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Response of Tapered Piles under Lateral Harmonic Vibrations

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ABSTRACT: This paper presents a new method for analysis tapered piles under lateral harmonic vibration. The behavior of tapered piles is assumed to be as elastic and linear. The soil consists of some elastic horizontal layers that they are homogeneous, isotropic, and linearly visco–elastic. The pile is divided to some segments and the differential equation for a given desirable segment is obtained and solved. Then the dynamic complex stiffness parameters are derived for the pile head. Parametric studies have been performed to investigate the influence of pile geometry, soil properties, and loading details on pile-soil system amplitudes. It has been found that under lateral harmonic vibrations, with increasing the pile taper angle, the resonant amplitude decreases. In addition, it has been concluded that under lateral harmonic vibrations, a tapered pile experiences lower amplitude than a cylindrical pile of the same length and material volume.

Keywords: Tapered Piles, Lateral Harmonic Vibration, Stiffness, Damping

1. INTRODUCTION

There are various methods for studying behavior of dynamically loaded uniform piles. These are continuum approach (Novak, [1]; Novak and Aboul – Ella, [2]; and Nogami, [3]), boundary element method (Kaynia and Kausel, [4]; Sen, Kausel and Banerjee, [5]), lumped–mass method (Penzien, [6]); and finite element solutions (Blaney, Kausel and Rosset, [7]; Wolf and Von Arx, [8]; chow, [9]). Novak presented an approximate continuum approach to account for soil-pile interaction: it is assumed that the soil is composed of a set of independent horizontal layers of infinitesimal thickness, which extend to infinity. As each plane is considered independent, this model may be viewed as a generalized Winkler model. The planes are homogeneous, isotropic, and linearly elastic, and they consider being in a plane strain state. Using Baranov's [10] solution for the horizontal soil reaction to a rigid circular disc with harmonic horizontal displacement (representing a pile cross section), Novak formulated the differential equation of the damped pile in horizontal vibration. He found the steady state (particular) solution for harmonic motion induced through pile ends, and used this solution to find the dynamic stiffness of the pile head for different boundary conditions.

The existent methods are few for analyzing tapered piles under dynamic loads, so recently it has been attracted many investigators. The most studies in this piles, related to vertical harmonic vibrations such as finite difference method (Saha and Ghosh, [11]) and mathematical method (Xie and Vaziri, [12]).

Investigations have been done on tapered piles includes: Full finite element, analytical solution and laboratory tests (Kurian and Moola, [13]), and centrifuge model tests (El Naggar and sakr, [14]). Field load tests were also conducted on tapered piles to investigate their load-carrying capacity (Rybnikov,) and indicate that bored cast-in-place tapered piles can have bearing capacity 20-30% higher than that for cylindrical piles with the same volume and same mean radius. The Ghazavi has also recently performed full-scale tests on a tapered pile driven into a cohesive soil profile in the field. These tests showed that, in long term, the tapered pile had 80% more capacity than a uniform pile of the same volume and length. Zil'berberg and Sherstnev [15] have reported from their field tests that driven tapered piles in sandy soils can give a stiffer and stronger axial response resulting in a 200-250% increase in bearing capacity when compared to the capacity of cylindrical piles with the same volume and mean radius. The response of these piles under lateral static loads was also investigated El Naggar and Wei, [16]. El Naggar and Wei [17] also conducted tests in a pressure chamber on tapered model piles subjected to uplift loads.

Ghazavi et al. [18] described the performance of tapered Pile during Pile driving and Ghazavi and Tavassoli [19] Study of Pile geometry on Pile driving. Also has proceeded analysis of kinematic seismic response of tapered piles (Ghazavi, [20]) and Response of tapered piles to axial harmonic loads and effect of angles on tapered piles has been discussed then obtained has been verified whit finite element methods and they were satisfactory (Ghazavi, [21]).

