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Load Settlement Relationships of Circular Footings Considering Dilatancy Characteristics of Sand

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ABSTRACT:In order to elucidate the mechanical properties of load-settlement relationships of spread foundations on sand ground, experimental and analytical study on strain hardening and dilatancy of sand is needed. Here we present the theoretical properties which were derived from circular footing model experiment and computer simulation. The model experiment was carried out through vertical loading on circular footing, and used relative density and tank dimensions as parameters.The quantitative relationship between load and settlement was analyzed through FEM simulation with SMP-Cam-Clay model, which is capable of estimating the dilatancy of sand. The results are summarized as follows;In both cases of dense and loose sand models, before reaching the ultimate load, the load-settlement relationship obtained from FEM simulation corresponded with that of obtained from the experiment.Afterreaching the ultimate load, "Terzaghi's bearing capacity line"corresponded with the load-settlement relationship obtained from the experiment.In the case of medium-dense sand models,some binding effect of a soil tank rectanglewas recognized. The effect was considered to be exerted by the positive dilatancy of the sand, which occurred steadily with the settlement progresses.

Keywords: Spread Foundation, Sand Ground, Load-Settlement Curve, Constitutive Equation, Dilatancy

1 INTRODUCTION

Although the quantitative load-settlement relationships of spread foundation have been experimentally well demonstrated on sand ground[1]–[4], the underlying mechanical properties of the relationships still remained to be elucidated.Especially, since little study has been done on the mechanical properties characterized through strain hardening and dilatancy of sand.Here we present the theoretical properties which were derived from circular footing model experiment and computer simulation. The model experiment was carried out through vertical loading on circular footing, and used relative density and tank dimensions as parameters. The quantitative relationship between load and settlement was analyzed through FEM simulation with SMP-Cam-Clay model.In addition, a part of the contents of this paper have been reported in [5],[6].

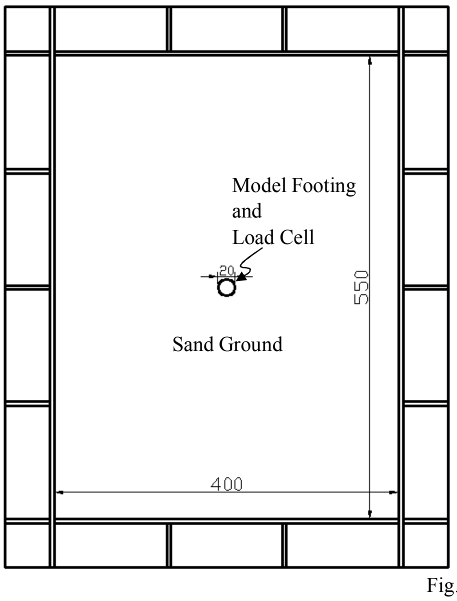
2 VERTICAL LOADING TESTOF CIRCULARFOOTING

2.1 Test Pit and Loading Equipment

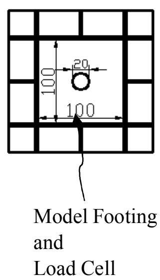
The model experiments were performed on Toyoura sand, with different relative densities. The Maximum and minimum densities of the sand, as determined by the standard procedure, are given in Table1. The loose sand models were built by pouring sand from containers. The density of the sand models is a unique-function of the height of free fall of sand. Medium-dense and dense sand models were built by vibration provided by an earthquake simulator.The soil tank was made of steel, which made the tank completely resistant to earth pressure.Teflon sheets with double layers of grease were set on the sides of the soil tank, in order to omit the frictioncaused by the experiment ground[7]. To study the effect of sand dilatancy on mechanical properties ofthe load-settlement relationship,soil tanks with different volumes were used in the experiments.The short side of the big soil tank was measured to be 400 millimeters in Fig.1-1 which shows the experimental device, whereas that of the small soil tank was 100 millimeters in Fig.1-2 which shows the experimental device.The model footing has a circular cross section with diameter of 20 millimeters, and is 80 millimeters in height. Moreover, the model footing is made of wood, and sandpapers are put on the tip[1]. In the following,

