% pre-peak to 10% post-peak). For both types of concrete mixes, the crack appears in the pre-peak regime.

At the peak load, the crack length for the concrete M1 is higher than those for concrete type M2. In order to assess the size effect on the crack length, the relative-Crack length evolution is plotted in Figs. 8 and 9. The ratio ( $L_{crack}/D$ ) where  $L_{crack}$  is the length of crack from the notch tip and  $D$  is the specimen ligament (distance from the notch tip to the top). For concrete type M1, at the peak load, the relative crack length for beam D2 and D1 are closer. However for beam D3 a size effect appears clearly. Also, the three beams seem to follow the same trend regarding the crack propagation.

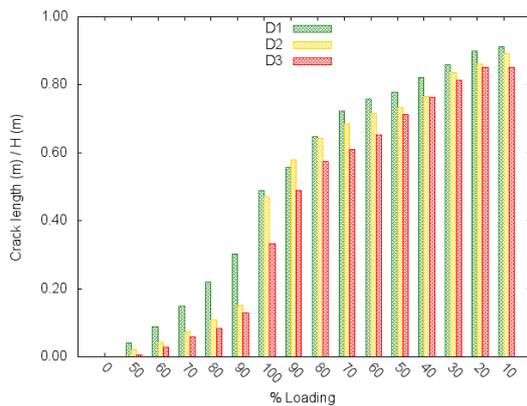


Fig. 8 Relative-Crack length evolution during loading (M1)

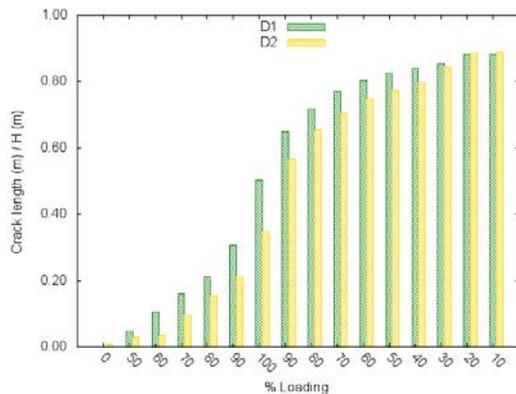


Fig. 9 Relative-Crack length evolution during loading (M2)

For beam D3, a less value of relative crack length is observed. Most of the size effect formulas (Bazant SEL [2], MMTS [11], etc.) based on Fracture Mechanics are derived on the basis of the equivalent linear elastic fracture mechanics where we supposed that the onset of crack growth corresponds to the peak Load. Also, according to these laws, the relative crack length should be same at the peak load whatever the structure size. This hypothesis is not confirmed using the mesoscopic approach. Also, experimental observations using digital image correlation

technique [9] give the same conclusions as the numerical ones, which confirm that, at the mesoscale level, the local behavior of concrete is well described.

For concrete M2, a size effect regarding the relative crack length is observed for beam D2 and D1. For this concrete type, beam D3 has not been tested experimentally.

### 5. SIZE EFFECT ON THE LOCAL BEHAVIOR –CRACK OPENING

As it has been outlined in the introduction, the crack opening computation is of great interest for design in civil engineering. For this reason, the size effect is also investigated regarding the crack opening (crack width).

The numerical method proposed by the authors (Matallah & al [4]) combined with the proposed meso-scale approach are used to evaluate the crack openings. Also, the numerical results are compared with the experimental ones.

