

## Ultimate Strength and Fatigue Life of Concrete Elements Reinforced with Carbon Fiber Sheet Based on Continuum Damage Mechanics

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**ABSTRACT:** Continuum Damage Mechanics (C.D.M) was applied to evaluate the mechanical behavior of concrete structural elements reinforced with carbon fiber sheet (CF-sheet) under quasi-static and cyclic loading. A constitutive equation for elasto-plastic damageable solids is formulated by using Drucker-Prager's equivalent stress and identified by the uniaxial compressive and tensile experimental results for concrete, tensile results for CF-sheet. And then finite element analyses are carried out for real-scale cantilever reinforced concrete slabs and concrete blocks with CF-sheet. Finally It was concluded that the application of C.D.M for concrete structural ultimate strength and fatigue durability comparing the experimental data and the calculation by 2-dimensional finite element method (F.E.M) was very useful.

**Keywords:** Ultimate strength, Fiber sheet, Fatigue life Concrete, Continuum Damage Mechanic

### 1. INTRODUCTION

Concrete is a standard brittle material in civil engineering. Reinforced concrete structures with steel bar resisting tension have been used for the infrastructures such as buildings, bridges and tunnels, due to its durability and economy. In recent years, special attention has been paid to CF-sheet to resist against earthquake repair damage and prevent separation failure [1]-[3]. The CF-sheet is of light weight (1/5 of steel), high strength (10 times of steel), high rigidity and high durability as well as easy to construct. Considering future development of CF-sheet, it is important to establish the analytical evaluation of the strength and durability of concrete structural members reinforced with CF-sheet.

The elasto-plastic model, the smeared crack model and the discrete crack model considering cracking in concrete have been proposed as the analytical model for reinforced concrete (RC) structures [4]. On the other hand, there has been much progress in the application of C.D.M [5] to metal materials. The mechanical deterioration is represented by the internal state variable  $D$  called damage variable and the strain energy release rate  $Y$  conjugate with  $D$  in damage mechanics. It is expected as a branch of mechanics essentially applicable to fatigue fracture and residual life prediction as well as static and dynamic strength analysis at the material test and structure level. The validity of the application of damage mechanics to concrete structures has been recognized through the researches on reinforced concrete rigid frames [6], short fiber-reinforced concrete [7] and high-strength concrete [8].

The development of the analysis program based on damage mechanics and its experimental validations are conducted in the present study for the purpose of establishing the method for ultimate strength and lifetime evaluation of the concrete structural members reinforced with CF-sheet. In the application of damage mechanics models to concrete and the CF-sheet, the two-dimensional elasto-plastic damage constitutive equation is formulated

by using Drucker-Prager's equivalent stress in order to consider the difference between the tensile and the compressive strength of concrete. The formulated constitutive equation is implemented in the two-dimensional finite element program. The material constants are identified by using the material test results under uniaxial tension and compression and applied to the ultimate strength and fatigue life analysis.

The formulation of the elasto-plastic damage constitutive equation and its identification are described in the section 2. The section 3 contains the experimental results for RC structural members under static and cyclic loading with CF-sheet. The results of finite element analysis are compared with the experimental results. An another equivalent stress is applied for the purpose of evaluating shear behavior for de-lamination or surface detachment between CF-sheet, epoxy resin and concrete in the section 4. The section 5 is the concluding remarks.

### 2. FORMULATION AND IDENTIFICATION OF ELASTO-PLASTIC DAMAGE CONSTITUTIVE EQUATION

#### 2.1 Elasto-Plastic Damage Constitutive Equation

The dissipation potential for the growth of plastic strains, which is the sum of the plastic potential and the damage potential, is expressed by the following equation:

$$F = F_p(\bar{\sigma}, \gamma; D) + F_D(Y; p, D) \\ = \bar{\sigma}_{eq} - \gamma - \sigma_y + \frac{S_1}{(S_2 + 1)(1 - D)} \left( \frac{Y}{S_1} \right)^{S_2 + 1} \quad (1)$$

Where  $F_p$  is the potential for the growth of plastic strains, which is a function of the effective stress  $\bar{\sigma}$ , the plastic hardening parameter  $\gamma$  and the scalar damage variable  $D$ .  $F_D$  is the potential for the evolution of damage, which is a function of the strain energy release rate  $Y$ , the equivalent plastic strain  $p$  and the damage variable  $D$ .