Table1 Specifications of the Experiment Ground

Density of Soil Particles ρ_s	2.558g/cm^3
Maximum Density $\rho_{d\,max}$	1.645g/cm^3
Minimum Density $\rho_{d\,min}$	1.337g/cm^3



The Ground Plan : ℓ 20B (Unit : mm)
Fig.1-1Experimental Device (ℓ 20B)



The Ground Plan : ℓ 5B (Unit : mm)
Fig.1-2Experimental Device (ℓ 5B)

Table2 Experiment Name

Name	Relative Density D_r	Soil Tank
L-20B	20～30%	ℓ 20B
L-5B	20～30%	ℓ 5B
M-20B	50～60%	ℓ 20B
M-5B	50～60%	ℓ 5B
D-20B	80～90%	ℓ 20B
D-5B	80～90%	ℓ 5B

short side length of the soil tank is named ℓ, diameter of the model footing is named B,the soil tank that ℓ is 400 millimeters is named ℓ 20B, and the soil tank that ℓ is 100 millimeters is named ℓ 5B. In other words, ℓ 20B expresses that ℓ is 20 times of B, ℓ 5B expresses that ℓ is 5 times of B.Experiment name is shown in Table2. In Table2, L, M, and D indicate the loose sand ground, the medium-dense sand ground, and dense sand ground, respectively. Also,20B indicates soil tank ℓ 20B, whereas 5Bindicates ℓ 5B.The binding effect caused by the soil tank was treated as a negligible factor throughout the experiment.This was confirmed when the result of separate experiment conducted on dense sand ground using ℓ as 800 millimeters corresponded to that of D-20B[5].Therefore, the binding effect was also considered to be negligible in the experiments of L-20B and M-20B.

2.2 Loading Procedure

300kN universal testing machine is used as a loading device. By vertical loading to the settlement rate 12.5μm/secon the model footing,the quantitative load-settlement relationship was determined.Inthe case of loose sand models and medium-dense sand models, the load measured by load cell installed at the top of the model footing divided by the bottom area of the model footing was defined as the load.Inthe case of dense sand models, the load measured by load cell installed at the bottom of the soil tankdivided by the bottom area of the model footing was defined as the load.Inall the experiments, the amount of relative displacement between the soil tank and the loading plate of a universal testing machine was defined as the settlement. In the following, the load is named*p*, the settlement is named*S*.In lastly, loading was performed until *S* reached 10 millimeters.

2.3 Test Results

2.3.1 In the Case of Loose Sand Models

L-20B and L-5B were performed three times,each. As a result, the *p*-*S* relationship showed no significant variation. The results of two typical cases are shown by both logarithms indications in Fig.2. In anyresults of L-20B and L-5B, *p* steadily increases as the settlement progresses, and bothresults appear to be consistent. Based on [3], theultimate load in *p*-*S* relationship was defined as a load at the point that shifts from a curve to a straight line.This point corresponds to an inflection point in log *p*-log *S*relationship.In anyresults of L-20B and L-5B, the inflection point was

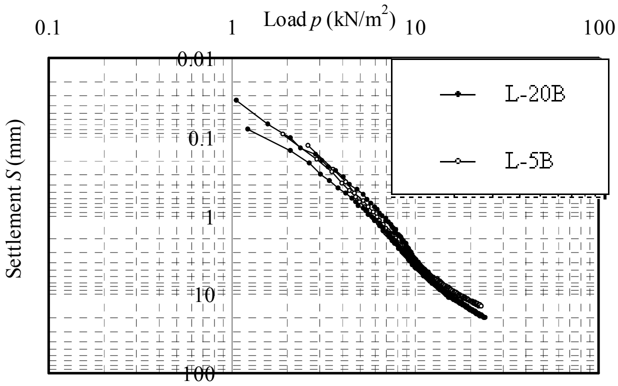


Fig.2log*p*-log*S*Relationships (In the Case of Loose Sand Models)