SSM (Segment by Segment Method) is based on continuum method of Novak elasto- dynamic approach for analysis of piles. The SSM has been applied to uniform piles under axial compressive loads (Ghazavi et al. [22]), uplift static loads (Ghazavi et al. [23]), and axial and lateral harmonic vibrations (Ghazavi, [24], Ghazavi and Dehghanpour [25]) in this paper SSM was applied to analyze dynamic behavior of tapered piles under lateral harmonic loads.

2. ANALYTICAL MODEL

The characteristic effects of surrounding soil on the pile response are determined with stiffness and damping parameters of soil – pile system. These effects can be taken in to account if a proper soil reaction is employed.

For analyzing tapered pile, it was idealized to some cylindrical segments with different diameter that connected together by rigidly at nodes. This idealization was used in tapered piles under harmonic axial vibration [11-12] and [21].

Surrounding soil reaction to the loaded tapered pile segments had been presented by $V_s(z, t)$ and $V_r(z, t)$, that shear resistance per unit length of the pile shaft and horizontal reaction at the horizontal annular projections of the pile shaft, respectively. Parameters z and t represent depth and time in order as shown in Fig l.

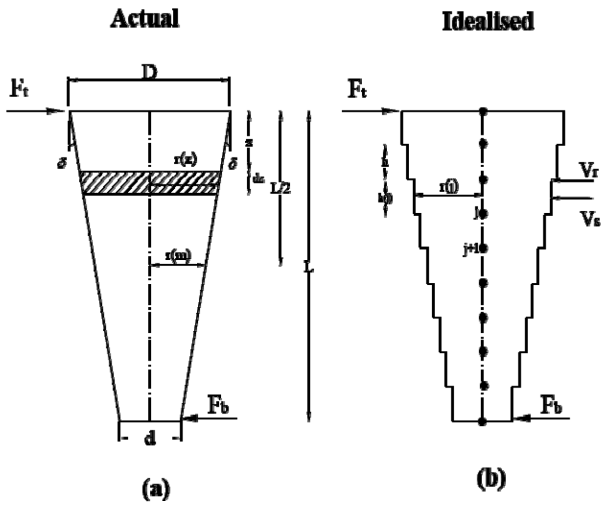

Fig.1 Idealization of tapered pile for dynamic analysis in laterally inhomogeneous media using segment by segment, (a) Actual pile; (b) Idealized pile

The soil response with time to motion of the pile toe, $F_b(t)$, is taken as that of a viscoelastic half – space to rigid, mass less, circular disc of radius r_b undergoing harmonic vibration. This can be expressed as:

$$F_b(t) = G_b r_b [C_{u_1}(a_{\circ b}, \nu, D) + iC_{u2}(a_{\circ b}, \nu, D)] u_b(t) \qquad (1)$$

Where G_b is the soil shear modulus at the pile toe, C_{ul} , C_{u2} are dimensionless complex parameters given in the form of polynomial expressions (Veletsos and Vbric, [26]), $u_b(t)$ is the toe horizontal displacement, r_b is pile radius at the pile tip, $a_{\circ b} = \frac{r_b \omega}{V_b}$, where V_b is shear wave Velocity of soil below the tip, ω is circular frequency, ν is Poisson's ratio, D is material damping.

The soil reaction on the pile is represented by springs and dashpots, which are modeled on elasto – dynamic theory. The interaction of the soil and the pile is then determined for each segment according to the characteristics of soil. This interaction can be demonstrated by a complex displacement, shear force, rotation and bending moment at the end of the adjacent segment. This procedure is performed from the lowest pile segment and extends to the next upper segment. This manner is continued to reach the topmost segment. That is why this procedure is called the SSM (Ghazavi [18-25]).

