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INTERNAL EROSION OF VOLCANIC COARSE GRAINED SOILS AND ITS EVALUATION

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ABSTRACT: This paper aims at revealing mechanical behavior of volcanic coarse grained soils subjected to seepage flow. In order to accomplish the purposes, a series of upward seepage tests was conducted to grasp piping phenomenon in compacted volcanic soils and to investigate the effects of differences in compaction conditions on its behavior. In the experiments, the movement of soil particles in seepage flow tests was observed using an X-ray CT scanner in detail. The test results showed that destabilization of soil structures due to seepage flow is changed depending on an increase of amount of finer soil particles, and that internal erosion is induced by loss of fine particles with the changes in void ratio. As a result, a significant variation in hydraulic conductivity was generated. Additionally, internal stability of volcanic soils under several geotechnical conditions was elucidated by empirical criterion. In the consideration of the results, it was shown that volcanic coarse grained soil including pumice particles with a low specific gravity was internally unstable. Based on the results, a geotechnical evaluation was discussed for the stability of soil structures such as embankments constructed by volcanic coarse grained soils.

Keywords: Volcanic coarse grained soils, Seepage flow, Erosion, Model test

1. INTRODUCTION

In Hokkaido Japan, there are over forty Quaternary volcanoes, and pyroclastic materials cover over 40 % of its area. Sedimentary structure, components, distributional areas and degrees of the weathering greatly differ with the depositional environment. Therefore, it is anticipated that the mechanical property of volcanic soil grounds will be diverged [1]. Such volcanic soils have been also used as useful construction materials, especially man-made earth structures (embankments and cut slopes, etc.). However, a large number of earthquake- and rainfall-induced failures of artificial slopes such as cut slopes or embankments have been reported in Hokkaido, Japan [1].

In this study, seepage performance of volcanic coarse grained soil and a stability evaluation for seepage flow-induced internal erosion were investigated. It has been well known that internal erosion leads to an increased hydraulic conductivity and induces a reduction of soil strength [2]. In addition, the transport of fine particles in soil matrix results in piping phenomenon or in collapse of soil structures [3]-[5].

The final goals of this study are to grasp seepage performance of compacted volcanic coarse grained soils such as embankments and to clarify the effects of geotechnical conditions (grain size distribution and fines content etc.) and compaction conditions (degree of compaction) on piping mechanisms of volcanic coarse grained soils. Firstly, a stability evaluation of volcanic soils was investigated based on empirical geotechnical criteria. Thereafter, a series of model tests was performed compacted volcanic soils using a conventional piping test apparatus. Finally, a simple method to evaluate critical hydraulic gradient was proposed on volcanic coarse grained soils.

2. TEST MATERIALS AND EVALUATION OF INTERNAL STABILITY

2.1 Test materials

Volcanic coarse grained soil erupted from the Shikotsu caldera in Sapporo city, Hokkaido was used in this study. The sample was specified into pumice flow deposits (the notation is *Spfl*), and was referred to as Komaoka volcanic soil ($K_{original}$ soil and K_{soil}). The sampling site is depicted in Fig.1. Index properties and grain size distributions of test materials and of Toyoura sand are shown in Table 1 and Fig.2, respectively. As shown in the table, specific gravities of test materials are lower than that of Toyoura sand. The fines content of $K_{original}$ soil range from 26.0 % to 42.6 %. The fines was classified into non-plastic material (N.P.) according to Atterberg limits (liquid limit is 44.5%, plastic limit is N.P.). On the other hand, K_{soil} means soil materials composed of soil particles smaller than 9.5 mm in original grain size. In addition, $K_{1.9}$, $K_{8.5A}$, $K_{8.5B}$, K_{40A} and K_{40B} are the gradation-controlled materials using K_{soil} to evaluate the effect of finer fractions on hydro-mechanical characteristics, where the subscript of 1.9, 8.5 or 40 means each fines content Fc (%), and the subscript of A and B indicates that test materials with A are controlled only by fines content and materials with B are controlled by not

Fig.1. Location of sampling site

Table 1. Index properties of test materials

Sample name	ρ_s (g/cm^3)	D_{50} (mm)	U_c	F_c (%)
$K_{original}$	2.48	0.27	43.0	26.0~42.6
K_{soil}	2.48	0.27	62.5	27
$K_{8.5A}$	2.48	0.50	10.6	8.5
$K_{8.5B}$	2.48	0.27	6.30	8.5
K_{40A}	2.48	0.16	42.8	40
K_{40B}	2.48	0.27	71.4	40
$K_{1.9}$	2.48	0.60	7.08	1.9
Toyoura	2.68	0.18	1.50	0