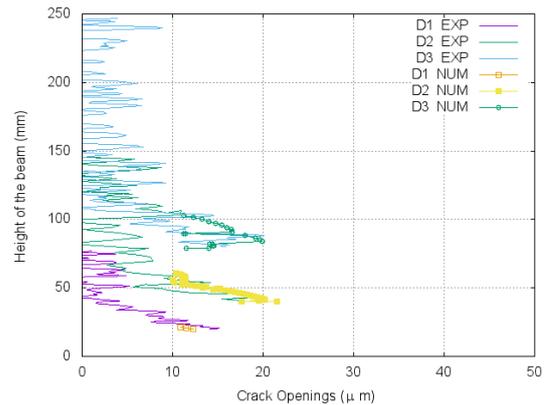


Fig. 10 Crack opening evolution at the peak- M1

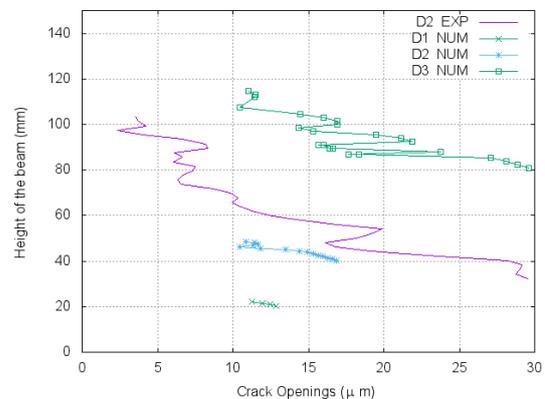


Fig. 11 Crack opening evolution at the peak-M2

Experimentally, the crack openings have been evaluated using the digital image correlation technique [9].

Figs. 10 and 11 present the crack opening evolution at the peak load for both type of concrete (M1 and M2). For concrete M1, the comparison shows a good agreement between the experimental values and the numerical ones with regards to the maximum value obtained at the notch tip. For concrete type M2, the comparison is limited to one beam D2. For this beam, numerically, the crack opening at the notch is about the half of the one obtained experimentally. Fig. 12 gives a comparison between numerical values both for concrete type of mix M1 and M2. For beam D1 and D2, the same values are obtained, however for large beam, a noticeable difference is observed. Also, we observe that for concrete M2 a size effect is more pronounced regarding the values of crack mouth at the notch.

Numerically, the crack opening evolution along the height of the beam has the same trend for both concrete type.

At the peak load, it seems that the size effect is not very pronounced regarding the crack opening values. However, as it has been outlined before, most of the size effect formulas (MMTS [11], Bazant SEL [2]) based on fracture Mechanics are derived on the basis of equivalent linear elastic fracture mechanics where we supposed that the onset of crack growth corresponds to the peak load.

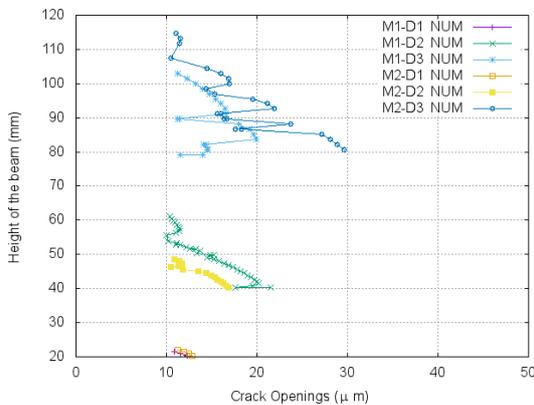


Fig. 12 Crack opening evolution at the Peak Comparison M1-M2

However recent studies [7][12], focused on the evolution of the FPZ length (the crack length is related to the FPZ length) shows that for small beams, at the peak load, the crack is not fully opened. The maximum load is obtained before the crack is fully open at the notch tip. However for large beams, a fully opened crack is developed at the peak load. At the peak load, for large beam the crack is completely open, ie, (the notch tip is stress-free). In the post-peak regime, the fracture

process is a steady crack propagation process. For small beams, the crack is still opening. Following these considerations, make a conclusion about the existing of a size effect on the crack width at the peak load seems to be a difficult statement.

