# QUANTUM PARTICLE SWARM OPTIMIZATION FOR ECONOMIC DISPATCH PROBLEM USING CUBIC FUNCTION CONSIDERING POWER LOSS CONSTRAINT

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**ABSTRACT:** In this paper, quantum computing (QC) inspired particle swarm optimization (QPSO) technique is utilized to solve economic dispatch (ED) problem, which has strong, robust and reliable search capability with powerful convergence properties. Here, authors use cubic criterion function to represent ED instead of the traditional quadratic function, to make the system robust against nonlinearities of actual power generators. Power balance, power loss and generator limit constraints are considered in this research work. To show the efficiency and robustness of the proposed method, authors have compared the obtained results with other algorithms like PSO and GA for ED problem on 3-unit and 5-unit power generating systems. The obtained results demonstrate QPSO's superiority over other methods in terms of providing quality solutions with significant amount of robustness and computationally efficiency.

*Keywords: Economic dispatch, Quantum particle swarm optimization, Cubic function, Power loss, Optimization, Quantum computing.* 

#### 1. INTRODUCTION

Economic dispatch (ED) is one of the most crucial problems in power generation system. The objective of ED is to find an optimal combination of power generation in order to minimize the total production cost satisfying all other constraints [1]. Many assumptions have been made in order to optimize ED in power systems and these assumptions are sometimes impractical to real systems and can no longer be considered. These assumptions are later ignored by the researchers and instead they consider different constraints such as transmission loss, generator limit constraint, emission of pollutants, uncertainty, reactive power dispatch, ramp rates, and integration of renewable energy (RE) generators [2], [3]. In this work, for simplicity, authors consider only three objectives such as power balance, transmission loss and generator limit constraints.

A significant amount of research have been done to get optimal solution for economic dispatch problem in power generation system. Various classical methods e.g. lambda iteration [4], gradient approach [5] etc. have been used to solve ED problems. But, these conventional methods cannot optimize ED problems efficiently if fuel-cost curves of the generating units are not piece-wise linear and monotonically increasing [6]. Generally, ED problem is represented using quadratic function. But, quadratic function can't represent the actual power response of generating unit accurately [7]. Thus, higher order polynomial function is preferred to counter this problem. To avoid complexities of higher order polynomial function, authors have used cubic function to represent ED problem in this paper. Khoa et al. [3] proposed a new Mean-Variance Mapping Optimization (MVMO<sup>S</sup>) method to solve ED problems using cubic function and showed that it performs better than PSO, genetic algorithm (GA) and firefly algorithm (FA).

The methods used previously in economic dispatch problem have evolved from traditional methods to heuristic methods, and finally to hybrid methods in solving optimization issues [8], [9]. Adhinarayanan and Sydulu [19] proposed PSO in to solve economic problem using cubic function, and showed that it improves the effectiveness of particle swarm optimization (PSO) in solving ED problems. PSO is considered as one of the modern swarm based heuristic algorithms for optimization problems in power systems [10], [11]. It is a population-based technique which is an alternative tool to genetic algorithms and this behavioral interaction technique gained popularity in control system applications [12]. It is computationally efficient and easier to implement compared to other evolutionary algorithms proposed in recent studies [13], [14].

One key edge is this algorithm has the capability to allocate memory for storage. Each particle stores the best solution and the solution is compared to that of the group's best solution to tackle optimization issues. PSO works best when there is no need to differentiate conditional variables and the constraints are visible throughout the process. In practical applications, however, PSO has defects such as premature convergence [10]. The disparities of PSO are prone to optimization issues and the quantum particle swarm optimization algorithm (QPSO) improves such shortcomings.

Quantum particle swarm optimization (QPSO) [15] is a new intelligent optimization algorithm which can be easily implemented into the control system optimization issues. The algorithm introduces quantum computing idea into PSO with the manipulation that the particles in the space have quantum behavior. The algorithm succeeds in producing quality and robust solution. The convergence characteristics of QPSO is also better than most other optimization algorithms found in the literature. It also retains the advantages of particle swarm algorithm [13].

