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DYNAMIC INTERACTION FACTOR OF PILED RAFT FOUNDATION SUBJECTED TO VERTICAL LOADING

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ABSTRACT: In this paper, a dynamic interaction factor between a raft and a pile group that constituted a piled raft foundation subjected to vertical loading is investigated. Because, if the dynamic interaction factor can be evaluated, the dynamic impedance of the piled raft foundation can be easily calculated from the dynamic impedance of a spread foundation and a pile foundation. The dynamic interaction factor is formulated as an arbitrary function in the frequency domain, based on the relationship of the dynamic impedance between the piled raft, the spread and the pile foundation obtained by parametric numerical analyses with various changes in the foundation configuration and the ground condition. Moreover, the coefficients used in the function have related the width of the foundation or the stiffness ratio of the spread and pile foundations. It is confirmed that the results of the simplified method using the interaction factor are in good agreement even for the dynamic impedance of piled raft foundation with conditions similar to the spread foundation or the pile foundation.

Keywords: Piled raft foundation, Dynamic impedance, Interaction factor, Dynamic soil-structure interaction

1. INTRODUCTION

An interaction effect between a building and a ground is important in evaluating the response of a building. Therefore, many studies on a dynamic impedance expressing the dynamic resistance including stiffness and damping of the spread foundation and pile group foundation have been conducted [1-4].

On the other hand, there are few studies on the dynamic impedance of a piled raft foundation. Emani and Maheshwari [5] investigated the influences of contacting the foundation bottom, the embedment of the foundation, and the number of piles on the vertical dynamic impedance by an analysis method using the finite element method (FEM) and the consistent infinitesimal finite element cell method (CIFECM). Nguyen et al. [6] evaluated the vertical dynamic impedance using the FEM incorporating the principles of multiphase models. Liu and Ai [7] investigated the influences of the ground-contacting raft, pile length, and layered ground on the vertical dynamic impedance. Also, numerical models based on a three-dimensional finite element analysis using the dynamic substructure method [8] or a method combining the FEM and the thin layer method [9] were used to evaluate the lateral and rotational dynamic impedance. In these studies, parametric analyses were carried out to investigate the dynamic impedance, but the effect of dynamic interaction between a raft and a pile group that constituted the piled raft foundation was not sufficiently clarified.

In a previous study, the author proposed a simplified method of estimating dynamic impedance for horizontal motion and rotational motion of a piled-raft foundation subjected to a building inertial force caused by an earthquake [10]. The characteristic of this method is to simply calculate the dynamic impedance of a piled raft foundation using three factors that are the dynamic impedance of a spread foundation, the dynamic impedance of a pile foundation, and the dynamic interaction factor between the raft and the pile group that constituted a piled raft foundation. An important point in this method is to evaluate the dynamic interaction factor. The dynamic interaction factor was formulated based on the relationship between the impedance of the piled raft, the spread and the pile foundation obtained by parametric numerical analyses in the frequency domain.

In the present paper, I attempt to formulate the vertical dynamic interaction factor between the raft and the pile group to simplify the calculation of the vertical dynamic impedance of the piled raft foundation subjected to vertical harmonic loading.

2. APPROACH FOR IMPEDANCE AND INTERACTION FACTOR

2.1 Outline of a Simplified Method

The piled raft foundation has two resistance components of a raft and a pile group. The static relationship between the load acting on the piled raft foundation, P_{PR}, and the displacement occurring at the foundation, U_{PR}, can be expressed by dividing the two components and considering the interaction between these components, as follows [11]:

$$P_{PR} = k_{PR} \cdot U_{PR} \quad (1)$$

$$P_{PR} = P_r + P_p \quad (2)$$

$$\begin{Bmatrix} U_r \\ U_p \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{k_r} & \dfrac{\alpha_{rp}}{k_p} \\ \dfrac{\alpha_{pr}}{k_r} & \dfrac{1}{k_p} \end{bmatrix} \begin{Bmatrix} P_r \\ P_p \end{Bmatrix} = \begin{Bmatrix} U_{rr} + U_{rp} \\ U_{pr} + U_{pp} \end{Bmatrix} \quad (3)$$

where k_{PR} is the overall stiffness of the piled raft foundation, U_r is the average displacement of the raft in the piled raft foundation, U_p is the average displacement of the pile group in the piled raft foundation, P_r is the total load carried by the raft in the piled raft foundation, P_p is the total load carried by the pile group in the piled raft foundation, k_r is the overall stiffness of the raft in isolation, k_p is the overall stiffness of the pile group in isolation, α_{rp} is the interaction factor of the raft on the pile group, and α_{pr} is the interaction factor of the pile group on the raft.