D_{50} : Mean grain size, Uc : Coefficient of uniformity

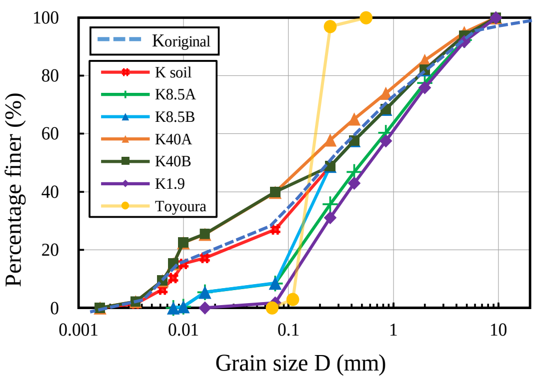

Fig. 2 Grain size distributions of test materials

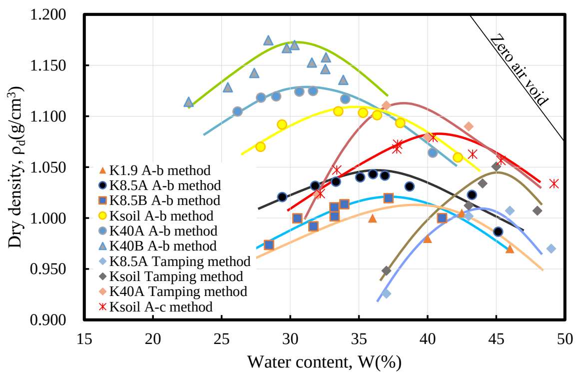

Fig.3 Compaction curves of Komaoka volcanic soils

only fines content but also mean grain size D_{50}, as shown in Fig.2. In this study, six kinds of volcanic soil were adopted as test materials for a series of upward flow tests. Fig.3 depicts compaction curves of Komaoka volcanic soils by a tamping method compared to those obtained from the A-b and A-c method of Japanese Geotechnical Society [6]. In the tamping method, test specimens were compacted using a tamper to four soil layers. The A-b, A-c method is the same compaction effort (=550 kJ/m^3) and the number of compactions and the degree of compaction by the tamping method are summarized in Table 2.

2.2 Stability evaluation of test materials based on empirical criteria

A large number of researches has been reported on internal erosion depending strongly on geotechnical conditions. For instance, an empirical criterion for evaluation of internal instability was presented by Kezdi A [7] based on the conception of filter rule [8]. Similarly, an evaluation to assess internal stability of granular soils was proposed by Kenney TC and Lau D [9], [10]. These criteria proposed by Kezdi A [7], Kenney TC and Lau D [9], [10] were synthesized by Li M and Fannin RJ [11]. Instability of soils due to internal erosion of soils

Table 2. Number of compactions and degree of compaction for each soil.

Sample notation	Desired dry density (g/cm^3)	Weight of roller (N)	Thickness of compacted layer (mm)	Number of compactions (per 1 layer)	Degree of compaction (%)
	1.048	2.65	50	120	95
K soil	0.972	2.65	50	75	88
	0.918	2.65	50	42	83
$K_{1.9}$	0.972	2.65	50	170	98
$K_{8.5A}$	0.972	2.65	50	130	93
$K_{8.5B}$	0.972	2.65	50	160	96
K_{40A}	0.972	2.65	50	32	86
K_{40B}	0.972	2.65	50	24	83

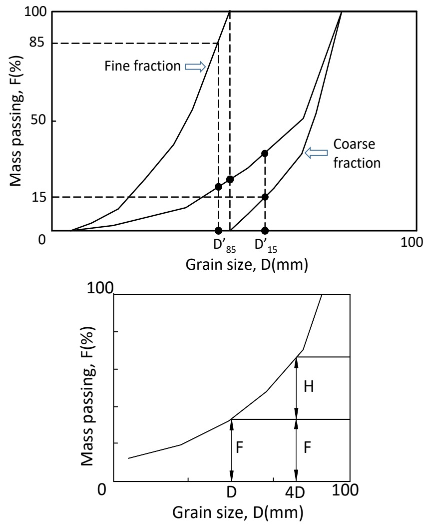

Fig.4 Stability evaluation by empirical criteria: (a) criterion by Kezdi A [7], (b) criterion by Kenney TC and Lau D [9], [10]