The principle of quantum mechanics claims that the PSO technique, applied to quantum space, is an approach within physics and quantum mechanics [16]. Latest progress in solving ED problems in a large number of units has been struck by the high computational time and growing nonlinearities of power generating systems. To reduce the computational efficiency, Meng et al. [17] proposed Quantum PSO using quadratic function to solve this cost problem. QPSO proved better for its stronger search ability and quicker convergence speed than other algorithms like GA and PSO. In this research, we have added dimension to the previous research by exploiting quantum computing technique in cubic cost function with the help of QPSO to solve ED problems. Next section of this paper presents the QPSO methodology, describing QPSO and its operation with flowchart and algorithm. After that, problem statement section briefly discusses about the economic dispatch and the constraints that have been considered in this paper. Finally, result and analysis section shows the obtained result and analysis in each part with tables and figures. The paper is concluded with discussion and conclusion sections, where the contribution, short-comings of this research, future direction are described to make further improvement in solving economic dispatch problem.

#### 2. THE QPSO METHODOLOGY

Quantum PSO is a new and efficient version of PSO which is basically the integration of quantum computing into PSO. Due to the introduction of quantum bit and quantum rotation gate along with implementation of self-adaptive probability selection and chaotic sequences mutation, QPSO demonstrates stronger search ability and quicker convergence speed. QPSO uses quantum bit and angle to depict the state of a particle rather than position and velocity used in the classical PSO. The performance and capabilities of the QPSO has gone beyond that of the classical methods, e.g. PSO, in terms of convergence speed and computational efficiency [18].



Fig. 1 Flowchart of standard quantum PSO

QPSO uses qubit to denote particles. The basic difference between qubit and classical bit used in PSO is that qubit can simultaneously stay in the superposition of two different quantum states,  $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$  (1) where  $\alpha$  and  $\beta$  are complex numbers that satisfy the following equation,  $|\alpha|^2 + |\beta|^2 = 1$  (2)

|0> represents spin up state and |1> represents the spin down state. From Eq.1, we can see one qubit is representing two state of information (|0> and |1>) simultaneously. This superposition state can also be expressed as,

$$\left|\psi\right\rangle = \sin\theta \left|0\right\rangle + \cos\theta \left|1\right\rangle \tag{3}$$

where  $\theta$  represents the phase of the qubit. The relation among  $\theta$ ,  $\alpha$  and  $\beta$  can be defined as,

$$\theta = \arctan \frac{\beta}{\alpha} \tag{4}$$

The structure of QPSO is depicted in fig. 1. Some of the main steps of QPSO are initialization of qubit encoding for particles, evaluation and changing particle forms, updating particles and decoding particles. Detail description of these steps are out of the scope of this paper. However, interested readers may check [19] for details.



Fig. 2 Flowchart of proposed quantum PSO to solve economic dispatch problem

The procedure for implementing the QPSO algorithm is given below as Algorithm 1.

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Algorithm 1. The QPSO algorithm.
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Initialize the population size (M), the positions and the dimensions of the particles;



Positions of all particles in the population, M, are initialized randomly. The fitness value for all particles are then calculated and evaluated according to the problem at hand. The personal best (*pbest*) position of every particle is updated if the current fitness value is found to be better. The best *pbest* among the particles is then assigned to global best (*gbest*) in the next step.

After the *gbest* is assigned, the velocity for all particles is determined. The calculated velocities are then updated to its data values and these values are compared to each other in iterations to get the best fitness value (target value). The best fitness value is considered as the criterion in the algorithm.

If the criterion is not satisfied, the fitness value of the particle is calculated again and the steps are repeated until there is no further update of best fitness value.

### 3. PROBLEM FORMULATION

The main objective of economic dispatch is to find an optimal combination of generated power in order to minimize the total generation cost while satisfying all other constraints. Economic dispatch using cubic function can be written as:

$$F_i(P_{gi}) = a_i P_{gi}^3 + b_i P_{gi}^2 + c_i P_{gi} + d_i$$
(3)

where,  $F_i$  is the fuel cost (in \$/h) of generating unit *i*.  $a_i, b_i, c_i$  and  $d_i$  are fuel cost coefficients of generating unit *i*. Additionally,  $P_{gi}$  and *n* are the real power generation of the *i*th unit (in MW) and the total number of generation units, respectively. Our goal is to minimize total fuel cost, which can be defined as:

$$Minimize, F_T = \sum_{i=1}^n F_i(P_{gi})$$
(4)

In this research, three constraints are considered to solve economic dispatch problem. The constraints are given below

Power Balance Constraint: The total output power should be equal to the total power demand plus transmission losses:

$$P = \sum_{i=1}^{n} P_i = P_D + P_L$$
 (5)

where P,  $P_D$  and  $P_L$  are total output power generated (in MW), total power demand (in MW) and transmission loss (in MW), respectively.