Next, the off-diagonal components of the soft matrix in Eq. (3) are equal ($\alpha_{pr}/k_r = \alpha_{rp}/k_p$). Assuming the displacements of the raft and the pile group are equal ($U_{PR}=U_r=U_p$), the following equation is obtained:

$$\frac{k_{PR}}{k_p} = \frac{1+(1-2\alpha_{rp})(k_r/k_p)}{1-\alpha_{rp}^{\ 2}(k_r/k_p)} \quad (4)$$

On the other hand, the dynamic impedance of the foundation subjected to a dynamic load is a complex stiffness that includes the stiffness and damping, so in the case of a dynamic problem, the above equation is a quadratic equation of complex coefficients. Hence, replacing the complex variables in Eq. (4) with K_{PR}, K_{SF}, K_{PG}, and α_d, respectively, for k_{pr}, k_r, k_p, and α_{rp}:

The simplified method can calculate the dynamic impedance of the piled raft foundation using the formulated dynamic interaction factor and the dynamic impedances of the spread foundation and the pile group foundation, as follows:

$$K_{m,PR}(f) = \frac{K_{m,PG}(f) + \{1-2\alpha_{d,m}(f)\}K_{m,SF}(f)}{1-\alpha_{d,m}^{\ 2}(f)\{K_{m,SF}(f)/K_{m,PG}(f)\}} \quad (5)$$

where K_{PR}, K_{SF}, and K_{PG} are the dynamic impedances of the spread, the pile and the piled raft foundation, respectively, and α_d is the dynamic interaction factor, f is the frequency, subscript m is the index of motion (For example, horizon, rotation).

A parametric numerical analysis is carried out to calculate the dynamic impedances of the piled raft foundation, the spread and pile group foundations are set to be the same as the corresponding dimensions of the piled raft foundation. The dynamic interaction factor α_d (=|α|・exp(iϕ_α)) is obtained by substituting these dynamic impedances into Eq. (6) and solving this equation.

$$\left(\frac{K_{m,PR}}{K_{m,PG}}\right)\alpha_{d,m}^{\ 2} - 2\alpha_{d,m} + \left(\frac{K_{m,PG}}{K_{m,SF}} - \frac{K_{m,PR}}{K_{m,SF}} + 1\right) = 0 \quad (6)$$

The displacement of the piled raft foundation, as shown in Fig. 1, can be expressed as the following equation:

$$U_{PR} = U_{rr} + U_{rp} = U_{rr} + \alpha_d \cdot U_{pp} \quad (7)$$

The dynamic interaction factor α_d, which is the dynamic interaction from the pile group to the raft, means that the displacement component U_{pp} due to the load carried by the group pile is multiplied by a coefficient, and the angle conversion is performed as shown in Fig. 2.

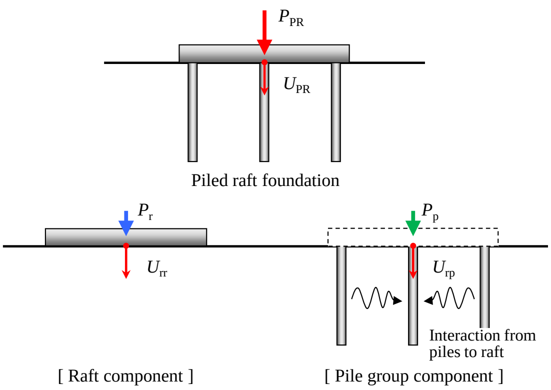
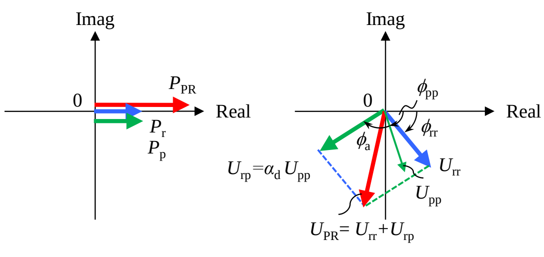