In the formulation of the constitutive equation, the yield function is assumed as follows:

$$\overline{f} = F_p - \overline{\sigma}_{eq} - \gamma - \sigma_y = 0 \quad (2)$$

$$\overline{\sigma}_{eq} = \sigma_{eq} / (1 - D) \quad (3)$$

$$\sigma_{eq} = \alpha I_1 + (J_2')^{1/2} \quad (4)$$

Where the following notations are used:  $\overline{\sigma}_{eq}$ ; effective Drucker-Prager's equivalent stress,  $\sigma_y$ ; the yield stress,  $\alpha$ ; the material parameter,  $I_1$ ; the first invariant of stress and  $J_2'$ ; the second invariant of deviatoric stresses. The yield function given in Equation (2) is assumed as the plastic potential. The following equation holds on the yield surface considering damage.

$$df = \left( \frac{\partial f}{\partial \sigma} \right)^T d\overline{\sigma} + \frac{\partial f}{\partial \gamma} d\gamma + \frac{\partial f}{\partial D} dD = 0 \quad (5)$$

The plastic strain increment  $d\varepsilon_p$  and the equivalent plastic increment  $dp$  are given by the following equations.

$$d\varepsilon_p = d\lambda \frac{\partial F}{\partial \sigma} = d\lambda \frac{\partial F_p}{\partial \sigma} \quad (6)$$

$$dp = -d\lambda \frac{\partial F_p}{\partial \gamma} \quad (7)$$

Where  $d\lambda$  is a proportional coefficient.

As the total strain increment in the elasto-pastic state is sum of the elastic strain increment and the plastic strain increment, the effective stress increment can be expressed as follows:

$$d\overline{\sigma} = C d\varepsilon_e = C(d\varepsilon - d\varepsilon_p) = C d\varepsilon - C d\lambda \frac{\partial F_p}{\partial \sigma} \quad (8)$$

Where  $C$  is the stress-strain matrix.  $d\varepsilon_e$  and  $d\varepsilon_p$  are the elastic strain increment and the plastic strain increment respectively. The plastic hardening parameter and its increment are assumed as follows;

$$\gamma = K p^n \quad (9)$$

$$d\gamma = n K p^{n-1} dp = H dp = H d\lambda \quad (10)$$

Where  $K$  and  $n$  are the material constants. The damage increment is obtained by the following equation:

$$dD = d\lambda \frac{\partial F}{\partial Y} = d\lambda \frac{\partial F_D}{\partial Y} \quad (11)$$

Substituting equation (8), (9) and (11) into equation (5) and solving it with respect to  $d\lambda$ ; the following equation can be obtained:

$$d\lambda = \frac{\left( \frac{\partial F_p}{\partial \sigma} \right)^T C}{H + \left( \frac{\partial F_p}{\partial \sigma} \right)^T C \frac{\partial F_p}{\partial \sigma} - \frac{\overline{\sigma}_{eq}}{1 - D} \frac{\partial F_D}{\partial Y}} d\varepsilon \quad (12)$$

The substitution of equation (12) into equation (8) leads to the following relation between the effective stress increment and the strain increment [9],[10]:

$$d\overline{\sigma} = \overline{C} d\varepsilon$$

$$= C \left[ 1 - \frac{\frac{\partial F_p}{\partial \sigma} \left( \frac{\partial F_p}{\partial \sigma} \right)^T C}{H + \left( \frac{\partial F_p}{\partial \sigma} \right)^T C \frac{\partial F_p}{\partial \sigma} - \frac{\overline{\sigma}_{eq}}{1 - D} \frac{\partial F_D}{\partial Y}} \right] d\varepsilon \quad (13)$$

Where  $\overline{C}$  is the effective stress-strain matrix considering the elasto-plastic damage. The relation between the stress increment and the strain increment is expressed by the following equation:

$$d\sigma = (1 - D) d\overline{\sigma} - \overline{\sigma} dD = D_{epd} d\varepsilon \quad (14)$$

Where  $D_{epd}$  is the tangential stress-strain matrix considering the elasto-plastic damage, which relates the stress increment with the strain increment. The damage evolution equation as given by the following equation is used in the present study [5].