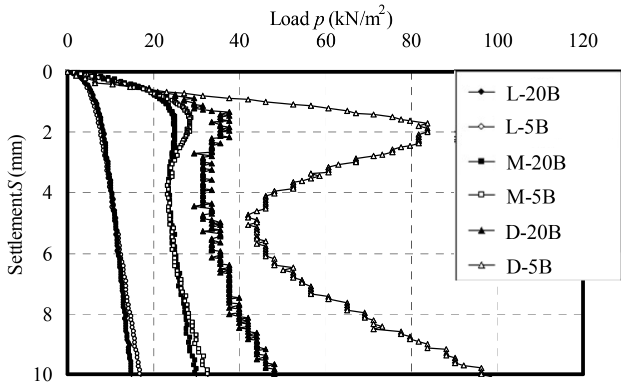


Fig.3*p*-*S*Relationships (Comparison of the Relative Density)

confirmed when *S* reached 3 millimeters, so the load atthis point was defined asthe ultimate load. Probably because the loose sand ground under the model footing exhibit contractile behavior,the elevation of sand surface around the model footing did not appear in anyresults of L-20B and L-5B.

2.3.2 In the Case of Dense Sand Models

D-20B and D-5B were performed three times, each. As a result, the *p*-*S* relationship showed no significant variation. The result of a typical case is shown in Fig.3. In anyresults of D-20B and D-5B, *p* steadily increases until *S* reaches about 2.0 millimeters. Bothresults are almost consistent until *S* reaches 1.0 millimeters. But the stiffness on settlement of D-5B becomes larger than that of D-20B from 1.0 millimeters to 2.0 millimeters. As a result, *p* of D-5B is about 2.4 times larger than that of D-20B as*S* reached about 2.0 millimeters. From 2.0 millimeters to 4.5 millimeters, *p* steadily decreases as the settlement progresses where the rate of decrease in *p* of D-5B is about 6 times larger than that of D-20B. In anyresults of D-20B and D-5B, *p*then steadily increases again as the settlement progresses, and the stiffness on settlement of D-5B becomes larger than that of D-20B. In addition, in bothresults of D-20B and D-5B, the ultimate load was confirmed when *S*reached about 2.0 millimeters.The elevation of sand surface around the model footing appearedwhen*S* reaches about 4.5 millimeters in theresult of D-5B.In this paper, the failure surface was decided to have reached the sand surface, based on the appearance of the elevation of the sand surface around the model footing[8].

2.3.3 In the Case of Medium-Dense Sand Models

M-20B and M-5B were performed three times, each. As a result, the *p-S* relationship showed no significant variation. The result of a typical case is shown in Fig.3. In anyresults of M-20B and M-5B, *p* steadily increases until *S* reaches about 1.6 millimeters. Bothresults are almost consistent until *S* reaches 0.8 millimeters. But the stiffness on settlement of M-5B becomes larger than that of M-20Bfrom 0.8 millimeters to 1.6 millimeters. As a result, *p* of M-5B is about 1.2 times larger than that of M-20B at*S* reached about 1.6 millimeters. From 1.6 millimeters to 4.0 millimeters, *p* steadily decreases as the settlement progresses where the rate of decrease in *p* of M-5B is about 3 times larger than that of M-20B. In anyresults of M-20B and M-5B, *p*then steadily increases again asthe settlement progresses, and bothresults appeared to be consistent. In addition, in bothresults of M-20B and M-5B, the ultimate load is confirmed when *S* reached about 1.6 millimeters.Such characteristics of the *p-S* relationship were similar to that of the *p-S* relationship which was derived fromD-20B and D-5B. The elevation of sand surface around the model footing did not appear in anyresults of M-20B and M-5B.Such behavior of sand was similar to whatwas derived fromL-20B and L-5B.

3 THE INFLUENCEOF SAND DILATANCYON P-S RERATIONSHIPS

3.1 In the Case of Dense Sand Models

In the case of dense sand models, sand ground under the footing seemed to exhibit a continuous behavior with positive dilatancy until *p* reaches the ultimate load, whereas the sand behavior under footing exhibits a discontinuous behavior with a sign of failure surfaces after *p* reached the ultimate load. Therefore,the FEM simulationwasperformed until *p* reaches the ultimate load, and thenthe rigid-plastic analysis based on the bearing capacity of Terzaghi's theory was performed [5].In the following, SD-20B expresses the analysis corresponding to D-20B, and SD-5B expresses the analysis corresponding to D-5B.