Fig. 1 Displacement and interaction between the raft and the pile group in the piled raft foundation



(a) (b)

Fig. 2 Representation of harmonic load and displacement in the piled raft foundation on the complex plane; (a) dynamic load shared by the raft and the pile group; (b) dynamic displacement

2.2 Numerical Analysis Method

Fig. 3 shows a schematic diagram of the analytical method. This method combines the finite element method and the thin layer method. The equation of motion for the foundation-soil system in the frequency domain is

$$\left(\left\lfloor K^{*}\right\rfloor - \omega^2[M]\right)\cdot\{U(\omega)\} = \{F(\omega)\} \tag{8}$$

where $\{U\}$ is the displacement vector of the foundation, $\{F\}$ is the external force vector acting on the foundation, ω is the circular frequency (= $2\pi f$), $[K^*]$ is the stiffness matrix, and $[M]$ is the mass matrix.

The stiffness matrix is given by

$$\left\lfloor K^{*}\right\rfloor = [K_r]+[K_p]+[K_{rp}]+[K_s]-[K_s{}^{ep}] \tag{9}$$

where $[K_r]$, $[K_p]$, $[K_{rp}]$, $[K_s]$, and $[K_s{}^{ep}]$ are the stiffness matrices of the raft, piles, pile head connections, free field soil, and soil that would fill the same volume as the pile bodies, respectively. The raft, piles, and pile head connections are modeled as a thin-plate element, beam elements, and spring elements, respectively. However, the stiffnesses of the raft and the pile head connections are assumed to be infinite in the present study. The soil is treated as the inverse matrix of the three-dimensional Green's functions derived from the excitation solution in the thin layer method [9, 12]. The ring excitation solution is used when the vertical axes of the points at which the excitation force acts coincide with the vertical axes of the points at which the vibration occurs; otherwise, the point excitation solution is used. Furthermore, the stiffness of each component is assumed to be represented by a linear viscoelastic model consisting of a spring and a dashpot mounted in parallel.

The mass matrix is given by

$$[M] = [M_r]+[M_p]-[M_s{}^{ep}] \tag{10}$$

where $[M_r]$ is the mass matrix of the raft (assumed to be zero in this study), $[M_p]$ is the mass matrix of the pile, and $[M_s{}^{ep}]$ is the mass matrix of the soil corresponding to the integral of the pile bodies.

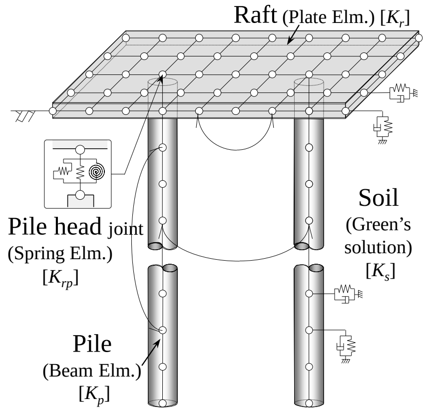


Fig. 3 Schematic of analytical method

2.3 Dynamic Loading and Dynamic Impedance

The external force $\{F\}$ acting on the foundation given in Eq. (8) is given by

$$V(f) = V_0\cdot\exp(i2\pi ft) \tag{11}$$

where V_0 is the amplitude of the vertical harmonic load applied to the foundation as shown in Fig. 4.

The steady-state vibration amplitudes of the vertical displacement W of the foundation under a harmonic load is calculated by frequency response analysis for frequencies ranging from 1 to 40 Hz in increments of 1 Hz using the following equations:

$$W(f) = W_0(f)\cdot\exp\{i2\pi ft - i\phi_v(f)\} \tag{12}$$

where W_0 is the amplitude of the vertical dynamic displacement at the foundation, and ϕ_v is the initial phase angle of the vertical dynamic displacement.