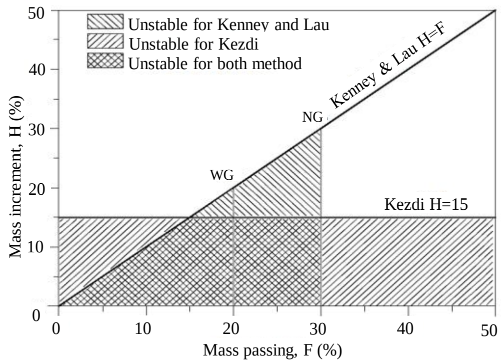

Fig.5 Synthesis of Kezdi A's and Kenney TC and Lau's criteria (Li M and Fannin RJ [11])

based on critical hydraulic gradient was also discussed in comparison with theoretical theory of Skempton AW and Brogan JM [12]. In the consideration of their researches, sand and gravels have been adopted as test materials. On the other hand, volcanic coarse grained soils containing a significant amount of fine particles with a low specific gravity [1], [13] have not been clarified on internal erosion or seepage performance yet although volcanic soils have been used as useful for construction materials.

In the previous study [14], a geotechnical evaluation on internal stability of volcanic coarse grained soils was investigated based on empirical geotechnical criteria [7]-[10]. For instance, evaluation methods on the internal instability of soil structures due to the piping phenomenon suggested in [7]-[10] are shown in Fig.4. According to Kezdi A criterion [7], the grain-size distribution curve was divided into coarse fraction and fine fraction at a random grain diameter and the coarse fraction worked as a filter for the fine fraction. As a result, soils were

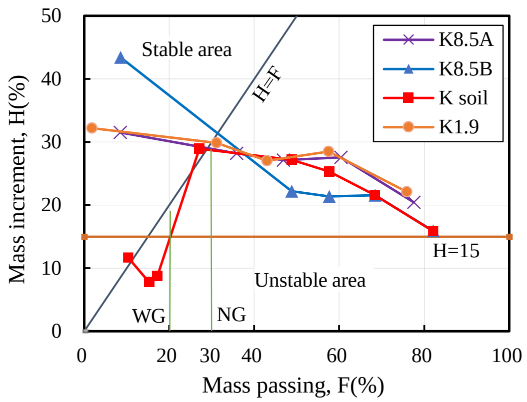

Fig.6 Internal stability assessment for Komaoka soils based on the empirical criteria [11]

evaluated as internally unstable if the maximum value of filter ratio of D'_{15}/D'_{85} was larger than four. In this criterion, D'_{15} is the diameter of 15% finer by weight in the coarse fraction, while D'_{85} is the diameter of the 85% finer by weight in the fine fraction (see Fig.4 (a)). In the Kenney TC and Lau D criterion [9], [10], an evaluation method of internal stability was proposed in terms of the mass passing H of grains having a diameter between D and 4D versus the mass passing F at grains size D, as shown in Fig.4(b). Soils were classified as internally unstable if the grading curve lay below a boundary by H = F and within $0 < F < 30(\%)$ for a primary fabric soil that was narrowly graded (NG) and $0 < F < 20(\%)$ for soils with a primary fabric which was widely graded (WG), respectively. In the [11], it was also suggested that "limit values to stability of $D'_{15}/D'_{85} = 4$ and $H/F = 1$ yield a unique point on the gradation curve where both criteria converge at H=15%". The comparison between two criteria is summarized in Fig.5.

Fig.6 summarizes the results of Komaoka volcanic soils based on the empirical criteria proposed by Li M and Fannin RJ [11]. It was apparent from that K_{soil} is classified into "well-graded type (WG)" under the original condition is judged to an unstable grading because its portion of shape curve lays in an unstable area. Conversely, the shapes of curves of $K_{8.5A}$, $K_{8.5B}$ and $K_{1.9}$ materials located in the stable zone of internally stable soil. On the other hand, K_{soil} with a fines content of more than 20% was classified into internal stable under Chang DS and Zhang LM criterion [15]. Furthermore, according to Skempton AW and Brogan JM [12], if finer fractions of soils exceed about 35%, coarse soil particles float in a matrix of fine grains. As a result, they cannot act as a filter for the fines. In the cases of K_{40A} and K_{40B}, coarse particles can be floated in a matrix of fine grains. Therefore, the internal instability of soils will proceed gradually by moving of volcanic fine particles having a low specific gravity under seepage flow.