3.1.1 In the Case of D-20B

[Until p reaches the ultimate load]

The comparison of the load-settlement relationship obtained from D-20B and SD-20B, and "Terzaghi's bearing capacity line" is shown in Fig.4. Here, "Terzaghi's bearing capacity line" is calculated as D_fS in(1), indicating the line that linked q_u as a parameter *S*.In Fig.4, both*p* of SD-20B and D-20Bare roughly consistent until *S* reaches about 1.0 millimeter.From 1.0 millimeter to 2.0 millimeters, *p* of SD-20B is larger than that of D-20B.

$$q_u = \alpha c N_c + \beta \gamma_1 B N_\gamma + \gamma_2 D_f N_q \quad (1)$$

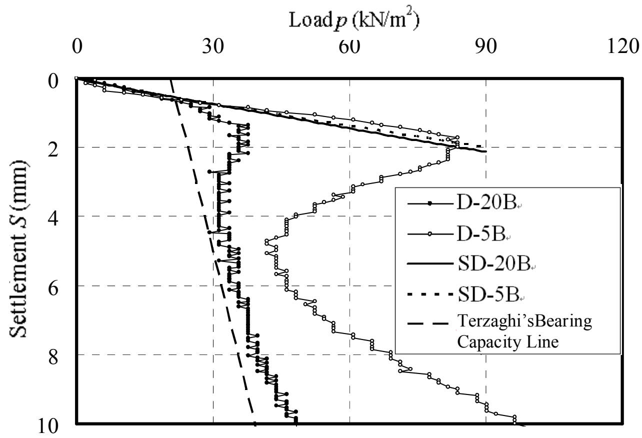


Fig.4 Comparison of the Load-settlement Relationshipobtained from D-20B and SD-20B, and "Terzaghi's Bearing Capacity Line"

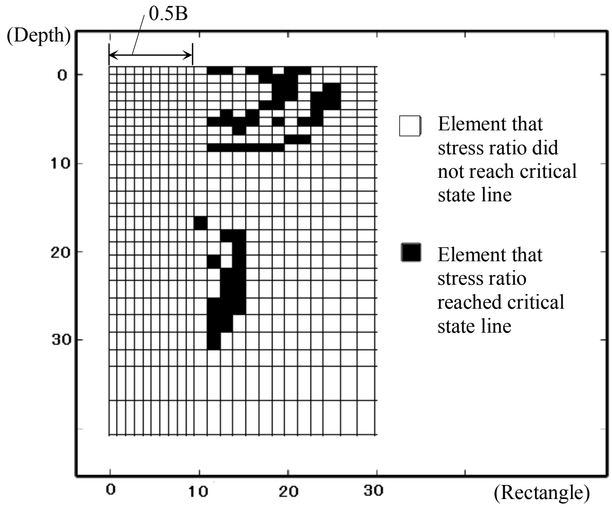


Fig.5The Distribution of Stress Ratio obtained fromSD-20B(Unit : mm)

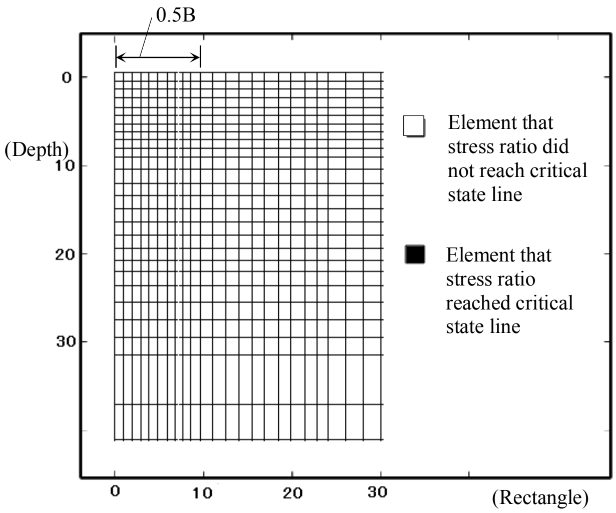


Fig.6 The Distribution of Stress Ratio obtained fromSD-5B(Unit : mm)