Power loss constraint: Power loss or transmission loss (in MW) can be defined as

$$P_{L} = \sum_{i=1}^{N} \sum_{j=1}^{N} P_{gi} B_{ij} P_{gj} + \sum_{i=1}^{N} B_{i0} P_{gi} + B_{00}$$
(6)

where  $B_{ij}$ ,  $B_{i0}$  and  $B_{00}$  are loss coefficient of George's formula, transmission loss constant of generating unit *i* and Kron's transmission loss constant, respectively.

Generator Limit Constraint: The output power generation of each power generating unit has its minimum and maximum value. The power generation should be between its maximum and minimum value. This inequality can be formulated as below:

$$P_{i,min} \le P_i \le P_{i,max} \tag{7}$$

where  $P_{i,min}$  and  $P_{i,max}$  are the minimum value and maximum value of power generating unit *i*, respectively.

## 4. RESULT AND ANALYSIS

Quantum particle swarm optimization (QPSO) technique is applied here to solve economic dispatch problem for 3-unit and 5-unit power generation systems [6], [20] using cubic function, where the total load demand is 2500 MW and 1800 MW, respectively. Authors have implemented this proposed algorithm in MATLAB R2015a and executed with Core<sup>TM</sup> i5-3470 CPU @ 3.20 GHz (4 CPUs), ~3.2GHz and 4GB RAM personal computer. Table 1 shows the parameter settings of QPSO. Total 100 number of runs are considered as a fair test of robustness and the average of the outcomes have been shown in this section.

Table 1 Settings of parameters for QPSO

Parameters	Values
Population Size	2000
Maximum Iteration	200
Number of Runs	100
Dimension	3

Tables 2 and 5 are showing the fuel cost coefficient values, minimum and maximum limit of each power generating unit and power loss coefficients for 3-unit and 5-unit system, respectively.

Table 2 Cubic cost function coefficients and power loss coefficients for 3-unit system [6]

Unit	1	2	3	
$a_i$	749.55	1285	1531	
$b_i$	6.95	7.05	6.531	
$c_i(10^{-4})$	9.68	7.38	10.4	
$d_i(10^{-8})$	12.7	6.45	9.98	
$P_{i,\min}$	320	300	275	
$P_{i,\max}$	800	1200	1100	
B coefficients				
1(10-7)	14	1.5	2.6	
2 (10-7)	1.5	6.5	2.4	
3 (10-7)	2.6	2.4	6.9	
$B_0(10^{-4})$	-76.6	-3.42	18.90	
$B_{00}$		4.0357		

Tables 3 and 6 are showing comparison of simulation results among GA, PSO and QPSO. From Table 3, it can be seen that QPSO outperforms GA and PSO in terms of finding optimal value for both 3-unit and 5unit systems. The obtained results are found to be highly robust and reliable.

 Table 3 Comparison of results (total cost) for 3-unit system without considering power loss

7	υ.		
Unit	GA [21]	PSO [21]	QPSO
1	725.02	724.99	726.866
2	910.19	910.15	908.4451
3	864.88	864.85	864.6898
Total Power, P	2500	2500	2500
(MW)			
Total Cost, $F_T$	22730.14	22729.35	22728.52
(\$)			

Table 4 Test system data for 3-unit system considering power loss

	QPSO
$P_{I}$	742.192
$P_2$	912.193
$P_3$	847.965
Total Power, P (MW)	2502.35
Total Cost, $F_T$ (\$)	22749.277
Standard Deviation	0.0743

Figs. 3 and 5 show that the proposed QPSO provides excellent convergence characteristics for both 3-unit and 5-unit systems. The convergence graphs in figs. 3 and 5 verify that QPSO provides steady, smooth and fast convergence for solving economic dispatch problem.



Fig. 3 Convergence curve of the QPSO for economic dispatch of 3-unit system considering power loss



Fig. 4 Total cost (in \$/h) vs number of runs curve for economic dispatch considering 3-unit system with power loss constraint using QPSO.