3. TEST APPARATUS AND TEST PROCEDURES

3.1 Test apparatus

A series of seepage flow tests using a conventional cylindrical apparatus with ϕ=100mm in diameter and 300mm in height and X-ray CT scanner was conducted to observe piping phenomenon (see Fig.7). Thereafter, the similar flow tests were carried out using a cylindrical cell having ϕ=150 mm in diameter and 450 mm in height to clarify the effect of fine particles and compaction conditions on piping phenomenon (see Fig.8 (a)). As aforementioned, test specimens were prepared by a tamping method (see Fig.8 (b)).

3.2 Test Procedures

Test specimens were prepared by changing grain size distribution of $K_{original}$ soil so as to achieve the desired values in Fig.2 and Table 1. After preparation of specimens, water was permeated into the specimens from porous disks on the bottom at a small differential head for eight hours so as to free from

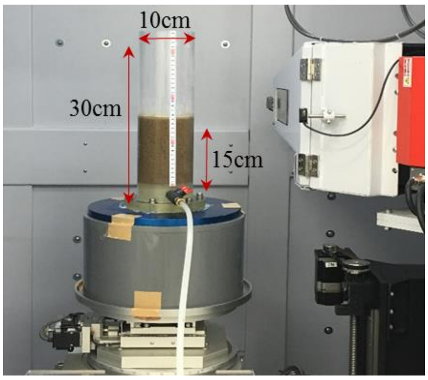

Fig.7 Test apparatus with ϕ=100mm in diameter

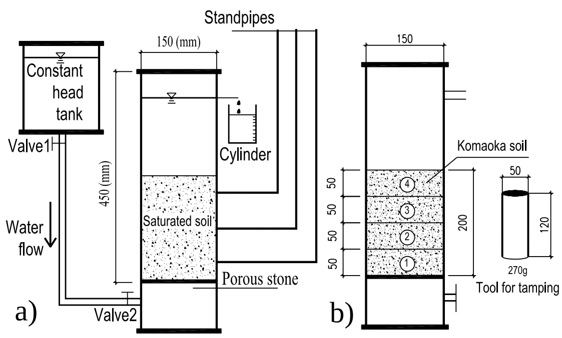

Fig.8 Schematic diagram of seepage flow test: (a) test apparatus (ϕ150mm), (b) preparation of test specimen

disturbance of initial fabric. In a series of upward seepage flow tests, the hydraulic gradient of 0.1-0.2 increased for each step. Variation in water head within the specimen was measured by three stand pipes at three different depths. For each step, a hydraulic gradient was kept after water level measurements of stand pipes were a constant (around 20 minutes). The tests were performed until boiling phenomenon to ensure internal erosion.

In this study, the onset of internal instability was primarily estimated on the basis of three attributes, namely the change in differential coefficient of the relationship among flow velocity (v) and hydraulic gradient (i), visual observation and using X-ray CT scanner. A flow velocity was directly measured using a graduated cylinder (accuracy is 10 ml).

4. OBSERVATION AND TEST RESULTS DISCUSSION

4.1 Observation on piping phenomenon of volcanic soils

Fig.9 shows typical photos for piping and boiling phenomena. In the preliminary experiment, a reliability of test apparatus used in this study was confirmed on Toyoura sand compared to that reported by Yoshimi Y et al. [16].

The definition of piping phenomenon was determined based on 2-dimensional Digital Image Correlation analysis using the X-ray CT images, DIC [17]. For example, Fig.10 (a) shows an example of X-ray CT image of model test with YZ vertical cross section which has an image resolution of 1504 x 1504 pixels. Using the X-ray CT images, deformation of soils was estimated by DIC analysis. Deformations were tracked using interrogation windows of 50 x 50 pixels (approximately 3.72 x 3.72 mm square) that were located on a grid with 20 pixels spacing (approximately 1.488 mm). Fig.10 (b) shows distribution of deformation vectors at YZ section calculated using the images of i=0.5 to 0.6. The movements of soil particles due to seepage flow can be clearly estimated.