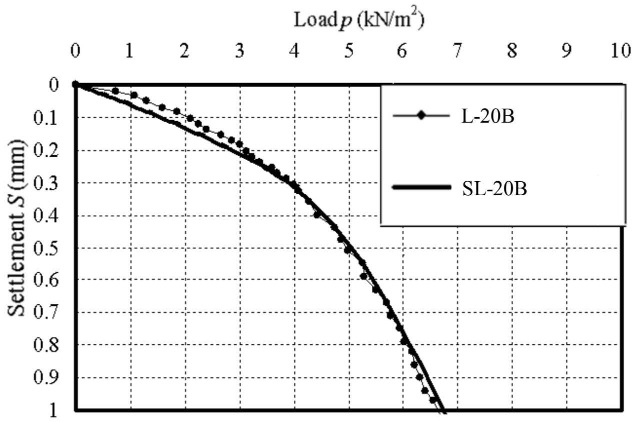


Fig.7 Comparison of the Load-settlement Relationship obtained from L-20B and SL-20B

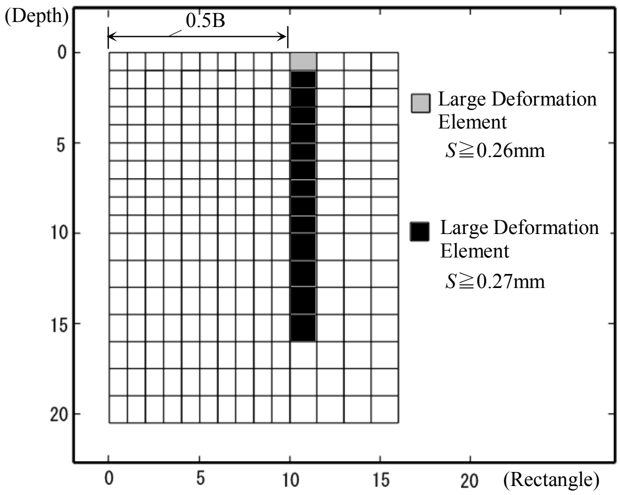


Fig.8TheDistribution of “Large Deformation Element” obtained from SL-20B(Unit：mm)

This is because the progressive failure accompanied by the generation of the failure surface occurs.It can be confirmed from Fig.5 which shows the distribution of stress ratio obtained fromSD-20B when S reached about 2.0 millimeters.In addition, the element that stress ratio reached critical state line occurred first when S reached about 1.0 millimeter.In Fig.5, it is confirmed that the element that stress ratio reaching critical state line steadily increases from 1.0 millimeter to 2.0 millimeters.Therefore, because the condition that the boundary of the element changes discontinuous when stress ratio reached critical state line is not considered in the case of SD-20B, p of SD-20B is considered to be larger than that of D-20B from 1.0 millimeter to 2.0 millimeters. From the above, it is considered thatsand behavior shifts from a continuous behavior based on SMP-Cam-Clay model to a discontinuous behavior accompanied by the generation of the failure surface when S reached about 1.0 millimeter, and that the sheared mass of sand appears under the footing when S reached about 2.0 millimeters.

[After p reached the ultimate load]

In Fig.4, from 2.0 millimeters to 4.5 millimeters, p decreases slightly as the settlement progresses. When S reached about 4.5 millimeters, the load-settlement relationship is asymptotic to“Terzaghi’s bearing capacity line”. Therefore, sand behavior is considered to result in a total failure at this stage.After S reached about 4.5 millimeters, p increases steadily along “Terzaghi’s bearing capacity line”as the settlement progresses, indicating the sand behaviorresulted in a continuousfailure.

3.1.2 In the Case of D-5B

[Until p reaches the ultimate load]

The comparison of the load-settlement relationship obtained from D-5B and SD-5B is shown in Fig.4. Because both p of SD-5B and D-5Bare roughly consistent until S reaches about 1.0 millimeter, the mechanical properties of the p-S relationship obtained from D-5B is considered to be similar to that of obtained from D-20B.From 1.0 millimeter to 2.0 millimeters, both p of SD-5B and D-5B are roughly consistent throughout. As a result, the ultimate load of D-5B is 2.4 times larger than that of D-20B. To analyze the cause, the comparative studies shown inFig.5 andFig.6 were conducted.Fig.6 shows the distribution of stress ratio obtained from SD-5B. In the case of SD-20B, the area recognized as sheared mass of sand appears under the footing. On the other hand, in the case of SD-5B, the area recognized as sheared mass of sand does not appear. In other words, in the case of D-5B, positive dilatancy exerts a marked binding effect of a soil tank rectangle and the dilatancy causes the increase of mean principal stress as a result. Therefore, from 1.0 millimeter to 2.0 millimeters, the ultimate load of D-5B is considered to be 2.4 times larger than that of D-20B.