Table 5 Cubic cost function coefficients and power loss coefficients for 5-unit system [20, 22]

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Unit	1	2	3	4	5
$a_i$	749.6	1285	1531	749.6	1285
$b_i$	6.95	7.05	6.531	6.95	7.05
$c_i(10^{-4})$	9.68	7.38	10.4	9.68	7.38
$d_i(10^{-8})$	12.7	6.45	9.98	12.7	6.45
$P_{i,\min}$	320	300	275	320	300
$P_{i,\max}$	800	1200	1100	800	1200

Tables 4 and 7 represent the best result obtained by QPSO for economic dispatch using cubic function considering power loss constraint.

When power loss constraint is considered, the total cost becomes higher than the cost without considering power loss.

Table 6 Comparison of results (in \$) for 5 units system without considering power loss

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Unit	GA	PSO	QPSO
1	320.00	320.00	320.51
2	343.74	343.70	346.47
3	472.60	472.60	482.95
4	320.00	320.00	320
5	343.74	343.70	330.08
P(MW)	1800	1800	1800
F <sub>T</sub> (\$)	18611.07	18610.4	18610.03

Table 7Test system data using QPSO for 5 unitssystem considering power loss

	QPSO
$P_{I}$	341.86
$P_2$	309.48
$P_3$	471.46
$P_4$	325.28
$P_5$	355.90
Total Power, P (MW)	1803.98
Total Cost, $F_T(\$)$	18648.17

Total 100 number of runs have been conducted to test the reliability and robustness of QPSO to solve ED problem. Figs. 4 and 6 shows that QPSO provides reliable and robust solutions for solving economic dispatch problem in both 3-unit and 5-unit system without considering power loss. The standard deviation found from the total runs is also negligible (the standard deviation for 3-unit and 5-unit systems are ~0.09147 and ~0.2218, respectively).



Fig. 5 Convergence curve of the QPSO for economic dispatch of 5-unit system considering power loss



Fig. 6 Total cost (in \$/h) vs number of runs curve for economic dispatch considering 5-unit system with power loss constraint using QPSO.

#### 5. DISCUSSION

This paper considers 3-unit and 5-unit systems to compare the obtained results with other methods like GA and PSO. For comparison purposes, authors have taken the same coefficients value and have not considered power loss. Simulation results show QPSO performs better to predict minimum total cost function for 3-unit and 5-unit systems than other methods like GA and PSO. QPSO is computationally more powerful and shows better convergence characteristics than PSO. The quality of the solutions are found to be reliable, robust and suitable. Authors have also shown the results for 3-unit and 5-unit system considering power loss. However, for cubic cost function, the authors couldn't manage data for large number of units and thus couldn't be able to present results for larger systems.

#### 6. CONCLUSION

In this paper, QPSO technique is presented to solve the ED problem using cubic function. QPSO technique is successfully implemented into ED problems considering 3-unit and 5-unit systems with power balance, power loss and generator limit constraints. Simulation results show its effectiveness in solving ED problems by demonstrating better and stable results. The obtained results are compared with PSO and GA which demonstrates QPSO superiority over these methods. To reduce the nonlinearities of power generating systems, cubic function is used to represent ED. In this paper, total 100 number of runs are considered as a fair test of robustness of the proposed method. The obtained results for the test systems confirm that the proposed method gives better global solution, is more robust and computationally powerful in solving the ED problems. To the best of the authors' knowledge, this is the first work on single objective economic dispatch problem with cubic function using QPSO. Authors' next work

is to include emission dispatch as another objective i.e. it will be a multiobjective problem, where the authors shall consider quantum integrated advanced computational intelligence-based methods e.g. quantum cuckoo search (QCS), quantum bat algorithm (QBA) etc. to test the feasibility of their use in this multiobjective power dispatch problem.

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#### 8. NOMENCLATURE

- $F_i$  Fuel cost (\$/h) of generating unit *i*.
- $a_i, b_i, c_i, d_i$ -Fuel cost coefficients of generating unit *i*
- Pi Real power generation of the *i*th unit
- n Total number of generation units
- P Total output power generated (in MW)
- $P_D$  Total power demand (in MW)
- $P_L$  Transmission loss (in MW)
- ED Economic dispatch
- FA Firefly algorithm
- GA Genetic algorithm
- MVMO<sup>S</sup> Mean-variance mapping optimization
- PSO Particle swarm optimization
- QBA Quantum bat algorithm
- QCS Quantum cuckoo search
- QPSO Quantum particle swarm optimization
- RE Renewable energy

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