$$dD = \left( \frac{Y}{S_1} \right)^{S_2} dp \quad (15)$$

Where  $S_1$  and  $S_2$  are the material constants. It is assumed that the damage evolves with an increase of the equivalent plastic strain. The strain energy release rate  $Y$  is a function of Young's modulus  $E$  and equivalent stress  $\sigma_{eq}$  as expressed by the following equation:

$$Y = \frac{\sigma_{eq}^2}{2E(1 - D)^2} \quad (16)$$

## 2.2 Identification of Material Parameters

The material constants of concrete used in the constitutive equation have been determined as shown in Table 1 by the curve-fitting technique for the uniaxial compressive test results given by displacement measuring method (Fig.1) and tensile results by using strain gauge method. In this study, the plus sign means tensile and minus means compressive respectively. Fig.2 and Fig.3 show the stress-strain curves of compression and tension. In Fig.2, one of the stress-strain curves for experiment sharply dropped after peak. It would be occurred by shear fracture due to eccentric loading after peak.  $\varepsilon_{pd}$  and  $D_{cr}$  in Table 1 shows the damage plastic strain threshold and the critical damage at crack initiation respectively. When the equivalent strain exceeds  $\varepsilon_{pd}$ ; the damage evolves according to equation (15). And the damage variable reaches  $D_{cr}$ ; the material fractures and the stress are released. The experimental results were shown until the tensile strength in Fig.3. The post peak behavior after tensile strength could not be obtained because of brittle fracture of concrete and strain gauge's breaking.

The material constants of the CF-sheet has been determined as shown in Table 2 from the tensile test result given in Fig.4.

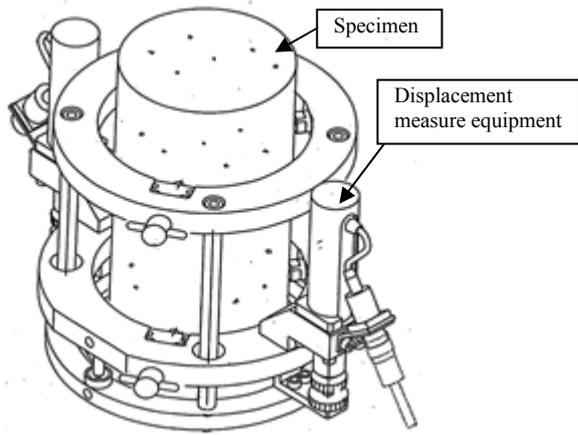


Fig.1 Displacement measure equipment (for compression)