Fig.11 depicts the relationship between

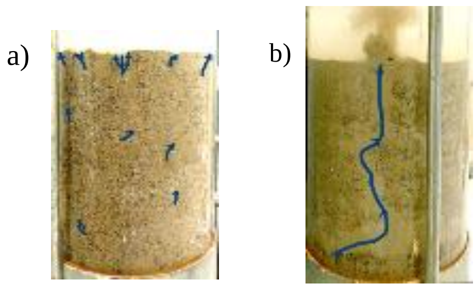

Fig.9 Observed particle migrations (a) Piping phenomenon, (b) Boiling phenomenon

percentage of soil particles movement based on DIC analysis and hydraulic gradient of K soil for Dc=85%. The percentage means the ratio of all movements of soil particles in YZ sections to the area of whole section, and is calculated by integrating soil movements in eight section layers (11 mm /a diameter). The A in the figure is the distance of particle movement, its range is determined as “0 to 0.075mm” or “more than 0.075mm” based on the maximum size of fine particles in the materials.

From Fig.11, it is evident that fine particles at the first stage of *i*=0.1 to 0.2 were gradually moved by water flow in the specimen. For example, 76.9%-84% of particles were moved for the range of 0 <A< 0.075 mm and A>0.075mm. Thus, piping phenomenon occurred in model test at around *i*=0.1. It is also clear from *i*=0.3 to *i*=0.6 that migration of particles due to water flow increased with the increase in hydraulic conductivity.

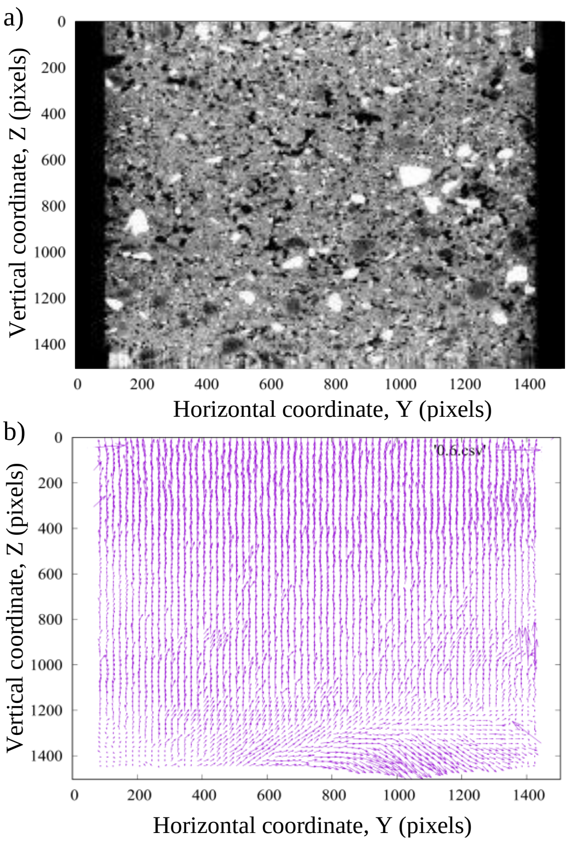

Fig.10 X-ray CT images and their DIC analysis: (a) internal image of model test, (b) particles movement at i=0.6