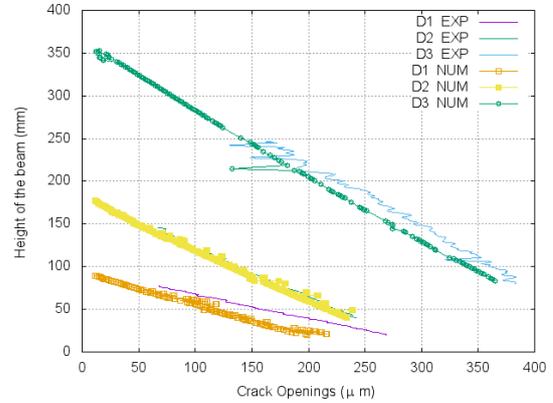


Fig. 13 Crack opening evolution at the end M1

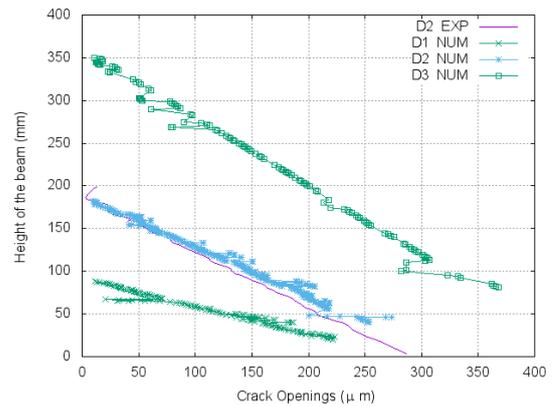


Fig. 14 Crack opening evolution at the end M2

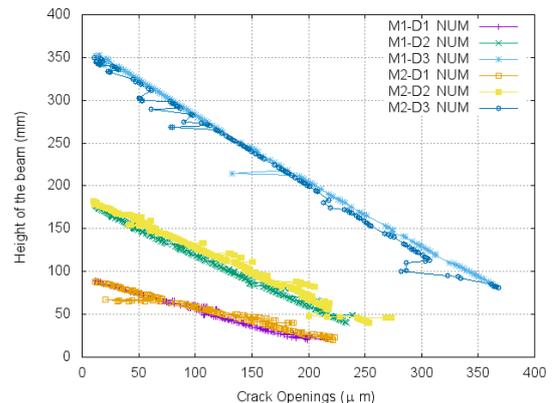


Fig. 15 Crack opening evolution at the end M1-M2

Another aspect is investigated. The crack width at the final stage of the failure process. Figs. 13 and 14 show the crack openings evolution at the end of the load-CMOD curve (at 10% of the post-peak

regime). Large beams present higher values compared with the smallest ones. Also, both concrete type M1 and M2 show the same trend (Fig.15).

## 6. CONCLUSIONS

In the present study, a numerical approach has been adopted in order to study the existing of a size effect of the cracking process of concrete. In the literature, size effect is mostly studied regarding the nominal strength. For design in civil engineering, the fracture process is a very important aspect to be considered in order to ensure a safe behavior for structure. Crack-length evolution and crack-opening evolution are two important processes that the designer should consider. In the present study, the existing of a size effect for these two fracture characteristics (crack length and crack opening) has been deeply investigated at a mesoscale level.

The numerical investigation shows that the concrete behavior is well described by a mesoscale approach. The size effect on the global behavior is well reproduced. The effect of aggregate size on the fracture behavior has been also confirmed at the meso-scale.

The local behavior has been also investigated. Crack length evolution and crack opening evolution are two interdependent processes. The numerical investigation reveals the existing of a size effect regarding these two parameters. Experimental observations (using digital image correlation) confirm this statement.

Establishing a relationship to extrapolate the size effect on the crack opening and the crack length is not an easy task. Most of size effect formulas suppose that the onset of crack growth corresponds to the peak Load. However, at the peak load, as it has been revealed in recent studies focusing on the FPZ evolution [7][12] (FPZ is linked to the crack length and crack width), the large beams and the small ones exhibit different cracking behavior due to other geometrical and material conditions. If a size effect must exist for crack openings and crack length, it would concern the whole crack opening process. Other investigations are needed in order to study this aspect.

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