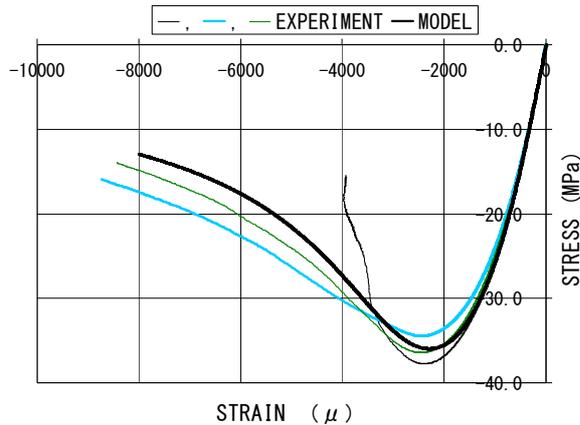


Fig.2 Compressive stress-strain curves for concrete

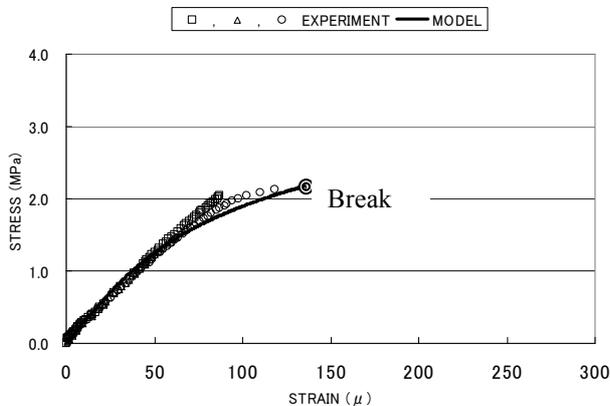


Fig.3 Tensile stress-strain curves for concrete

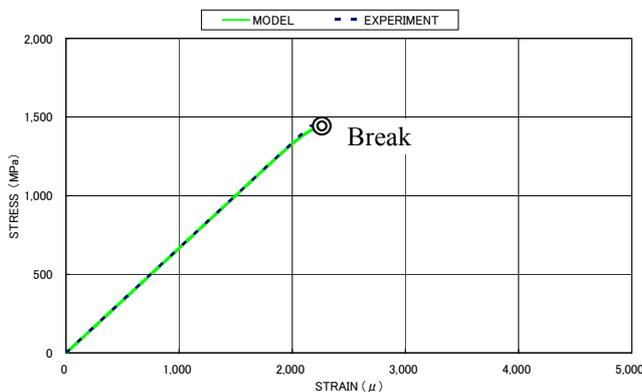


Fig.4 Tensile stress-strain curves for CF-sheet

Table 1 Material constant for concrete

E (Young's modulus)	$29.7 \times 10^3$ MPa
$\nu$ (Poisson's ratio)	0.17
$\alpha$ (equivalent stress)	0.72
$\sigma_y$ (yield stress)	0.75 MPa
K (plastic hardening)	40.0 MPa
n (plastic hardening)	0.215
$S_{p1}$ (damage parameter)	$0.215 \times 10^{-3}$ MPa
$S_{p2}$ (damage parameter)	1.55
$\epsilon_{pd}$ (damage threshold)	0.00
Dcr (critical damage)	$2.75 \times 10^{-5}$

Table 2 Material constants for CF-sheet

E (Young's modulus)	$668.0 \times 10^3$ MPa
$\nu$ (Poisson's ratio)	0.2
$\alpha$ (equivalent stress)	0.0
$\sigma_y$ (yield stress)	1200.0 MPa
K (plastic hardening)	1950.0 MPa
n (plastic hardening)	0.0955
$S_1$ (damage parameter)	0.055 MPa
$S_2$ (damage parameter)	1.755
$\epsilon_{pd}$ (damage threshold)	0.00
Dcr (critical damage)	0.0146

### 3. STATIC AND CYCLIC LOADING TESTS FOR RC STRUCTURAL MEMBERS

The real size clamped RC Slab structure used in the experiments is schematically shown in Fig.5.

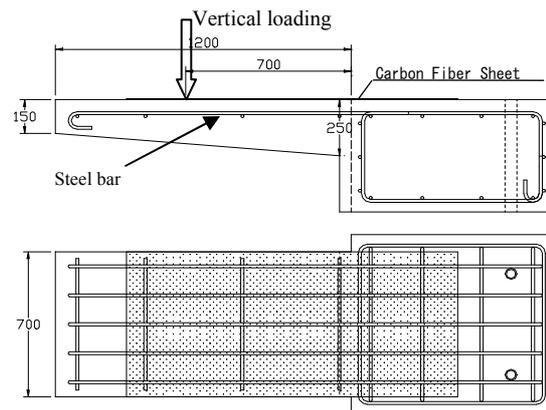


Fig.5 Test specimen (unit; mm)

This structure is a part of the bridge which has been constructed in the early of 1970's. It needs some repair and reinforce to use as a bridge. The static and cyclic loading test as a traffic load was carried out, and the ultimate strength and the effect of the strength after 2-million cyclic loading were evaluated from this test and numerical analysis.