[After p reached the ultimate load]

In Fig.4, from 2.0 millimeter to 4.5 millimeters, p decreases slightly until p reaches half of the ultimate load. Such characteristic of the p-S relationship is not recognized in the case of D-20B. Because the failure surface appeared at the sand surface when S reached 4.5 millimeters, the effect of positive dilatancy which occurred from 1.0 millimeter to 2.0 millimeters is considered to disappear from 2.0 millimeters to 4.5 millimeters.

3.2 In the Case of Loose Sand Models

In the case of loose sand models, shear failurewhich occurs and progressesbelow edge of the footing is considered to have a significant influence on the mechanical properties of the p-Srelationship.Therefore, the element that stress ratio reached critical state line and shear strain excelled was defined as “Large deformation element”, and regarded it as a gap element in each step, and the FEM simulation was performed [6]. In the following, SL-20B expresses the analysis corresponding to L-20B, and SL-5B expresses the analysis corresponding to L-5B. In addition, because both results of SL-20B and SL-5B were almost consistent, only the results of SL-20B are described as follows.

[Until p reaches the ultimate load]

The comparison of the load-settlement relationship obtained from L-20B and SL-20B is shown in Fig.7. In Fig.7, both p of SL-20B and L-20Bare almost consistent until S reaches about 1.0 millimeter. To investigate the stress condition in the sand ground, the distribution of “Large deformation element” wasobtained from SL-20B as shown in Fig.8. In Fig.8,

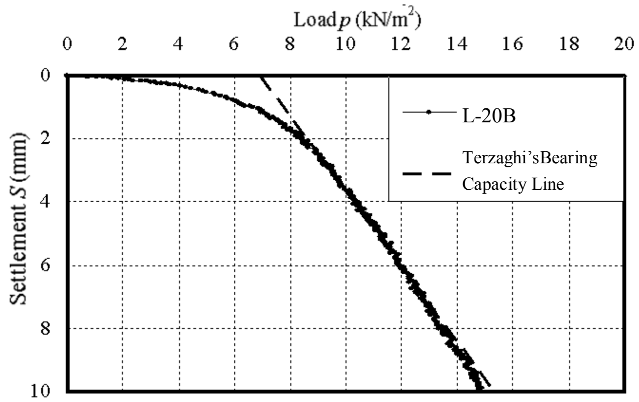


Fig.9 Comparison of the Load-settlement Relationship obtainedfrom L-20B and "Terzaghi's Bearing CapacityLine"

Table3Comparison of theUltimate Load by the Relative Density

Experiment Name	The Ultimate Load (kN/m^2)	R_u
L-20B	9.51	0.96
L-5B	9.17	
M-20B	25.01	1.15
M-5B	28.70	
D-20B	35.66	2.35
D-5B	83.91	

"Large deformation element" appears below edge of thefooting first when S reached 0.26 millimeters, and after Sreached 0.27 millimeters, it appears directly below "Large deformation element" which occurred when S reached 0.26 millimeters. Because both p of SL-20B and L-20B are almost consistent until S reaches about 1.0 millimeter, the sand ground under the footing is considered to exhibit a behavior based on SMP-Cam-Clay model until S reaches about 0.26 millimeters.In other words, sand ground under the footingis considered to exhibit a contractile behavior with negative dilatancy. From 0.26 millimeters to 1.0 millimeters, it is considered that sand ground under the footing exhibits a contractile behaviorcontinuously with negative dilatancyand that shear failure occurs and progresses at the same time below edge of the footing.