In the analysis, it is assumed that the soil reaction associated with a given soil layer is identical to that of an infinite rigid pile undergoing a uniform displacement of the same properties as the soil of that layer. This assumption is essential to the solution and will be examined subsequently using other, existing solutions. This assumption has also been used by other researcher [2]. In one dimensional finite element analysis of cylindrical piles under torsional vibrations, Novak and Howell also used the same assumption. A somewhat similar assumption was also made and by Mylonakis and Gazetas [30] for axially loaded cylindrical piles in a layered soil profile. It is note that Novak and Aboul – Ella used finite element method (FE) for analysis of piles embedded in inhomogeneous soil and subject to lateral harmonic vibrations. By considering the typical embedded segment j at depths, shown in Fig 1 and on the basis of the above assumption, the following governing dynamic differential equation of a soil – pile system subjected to harmonic lateral load can be obtained [1]:

$$m_{pj} \frac{\partial^2 u_j(z,t)}{\partial t^2} + C_{pj} \frac{\partial^2 u_j(z,t)}{\partial t} + E_{pj} I_{pj} \frac{\partial^4 u_j(z,t)}{\partial z^4} + G_{sj} S_{uj} u_j(z,t) = 0 \qquad (2)$$

Where m_{pj} the pile mass per unit length, C_{pj} is the damping coefficient of the pile material, $E_{pj}I_{pj}$ is the bending stiffness of the pile segment j, G_{sj} is the shear modulus of the soil layer surrounding the pile segment j , u_j (z, t) is the local time – dependent complex amplitude at depth z from the top of segment j and S_{uj} is Complex dimensionless soil resistance parameter defined elsewhere [1] as a function of Poisson's ratio and dimensionless frequency, $a_{\circ j} = {r_{oj}\omega}/{V_{sj}}$ Here r_{oj} is the pile segment radius, ω is the circular frequency, and V_{sj} the shear Wave velocity of the soil surrounding the pile segment j.

The four terms in (2) represent the inertia force due to lumped mass of the pile, the damping force of pile material, the lateral interaction between pile segments, and the soil resistance, respectively. For harmonic vibration, the local displacement u_j (z, t) is given by:

$$u_j(z,t) = u_j(z)_e^{i\omega t} \qquad (3)$$

Where $u_j(z)$ is the complex amplitude at depth z from the top of segment j and ω is the excitation frequency:

$$u_j(z) = u_{1j}(z) + iu_{2j}(z) \qquad (4)$$

Combining (2) and (3) gives

$$E_{pj} I_{pj} \frac{\partial^4 u_j(z)}{\partial z^4} + u_j(z)[G_{sj} S_{u1} - m_{pj}\omega^2 + i(C_{pj}\omega + G_{sj} S_{u2})] = 0$$

(5)

The above equation can be solved explicitly. The solution for the displacement at a point at vertical distance z below the upper node of segment j is given by:

$$u_j(z) = A_j Cosh(\zeta_j \frac{z}{h_j}) + B_j \sinh(\zeta_j \frac{z}{h_j}) + C_j \cos(\zeta_j \frac{z}{h_j}) + D_j \sin(\zeta_j \frac{z}{h_j})$$

(6)

Where

$$\zeta_j = h_j \sqrt[4]{\frac{1}{E_{pj} I_{Pj}} [m_{pj}\omega^2 - G_{si} S_{uq} - i(C_{pj}\omega + G_{sj} S_{u2})]}$$

(7)

Where A_j, B_j, C_j and D_j are integration constants determined using appropriate boundary conditions.

If the displacement, rotation, shear force and bending moment transmitted by the pile at node 2 of segment j are know, the integration constants A_j, B_j, C_j and D_j can be calculated. Thus, the displacement, rotation, bending moment and shear force at node 1 of segment j are respectively given by (8a-8d).

3. PARAMETRIC STUDIES FOR PILE GEOMETRY EFFECT

In this section, four type of piles whit difference geometry under lateral harmonic load and have same length and volume had been studied and the results are compared. Properties of tapered piles and soil have presented in Table 1. All piles are 10m of length and there volumes are 1.36 m^3. Pile C is cylindrical. Pile T-C consists of a top tapered segment with 5m length and a lower cylindrical segment with 5m length. Pile C-T has top cylindrical part with 5m followed by a tapered part with 5m length. Pile T is tapered. Taper angles of piles are 0.5° and 1.5° that has been shown in Fig 2 and Table 2.