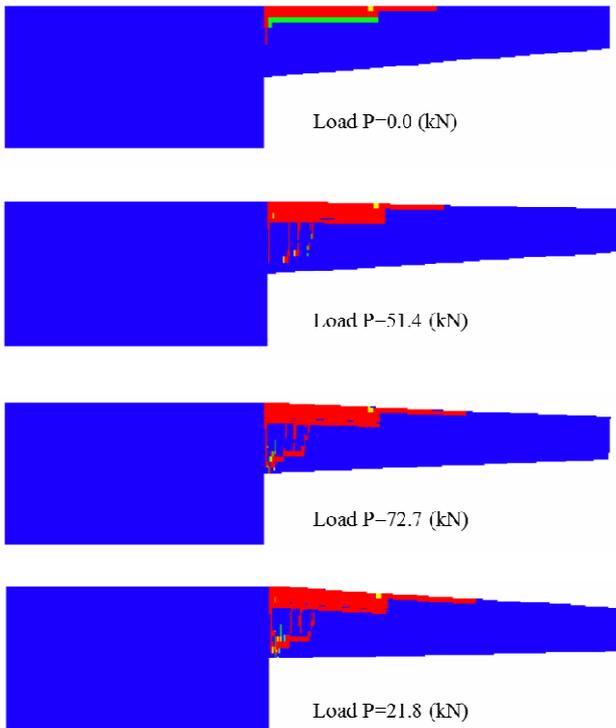


Fig.13 Cracking distribution by F.E.M (after cyclic load)

Fig.8 is a specimen .Fig.9 and Fig.10 are damage states of specimen after quasi-static loading test and previous cyclic loading test, respectively. The CF-sheet breaking mode was found as a final fracture mode. The relationship between load and displacement at loading- point and the stress-strain relationship about CF-sheet are shown in Fig.11.and Fig.12. If the stress reached tensile strength of CF-sheet, The fiber breaking was occurred simultaneously, and the stress was released suddenly. This phenomenon was typical behavior of high strength CF-sheet. It was found that the reinforcement with CF-sheet led to brittle fracture .

The concrete cracking under the CF-sheet as well as the vertical crack at the root have corresponded well with the experimental result as shown in Fig.7 , Fig.9 and Fig.10. The number of vertical cracks at the root is one in the experiment and is two in the analysis. This difference is probably due to the modeling of the steel bars, which is considered as the equivalent layer area model.

The experimental observation for the load-displacement curve, in which the brittle fracture behavior with a sudden load drop takes place after the fracture of the CF-sheet, has also well been simulated as shown in Fig.11. The calculated ultimate load of 82.5 (kN) has agreed well with the experimental value of 77.0 (kN) [11].

The result of same ultimate load after 2-million cyclic loading expressed the sine curve which has 28.3 (kN) of amplitude and 2 (Hz) of frequency as a design traffic load is also shown in Fig.11. The calculated ultimate load of 72.7 (kN) has agreed well with the experimental value of 61.8 (kN). It was found that the ultimate strength due to affection of pre-cyclic load., which may reduce about 12 %the ultimate strength more than that of intact specimen by numerical analysis. And the experimental result was

approximately 20 %bigger than the numerical results. The cracking distribution by F.E.M after 2 million cyclic loading was shown in Fig.13. According to this result, the concrete cracking beneath CF-sheet was different from the experimental result (refer to Fig.10.). But, it was found that the principal cracking was coincident with the experiment. C.D.M is valid method to evaluate the damage due to fatigue loading and also can visualize the meso cracking of concrete structures.

#### 4. ADHESIVE FRACTURE OF CONCRETE BLOCK REINFORCED WITH CF-SHEET

In this Section, The adhesive fracture mode such as de-lamination , surface detachment between the CF-sheet, epoxy resin and concrete were discussed.

##### 4.1 A New Constitutive Equation for shear behavior

Most of ultimate strength of concrete structures reinforced with the CF-sheet has been determined by adhesive fracture mode between the CF-sheet, epoxy resin and concrete surface.

Therefore, a new equivalent stress was introduced to evaluate the mode as follows [12];

$$\sigma_{eq} = \alpha I_1 + (J_2')^{1/2} + \beta \langle \sigma_{max} \rangle + \delta |\tau_{max}| \quad (17)$$

Where the following notations are used:  $\beta$  and  $\delta$ ; the material parameter,  $\langle \rangle$ ; Macaulay's brackets,  $\sigma_{max}$ ; maximum principle stress,  $\tau_{max}$ ; maximum shear stress. Other parameters are same in equation (4).

$\beta$  and  $\delta$  were determined by the adhesive strength test shown in Fig.14. In this test, the adhesive length was changed 20,40,80 (mm) (refer to Table 3).