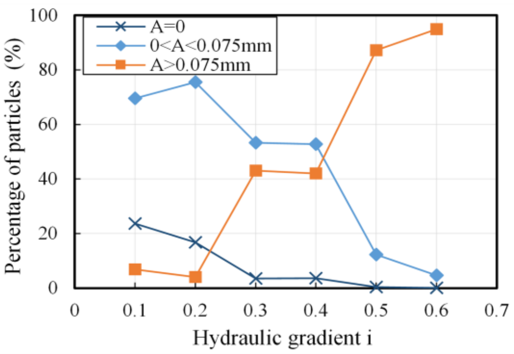

Fig.11 Diagram of distance of particle movement, A based on DIC analysis

4.2 Seepage performance of compacted volcanic soils

Fig.12 shows the relationship between hydraulic gradient and flow velocity, hydraulic conductivity (permeability) of K_{soil} with the different degree of compactions are 83%, 88% and 95%. As shown in this figure, internal erosion occurs at *i*=0.2 in all the samples. The erosion process generates the movement of fine particles within pores of coarse particles. In the K_{soil} with the compaction degree of 83%, a little fine particles was lost from the test specimen at *i*=0.2. This induces an increase of void ratio with an increase of hydraulic conductivity until *i*=0.4. An approximate linear relationship between hydraulic gradient and hydraulic conductivity was also recognized from *i*=0.4 to *i*=0.8, and internal erosion was not observed at that time. Contrarily, the permeability of the K_{soil} with D_c=88% and 95% decreased until *i*=0.4. This can be explained by a decrease in pore size in the specimens due to halting of fine particles. As a result, it works as parts of the filter fabric. Thereafter, fine particles were washed out by seepage flow for over *i*= 0.4. This means an increase in effective porosity, namely an increase of permeability (see Fig.12(b)). After that, “boiling” was induced.

4.3 Influence of fines on piping phenomenon

Particle crushing of volcanic coarse grained soils is significant for an evaluation of mechanical behavior of man-made soil structures and affects seepage performance. Actually, it has been reported that fines content for Komaoka volcanic soils under original conditions ranges from 26.0% to 42.6%, and volcanic soils generated from Shikotsu caldera indicate a high crushability [13]. If particle crushing is induced due to external forces, fine particles will increase gradually. In order to clarify the influence of fine particles on the piping phenomenon, a series of seepage flow test was similarly performed on $K_{8.5A}$, K_{soil} and K_{40A} soils.

Fig.13 shows the results of each test case. As shown in the figures, an increment of fine particles in Komaoka soil affects seepage performance of the compacted soils. For example, piping phenomenon occurred at early stage (at *i*=0.2 with K_{40A}, $K_{8.5A}$, Ksoil and at *i*=0.8 with $K_{1.9}$), and values of critical hydraulic gradients at boiling phenomenon for $K_{1.9}$, $K_{8.5A}$, Ksoil and K_{40A} were 1.4, 1.4, 1 and 0.6, respectively (see Fig.13 (a)). This is due to the difference in the degree of compaction. Because the maximum value of dry density ($\rho_{d\ max}$) of the compacted soils is changed depending on the amount of fines content. In addition, the variation in hydraulic

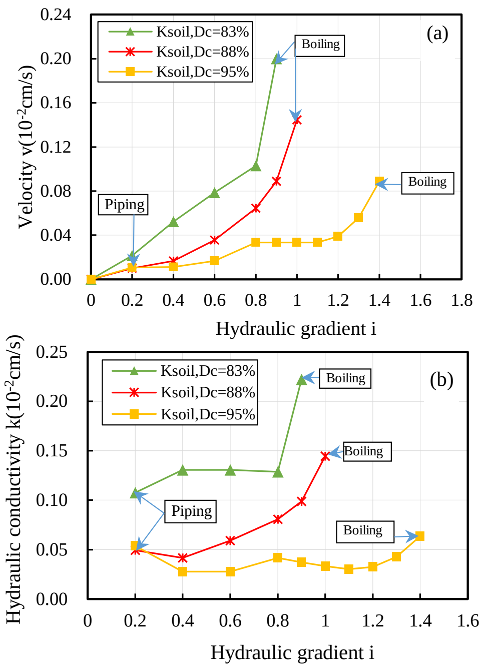

Fig.12 Change in results of K_{soil} with different compaction degrees: (a) velocity, (b) hydraulic conductivity.

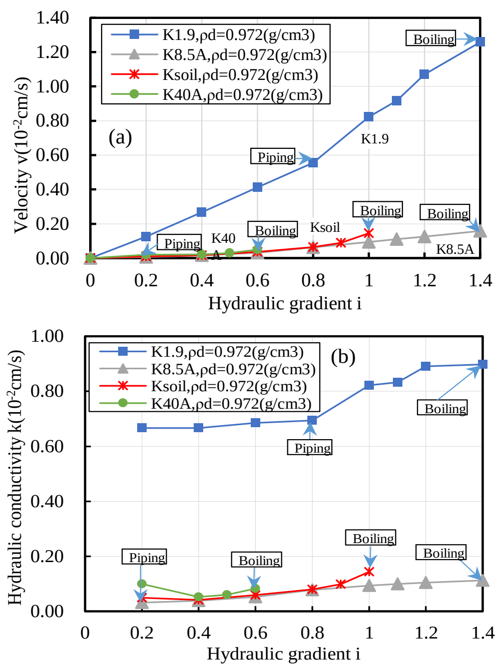

Fig.13 Test results of K_{soil}, $K_{8.5A}$ and K_{40A}: (a) velocity, (b) hydraulic conductivity.