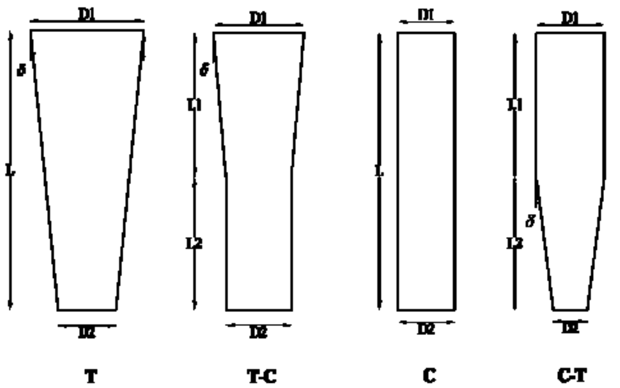

Fig. 2 Pile configurations

$$u_{j1}=\frac{V_{j2}h_j^3(\sinh\xi_j-\sin\xi_j)+\xi_j^2E_{p_j}I_{p_j}\theta_{j2}h_j(\sinh\xi_j+\sin\xi_j)+\xi_j^3E_{p_j}I_{p_j}u_{j2}(\cosh\xi_j+\cos\xi_j)+M_{j2}h_j^2\xi_j(\cos\xi_j-\cosh\xi_j)}{2E_{p_j}I_{p_j}\xi_j^3}$$

$$\theta_{j1}=\frac{V_{j2}h_j^3(\cos\xi_j-\cosh\xi_j)-\xi_j^2E_{p_j}I_{p_j}\theta_{j2}h_j(\cosh\xi_j+\cos\xi_j)+\xi_j^3E_{p_j}I_{p_j}u_{j2}(\sin\xi_j-\sinh\xi_j)+M_{j2}h_j^2\xi_j(\sinh\xi_j+\sin\xi_j)}{2E_{p_j}I_p\,\xi_j^2h_j}$$

$$M_{j1}=\frac{V_{j2}h_j^3(\sinh\xi_j+\sin\xi_j)+\xi_j^2E_{p_j}I_{p_j}\theta_{j2}h_j(\sinh\xi_j-\sin\xi_j)+\xi_j^3E_{p_j}I_{p_j}u_{j2}(\cosh\xi_j-\cos\xi_j)-M_{j2}h_j^2\xi_j(\cosh\xi_j+\cos\xi_j)}{-2\xi_jh_j^2}$$

$$V_{j1}=\frac{-V_{j2}h_j^3(\cosh\xi_j+\cos\xi_j)-\xi_j^2E_{p_j}I_{p_j}\theta_{j2}h_j(\cosh\xi_j-\cos\xi_j)-\xi_j^3E_p\,I_p\,u_{j2}(\sinh\xi_j+\sin\xi_j)+M_{j2}h_j^2\xi_j(\sinh\xi_j-\sin\xi_j)}{-2h_j^3}$$

(8a)- (8d)

Table1. Properties of tapered piles and soil

Radius of equivalent circular pile, r_{eq}^*	0.208 m
Shear wave velocity in soil, V_s	84m/s
Soil Poisson's ratio, ν_s	0.45
Soil unit weight, γ_s	17.5 kN/m^3
Pile modulus of elasticity, E_p	1.962×10^7 kN/m^2
Soil modulus of elasticity, E_s	3.58×10^4 kN/m^2
Pile unit weight, γ_p	25 kN/m^3

(* r_{eq} is radius of cylindrical pile of the same volume and length as tapered pile)

Table 2 Dimension of tapered piles for Fig 3.