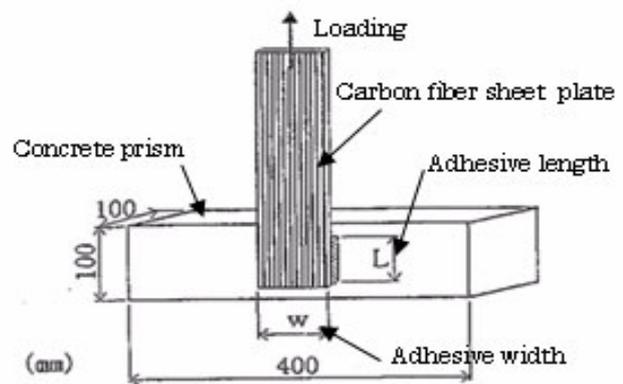


Fig.14 Adhesive strength test

Table 3 Test cases

CASE	Adhesive length L (mm)	Adhesive width W (mm)
CASE1	80	80
CASE2	40	80
CASE3	20	80

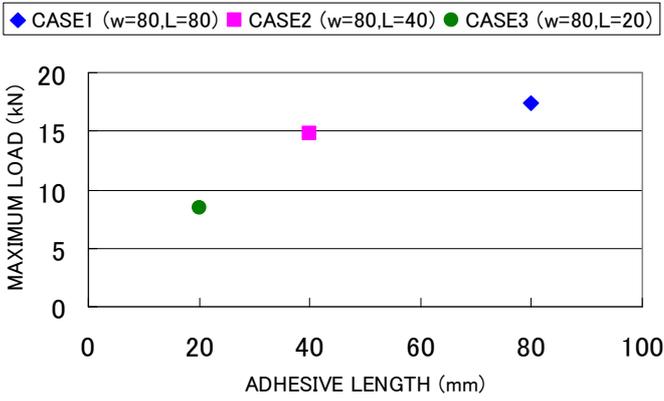


Fig.15 Maximum load and adhesive length

Table 4 Material property

E (Young's modulus)	30,000 (MPa)
$\nu$ (Poisson's ratio)	0.17
$\alpha$ (plastic potential)	0.85
$\beta$ (plastic potential)	0.035
$\delta$ (plastic potential)	0.01
$\sigma_y$ (yield stress)	1.50 (MPa)
$\sigma_f$ (fatigue limit stress)	1.875 (MPa)
K (plastic hardening)	45.0 (MPa)
n (plastic hardening)	0.175
$S_{pf1}$ (damage parameter)	$9.25 \times 10^{-4}$ (MPa)
$S_{pf2}$ (damage parameter)	1.55
$S_{e1}$ (damage parameter)	$4.25 \times 10^{-3}$ (MPa)
$S_{e2}$ (damage parameter)	5.375
$\epsilon_{pD}$ (damage threshold)	0.00
$D_{cr}$ (critical damage)	$7.45 \times 10^{-2}$

The relationship between maximum quasi-static load and adhesive length is shown in Fig.15. The load becomes bigger corresponding to the adhesive length, but the relation is nonlinear. It was found that there is a certain effective length of the CF-sheet to resist the shear deformation in quasi-static loading condition.

#### 4.2 Adhesive Fracture Analysis under Cyclic Loading

The damage evolution would be hypothesized following equation. The plastic damage is adopted the former relation of equation (18) and the elastic damage is adopted the later relation of equation (18), respectively.

$$dD = \left( \frac{Y}{S_{pf1}} \right)^{S_{pf2}} dp, \quad dD = \left( \frac{Y}{S_{e1}} \right)^{S_{e2}} de \quad (18)$$

Where  $S_{pf1}$  and  $S_{pf2}$  are material constants regarding plastic fatigue and  $dp$  is the increment of equivalent plastic strain.  $S_{e1}$  and  $S_{e2}$  are material constants regarding elastic fatigue.

$de$  is the increment of equivalent elastic strain [12].

The CF-sheet and epoxy resin were hypothesized as linear elastic material in this section [12]. The 2-dimensional model for F.E.M and the applied cyclic loading, which has

5 (Hz) of frequency for CASE3 are shown in Fig.16 and Fig.17. Finally, the material property was shown in Table 4.