[After p reached the ultimate load]

In Fig.7, the p-S relationship obtained from SL-20B showed a good correspondence with that ofobtained from L-20B until S reaches 1.0 millimeter. After S reached 1.0 millimeter, the simulation of the p-S relationship was carried out based on "Terzaghi's bearing capacity line," since SL-20B was difficult to perform as p of L-20B approached the ultimate load.Here, the relative density D_r was determined to be consistent with q_u when S reached 3.0 millimeters and the ultimate load of L-20B. The relationship between D_r and internal friction angle ϕ was estimated from [9]. As a result, D_r=59%, ϕ =38.9°, γ_1=γ_2=1.52g/cm^3 were obtained. These values correspond to the constants representing the mechanical properties of the medium-dense sand ground. The comparison of "Terzaghi's bearing capacity line" using these values andthe load-settlement relationship obtained from L-20B is shown in Fig.9.In Fig.9, it was obtained the interesting results that "Terzaghi's bearing capacity line" shows a good correspondence with the$p$-S relationshipobtained from L-20Bafter S reached 3.0mm millimeters.

3.3 In the Case of Medium-dense Sand Models

A comparison ofthe ultimate load in the case of each sand model is shown in Table3. InTable3, R_u is the ratio of the ultimate loadin the case of using ℓ 5B against ℓ 20B. Some binding effect of a soil tank rectangle is recognized on R_u in the case of medium-dense sand model. A binding effect of a soil tank rectangle becomes marked by positive dilatancyas described in Section 3.1.2. Therefore, in the case of medium-dense sand model, because the condition of dilatancy occurrence is located between positive dilatancyand negative dilatancy, some binding effect of a soil tank rectangle recognized on R_u is considered to be exerted by positive dilatancy which occurred steadily as thesettlement progresses.

4 CONCLUSIONS

In this paper, we present the theoretical properties of the load-settlementrelationshipon sand ground, which were derived from circular footing model experiment and computer simulation. To studythe effect of sand dilatancy on the mechanical properties of the load-settlementrelationship, the model experiment was carried out on relative density andtank dimensions asparameter. The quantitative relationship between load and settlement was analyzed through FEMsimulation with SMP-Cam-Clay model, which is capable of estimating the dilatancy. The results are summarized as follows;

1) In the case of dense sand modelswith large soil tank(D-20B),it is considered that sand behavior shifts from a continuous behavior based on SMP-Cam-Clay model to a discontinuous behavior accompaniedby the generation of the failure surfacewhen S reached about 1.0 millimeter, and that the sheared mass of sand appears under the footing when S reached about 2.0 millimeters. When S reached about 4.5 millimeters,sand behavior is considered to result in a total failure.After S reached about 4.5 millimeters, sand behavior is considered to result in a continuousfailure.
2) In the case of dense sand modelswith small soil tank (D-5B), until S reached about 1.0 millimeter, the mechanical properties of the p-S relationship obtained from D-5B is considered to be similar to that of obtained from D-20B.From 1.0 millimeter to 2.0 millimeters, it is considered that positive dilatancy exerts a marked binding effect of a soil tank rectangle and the dilatancy causes the increase of mean principal stress. Accordingly, the ultimate load of D-5B was 2.4 times larger than that of D-20B. From 2.0 millimeter to 4.5 millimeters,the of positive dilatancy which occurred from 1.0 millimeter to 2.0 millimeters is considered to disappear.When S reached about 4.5 millimeters, mechanical properties of D-5B are considered to be similar to that of D-20B.

3) In the case of loose sand models with large and small soil tank (L-20B and L-5B), sand ground under the footing is considered to exhibit a behavior based on SMP-Cam-Clay model until S reaches about 0.26 millimeters.In other words, sand ground under the footingis considered to exhibit a contractile behavior with negative dilatancy. From 0.26 millimeters to 1.0 millimeters, it is considered that sand ground under the footing exhibits a contractile behavior continuously with negative dilatancyand that shear failure occurs and progresses at the same time below edge of the footing.After S reached 3.0millimters, the results showed that, with the values correspond to the constants representing the mechanical properties of medium-dense sand ground, "Terzaghi's bearing capacity line" seemed to correspond with the p-S relationship obtained from L-20B.
4) In the case of medium-dense sand models (M-20B and M-5B),some binding effect of a soil tank rectanglewas recognized on R_u which shows the ratio of the ultimate loadin the case of using ℓ 5B against ℓ 20B.This is considered to be exerted by positive dilatancy which occurred steadily as thesettlement progresses, because the condition of dilatancy occurrence is located between positive dilatancy and negative dilatancy.

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