<table>
<tr><td>Piles</td><td>Taper Angle</td><td colspan="4">$\delta = 0.5^\circ$</td><td colspan="4">$\delta = 1.5^\circ$</td></tr>
<tr><td colspan="2">T</td><td colspan="2">L=10m</td><td>D_1=0.5m</td><td>D_2=0.326m</td><td colspan="2">L=10m</td><td>D_1=0.65m</td><td>D_2=0.126m</td></tr>
<tr><td colspan="2">C</td><td colspan="2">L=10m</td><td>D_1=0.416m</td><td>D_2=0.416m</td><td colspan="2">L=10m</td><td>D_1=0.416 m</td><td>D_2=0.416m</td></tr>
<tr><td colspan="2">TC</td><td>L_1=5 m</td><td>L_2=5 m</td><td>D_1=0.48m</td><td>D_2=0.393 m</td><td>L_1=5 m</td><td>L_2=5 m</td><td>D_1=0.603 m</td><td>D_2=0.342 m</td></tr>
<tr><td colspan="2">CT</td><td>L_1=5 m</td><td>L_2=5 m</td><td>D_1=0.4369 m</td><td>D_2=0.3497 m</td><td>L_1=5 m</td><td>L_2=5 m</td><td>D_1=0.4729 m</td><td>D_2=0.2111 m</td></tr>
</table>

SSM was applied with assumption pinned–ended for all of above piles and Fig. 2 illustrate the dimensionless amplitude lateral versus the excitation frequency for piles with taper angles $\delta = 0.5^{\circ}$ that results have been shown in Fig. 3 the T pile have the least lateral and rotary response amplitude and after T Pile. There are T –C, C-T and C respectively.

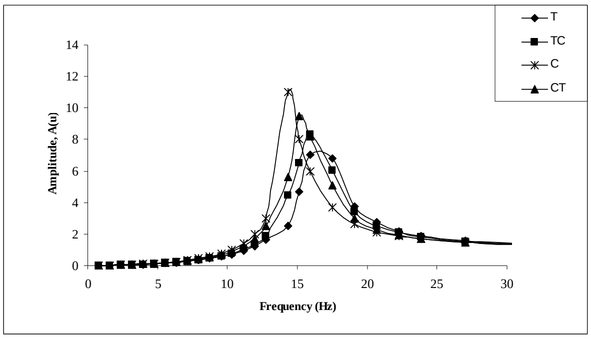

Fig. 3 Variation of lateral dimensionless amplitude versus frequency for T pile ($\delta = 0.5^{\circ}$)

It is noted that in the C piles, the resulted values of are exactly the same as those reported by Novak [1].
In Fig. 4, the dimensionless amplitude lateral versus the excitation frequency had been compared based on taper angle in T pile and observed with increasing taper pile, dimensionless response amplitudes will decrease.

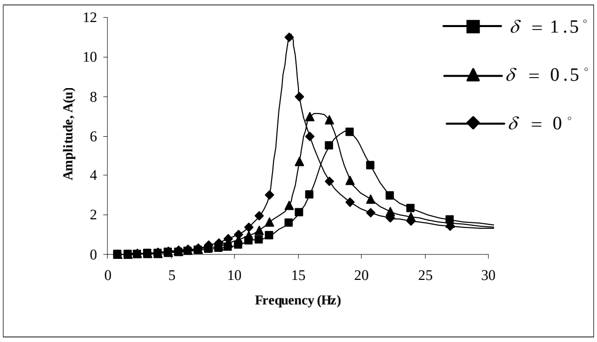

Fig. 4. Comparison variation of lateral dimensionless amplitude versus frequency for T pile according to taper angle

5. CONCLUSION

A simple approach, called SSM, has been presented in this paper for determination of stiffness and damping parameters of laterally loaded tapered piles subjected to harmonic vibrations. The soil-pile interaction in this method is modeled within each segment and applied via the segment nodes to the analysis of the adjacent segment. Therefore, the stiffness and damping parameters for the whole pile-soil system are determined. According to results SSM, T, TC, CT and C piles respectively have the least lateral and rotary response amplitude and observed that for tapered piles of the same volume and length under lateral harmonic vibrations, with increasing the taper angle, the resonant frequency increases slightly. However, the reduction of the amplitude is more pronounced.
The SSM is an efficient and simple method for analysis of tapered piles under harmonic vibration. In particular, the effects of the soil in homogeneity in the vertical direction even with complicated stratifications can be easily captured. This method involves less computational work than available numerical method based on the FE.

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