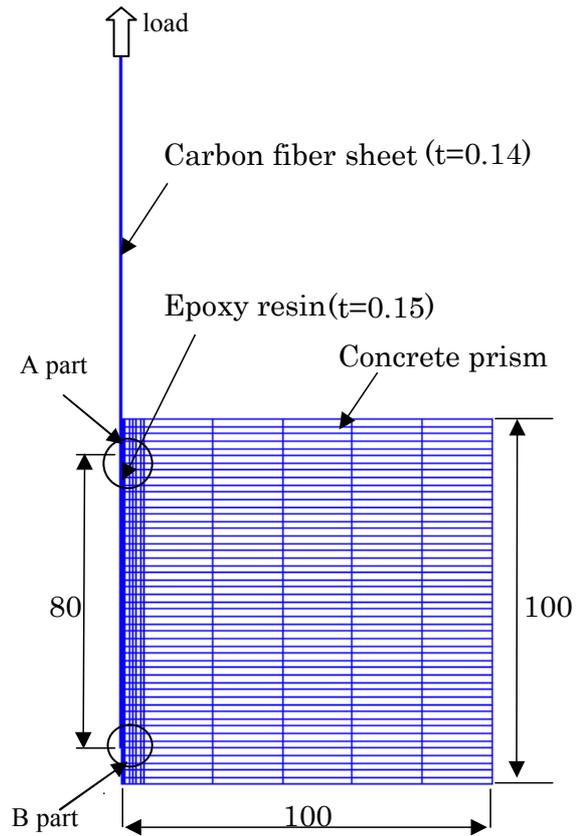


Fig.16 2-dimensional model for F.E.M (unit; mm)

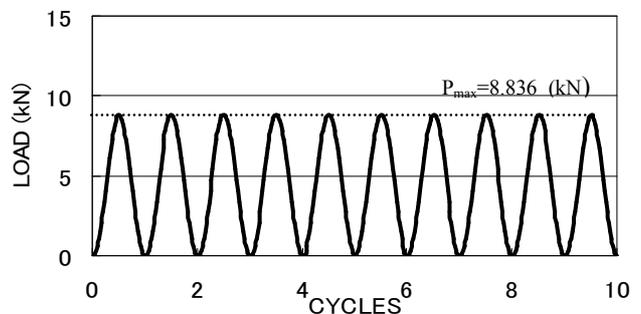


Fig.17 The applied cyclic loading (for CASE3)

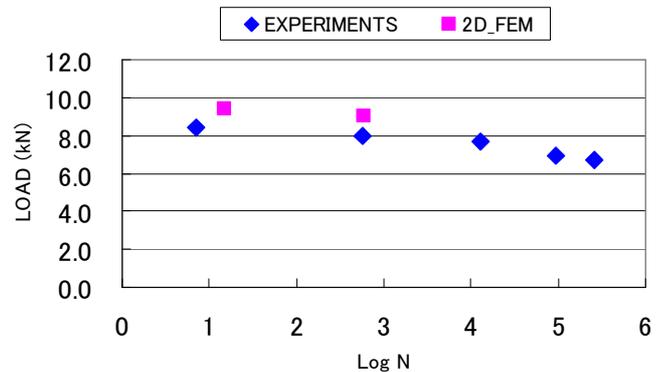


Fig.18 Relation fatigue fracture number and  $P_{max}$  (CASE1)

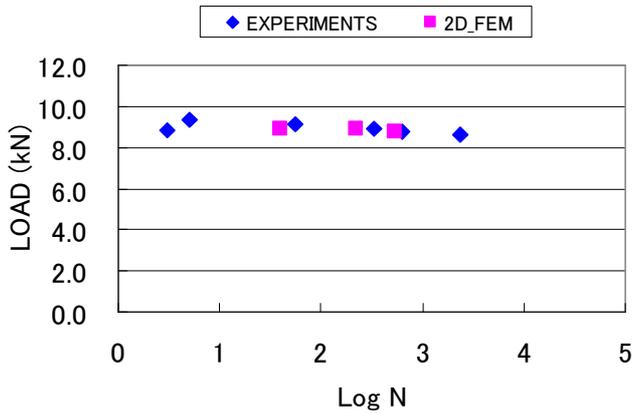
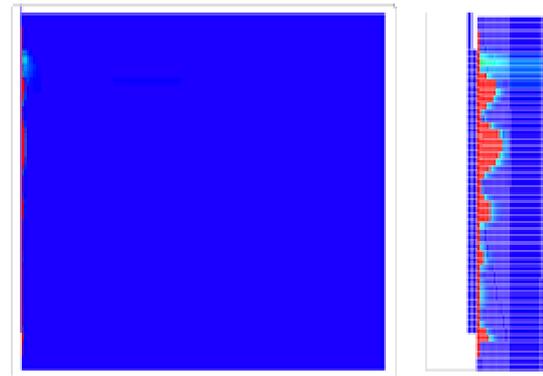