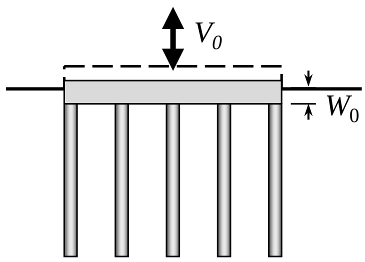


Fig. 4 Schematic of load application to determine the dynamic impedance and dynamic displacement

The dynamic impedance is defined as the ratio of the harmonic load acting on the foundation to its vibration amplitude. Hence, the magnitude of the dynamic impedance K_v in the frequency domain is expressed by

$$K_V(f) = \frac{V(f)}{W(f)} = \left|\frac{V_0}{W_0(f)}\right| \cdot \exp\{i\phi_V(f)\} \tag{13}$$

Eq. (13) is transformed as follows using Euler's formula. The real part of the impedance functions represents the stiffness characteristics, whereas the imaginary part represents the damping characteristics:

$$K_V(f) = |K_V(f)| \cdot \cos\{\phi_V(f)\} + i \cdot |K_V(f)| \cdot \sin\{\phi_V(f)\} = \mathrm{Re}(K_V(f)) + i \cdot \mathrm{Im}(K_V(f)) \tag{14}$$

3. DYNAMIC INTERACTION FACTOR

3.1 Analytical Model

Fig. 5 shows the models used in the present study. A semi-infinite soil is assumed, and the foundation slab is defined as a rigid, massless, flat square that is not embedded in the soil. The piles are cast-in-place concrete piles with diameters of d = 1 m. The raft-pile head connection is represented as a rigid connection. The behavior of the soil-pile system assumes perfect contact between pile and soil. The investigated parameters are the number of piles n, the pile spacing s/d, the pile length L/d, and the pile-soil stiffness ratio E_p/E_s, as listed in Table 1. In addition to piled raft (PR) foundations, two other foundation types, spread foundations (SF) and pile group (PG) foundations, are analyzed. The pile group foundation has the same pile length as the piled raft foundation, but the raft is not in contact with the soil. In each case, the dimensions of the spread and pile group foundations are set to be the same as the corresponding dimensions of the piled raft foundation.

3.2 Vertical Dynamic Interaction Factor

The amplitude and initial phase angle of the dynamic interaction factor obtained by parametric numerical analyses are shown as the label Ana in Figs. 6 and 7. Fig. 6 shows the results for different numbers of piles (Cases 1 through 3 in Table 1) and Fig. 7 shows the results for different pile spacings (Cases 2, 4, and 5 in Table 1). The distribution of the dynamic interaction factor when the pile length and the pile-soil stiffness ratio are changed (Cases 6 through 11 in Table 1) is similar to the distribution in Case 2. The horizontal axis of each plot represents the frequency normalized by the pile spacing (s) and the shear wave velocity (V_s) of the soil.

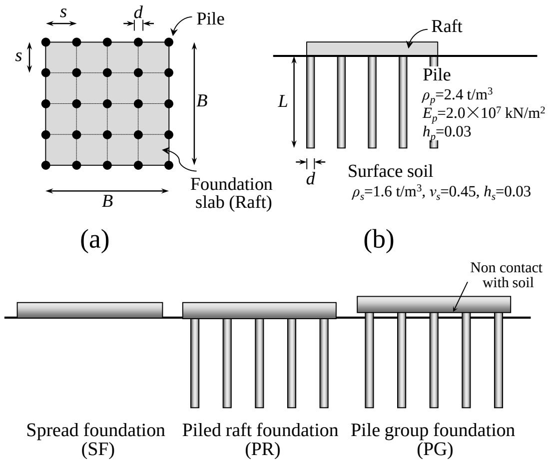


Fig. 5 Analytical model; (a) plan view; (b) elevation view; (c) foundation type

Table 1 Analytical parameters

Case	n	s/d	L/d	E_p/E_s
1	9	5	25	1.10×10^2 (200)
2	25	5	25	1.10×10^2 (200)
3	49	5	25	1.10×10^2 (200)
4	25	7.5	25	1.10×10^2 (200)
5	25	10	25	1.10×10^2 (200)
6	25	5	12.5	1.10×10^2 (200)
7	25	5	37.5	1.10×10^2 (200)
8	25	5	50	1.10×10^2 (200)
9	25	5	25	7.04×10^1 (250)
10	25	5	25	1.96×10^2 (150)
11	25	5	25	4.40×10^2 (100)

Note: Value in parentheses is a shear wave velocity of surface soil, V_s.