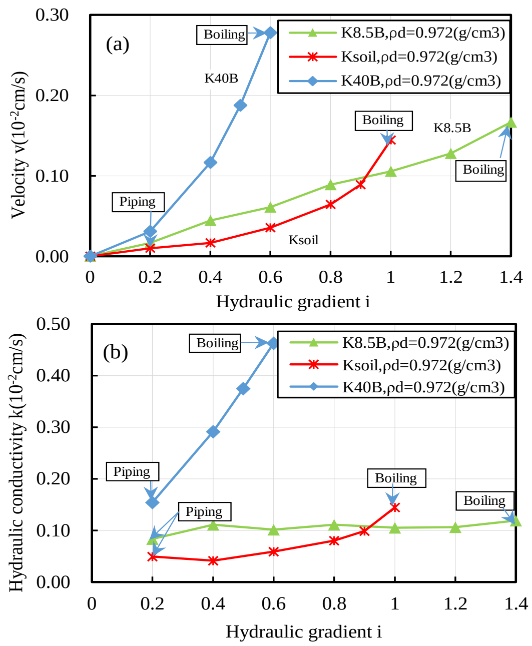

Fig.14 Test results of K_{soil}, $K_{8.5B}$ and K_{40B}: (a) velocity, (b) hydraulic conductivity.

conductivity with difference in fines content is also confirmed for $K_{1.9}$, $K_{8.5A}$, K_{40A} and K_{soil} (see Fig.13 (b)), this can be explained by fine particles are clogged into coarse grained particles, and are released. Consequently, the loss of fine particles seems to increase or decrease hydraulic conductivity. In the piping test for $K_{1.9}$ sample, a velocity of flow is higher than other cases. The reason is that pore network in coarse particles cannot hunt fine particles.

On the other hand, there are differences in velocity and hydraulic conductivity for $K_{8.5A}$, K_{40A} compared to those for $K_{8.5B}$, K_{40B} (see Fig. 14). This indicates an influence of an absence of intermediate particle sizes.

From the results, it can be said that the estimations of the amount of fine particles and grain distribution curves are very important, especially for volcanic soils including soil particles with low specific gravity.

4.4 Evaluation of internal stability of Komaoka volcanic soils using critical void ratio

In general, a number of volcanic soils contains a relatively high amount of pumice soil particles. Nakata T and Miura S [13] investigated that features of volcanic coarse grained soils to characterize its mechanical properties. In particular, volcanic soil particles containing void inside were modeled, as shown in Figs.15 and 16. According to their results, volcanic coarse grained soils in Hokkaido were composed of "opening intra-particle void" under a high rate, which leads to be particle breakage and

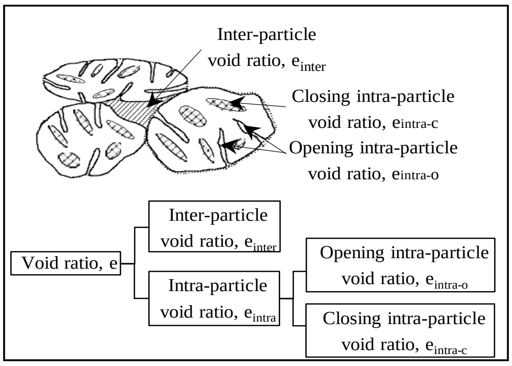

Fig. 15 Schematic of void ratio in volcanic soil (revision of [13])

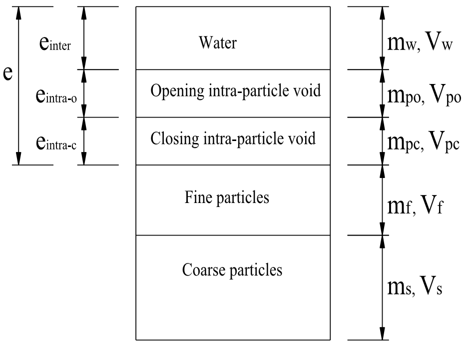

Fig.16 Schematic of saturated volcanic soil model

changes in seepage performance. Therefore, it is important to evaluate the effect of critical void ratio on hydraulic mechanical properties of volcanic coarse grained soils. Kimura M et al. [18] indicated the relationship between opening intra-particle void ($e_{\text{intra-o}}$) and dry density of Komaoka volcanic soil (*Spfl*).