Fig.19 Relation fatigue fracture number and  $P_{max}$  (CASE2)



(On the 15th cycle)

Fig.21 Damage evolution of the concrete surface (CASE1)

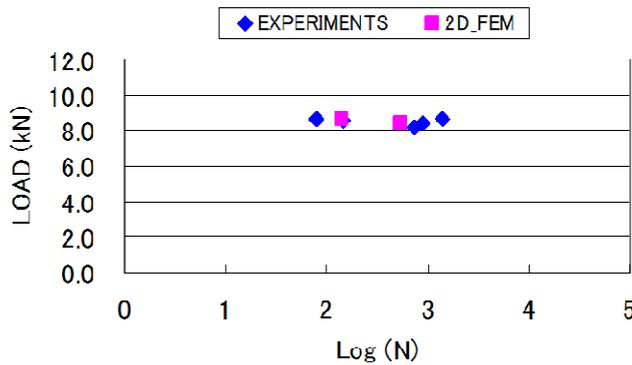
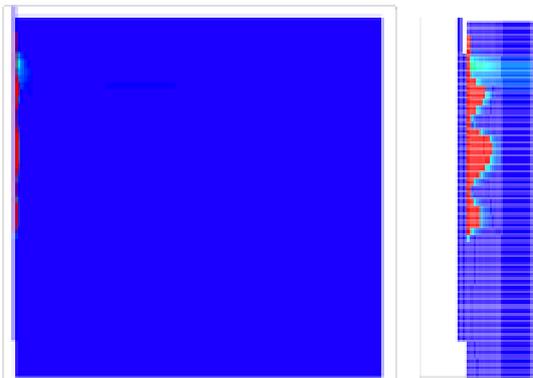


Fig.20 Relation fatigue fracture number and  $P_{max}$  (CASE3)

The relationship between cyclic fracture number and maximum load is shown in Fig.18, Fig.19 and Fig.20. Herein, the diamond and square dots remark the experimental and analytical results, respectively. In these cases, the analytical results are in a good agreement comparable with the experimental results. Finally, the damage distribution is shown in Fig.21. It was found that the adhesive fracture would be evolved from the root (A part) to the tip (B part) corresponding to cyclic number. In Fig.21, the red area reveals the evolution of de-lamination or adhesive fracture. Herein, the damage zone is enlarged to show up the detachment from concrete surface (refer to the right side of Fig.21).



(On the 9th cycle)



(On the 11th cycle)

## 5. CONCLUSION

In this study, the 2-dimensional F.E.M implementing the elasto-plastic damage constitutive equation based on C.D.M has been applied to the damage and fracture analysis of the RC structural member and the block reinforced with CF-sheet.

The following conclusions have been obtained:

- (1) The stress-strain relations of the concrete and the CF-sheet have been successfully identified by the elasto-plastic damage constitutive equation using Drucker-Prager's equivalent stress.
- (2) The calculated damage zones have corresponded well with the experimental crack propagation in the RC slab from which it is seen that the damage mechanics model is valid for the damage evaluation of the concrete structural member reinforced with the CF-sheet.
- (3) The adhesive fracture mode has been identified by an another equivalent stress and which has corresponded well with experimental fatigue fracture number. The validity of present analysis has especially been demonstrated by the fact that adhesive fracture has well been simulated.
- (4) The repair and the maintenance to increase the lifetime are important technologies for concrete structures. The quantitative evaluation of the chemical aging, the mechanical damage accumulation due to earthquakes and fatigue and their coupling behaviors is necessary

for the appropriate maintenance. Therefore, C.D.M will be a valid method to evaluate the lifetime and the deterioration of ultimate ability of concrete structures.

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