The vertical dynamic interaction factor is related to the frequency of the external load. As shown in these figures, the amplitude of the dynamic interaction factor $|\alpha_v|$ reaches a peak at some frequency and then approaches zero as the frequency increases further. With the decrease in the number of piles (n) and the pile spacing (s/d), the normalized frequency and the peak value when the amplitude reaches peak increase. The frequency at the inflection point corresponds to the frequency at which the impedance of the pile foundation peaks. The value of α_d reflects a dynamic phenomenon in which the displacement of a pile due to a wave propagating from an adjacent pile through the soil is delayed more than the displacement of the pile caused directly by the applied load. The initial phase angle $\phi_{\alpha,v}$ increases with increasing frequency, passes through an inflection point and converges to a value. The convergence value becomes smaller

(15)

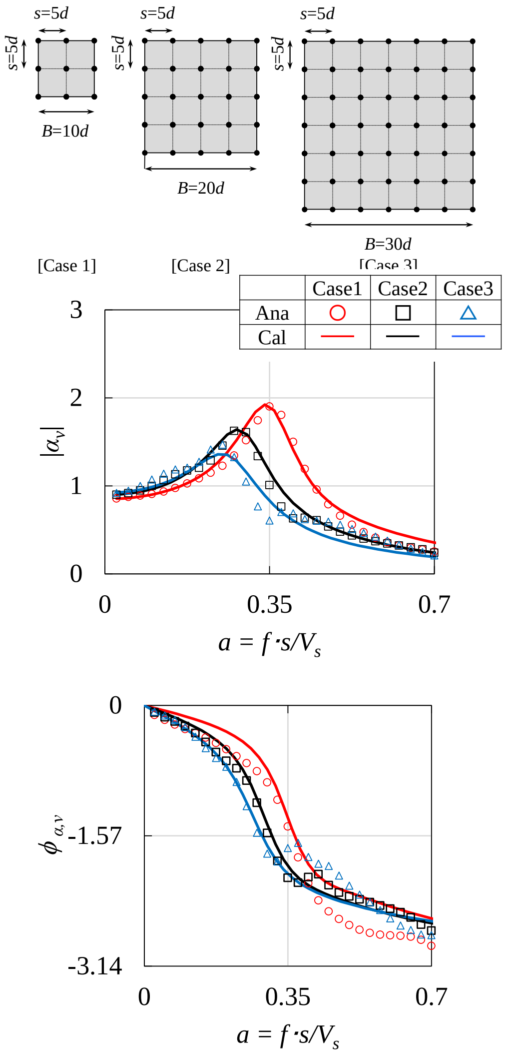


Fig. 6 Variation of dynamic interaction factor with normal frequency (a=$f{\cdot}s/V_s$) for the cases of n = 9; n = 25 and n = 49 (Cases 1, 2 and 3; s/d = 5, L/d = 25, E_p/E_s = 1.10×10^2)

for the vertical impedance than for the horizontal impedance presented in [10]. Moreover, the amplitude appears to be two peaks in cases 2 and 3. It can be said that the distribution of vertical dynamic interaction factors is slightly different from that of the horizontal dynamic interaction factors. The reason is that the interaction effect due to the out-of-phase of piles become larger for the vertical vibration than for the horizontal vibration.

Therefore, to express the above distributions of vertical dynamic interaction factors, the amplitude and the phase are formulated as so as to have two periodicities.

$$|\alpha_v(a)| = \xi \cdot \frac{a_{i1}{}^2 \cdot a_{i2}{}^2}{a_{i1}{}^2 + a_{i2}{}^2} \cdot \sqrt{(\chi_{11}+\chi_{21})^2 + (\chi_{12}+\chi_{22})^2}$$