Based on their results, the actual void ratio was evaluated for internal stability of Komaoka volcanic soils under seepage flow. The following Eqs. to calculate soil parameters on void ratios for Komaoka volcanic soils are proposed;

$$\mathrm{e} = e_{inter} + e_{intra-o} + e_{intra-c} = (V_w + V_{pc} + V_{po})/(V_f + V_s) \quad (1)$$

where e is void ratio, $e_{\text{intra-o}}$ is opening intra-particle void, $e_{\text{intra-c}}$ is closing intra-particle void, as shown in Figs.15 and 16. Evaluation of both opening and closing intra-particle voids will be significant for piping phenomenon. In this study, an effective void ratio e' was adopted under seepage flow.

$$e' = e_{inter} = e - e_{intra-o} - e_{intra-c} \quad (2)$$

According to [18], these parameters was empirically given as follows;

$$e = 1.78(\rho_d)^{-0.652};\ e_{intra-c} = 0.04;$$
$$e_{intra-o} = 1.49(\rho_d)^{-0.195} \quad (3)$$

As a result, effective critical hydraulic gradient is expressed follows;

$$i_{c\_intra} = \frac{G_s - 1}{1 + e'} \quad (4)$$

From Eq. (4), it is apparent that the effective critical hydraulic gradient $i_{\text{c\_intra}}$ is larger than those for theoretical ones, as shown in Table 3.

Based on test results, an experimental critical hydraulic gradient $i_{\text{c\_exp}}$ for each test case is summarized in Table 3 compared with theoretical critical hydraulic gradient i_{c} and effective critical hydraulic gradient $i_{\text{c\_intra}}$. As shown in Table 3, i_{c} in some cases (K_{40A} and K_{40B}) is almost the same $i_{\text{c\_exp}}$. On the other hand, $i_{\text{c\_intra}}$ of K_{soil}, $K_{1.9}$, $K_{8.5A}$, and $K_{8.5B}$ are similar to $i_{\text{c\_exp}}$. From the results, it can be said that effective void ratio is available for evaluation of boiling phenomenon for the cases less than 40% of finer contents. For more than 40%, theoretical method will be useful. At any rate, several evaluations will be required for investigation of stability of volcanic soil structures such as embankments.

Table 3 Critical hydraulic gradient for each case.

Sample name	Void ratio (e)	Effective void ratio (e')	Effective critical hydraulic gradient ($i_{\text{c\_intra}}$)	Theoretical critical hydraulic Gradient (i_{c})	Experimental critical hydraulic gradient ($i_{\text{c\_exp}}$)
K soil	1.81	0.27	1.17	0.53	1.0
	1.73	0.25	1.18	0.54	1.4
	1.89	0.37	1.08	0.51	0.9
$K_{1.9}$	1.81	0.27	1.17	0.54	1.4
$K_{8.5A}$	1.81	0.27	1.17	0.54	1.4
$K_{8.5B}$	1.81	0.27	1.17	0.54	1.4
K_{40A}	1.81	0.27	1.17	0.54	0.6
K_{40B}	1.81	0.27	1.17	0.54	0.6

5. CONCLUSIONS

From the results of series of piping test, the following conclusions were derived;
1) Komaoka volcanic soil under the original condition classified into "WG" was evaluated to an unstable grade on piping phenomenon based on empirical criteria.
2) Piping phenomenon early occurs in the Komaoka volcanic soil ranging from i= 0.1 to i= 0.2. However, after the onset of internal erosion, movements of fine particles were clogged in pore network of particles. Thereafter, fine particles was washed out by water flow for larger hydraulic gradient.
3) Piping phenomenon behavior significantly differed depending on compaction conditions.
4) Critical hydraulic gradients in volcanic soils were estimated by theoretical ones depending on the amount of fine particles in the soils. It is important to grasp the amount of fine particles and the shapes of grain distribution curve, especially in volcanic soils including soil particles with low specific gravity and intra-particle void.

6. ACKNOWLEDGEMENTS

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