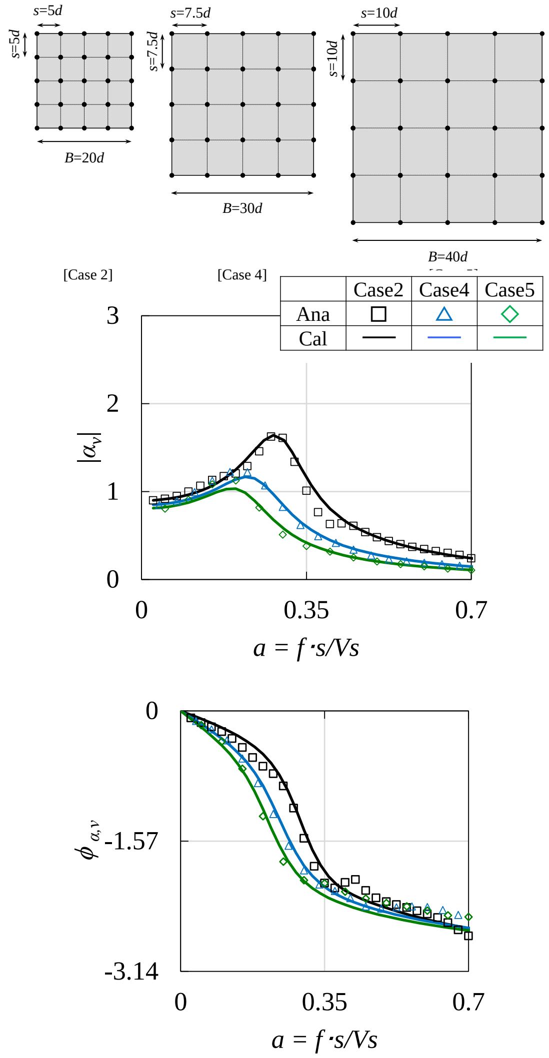


Fig. 7 Variation of dynamic interaction factor with normalized frequency (a=$f{\cdot}s/V_s$) for the cases of s/d = 5; s/d = 7.5 and s/d = 10 (Cases 2, 4 and 5; n = 25, L/d = 25, E_p/E_s = 1.10×10^2)

$$\phi_{\alpha,v}(a) = -\tan^{-1}\left(\frac{\chi_{11}+\chi_{21}}{\chi_{12}+\chi_{22}}\right) \quad (16)$$

where

$$\chi_{11} = \frac{\eta_1 \cdot a_{i1} \cdot a}{(\eta_1 \cdot a_{i1} \cdot a)^2 + (a_{i1}{}^2 - a^2)^2},$$

$$\chi_{12} = \frac{a_{i1}{}^2 - a^2}{(\eta_1 \cdot a_{i1} \cdot a)^2 + (a_{i1}{}^2 - a^2)^2},$$

$$\chi_{21} = \frac{\eta_2 \cdot a_{i2} \cdot a}{(\eta_2 \cdot a_{i2} \cdot a)^2 + (a_{i2}{}^2 - a^2)^2},$$

$$\chi_{22} = \frac{a_{i2}{}^2 - a^2}{(\eta_2 \cdot a_{i2} \cdot a)^2 + (a_{i2}{}^2 - a^2)^2},$$

a (= $f{\cdot}s/V_s$) is the normalized frequency, s is the pile spacing, V_s is the shear wave velocity of soil, a_{i1} is value of a when $\phi_{a,v}$ = $-\pi/2$, $\xi = |\alpha_v(0)|$, η_1 is a positive real number satisfying the following fourth-order equation (Eq. (17)), which is a modification of Eq. (15) when $a = a_{i1}$, $a_{i2} = \kappa \cdot a_{i1}$, and $\eta_2 = \lambda \cdot \eta_1$.

$$-\left\{\frac{|\alpha_v(a_{i1})|\cdot(1+\kappa^2)\cdot\lambda}{\xi\cdot\kappa}\right\}^2\cdot\eta_1^4 + \left[\begin{array}{l}\kappa^2\cdot\lambda^2+2\kappa\cdot\lambda+1\\ -\left\{\dfrac{|\alpha_v(a_{i1})|\cdot(1+\kappa^2)\cdot(\kappa^2-1)}{\xi\cdot\kappa^2}\right\}^2\end{array}\right]\cdot\eta_1^2 + \left(\kappa^2-1\right)^2 = 0 \quad (17)$$

where $\kappa = \sqrt{2}$ and λ = 2.5. These values are determined to fit the interaction factor obtained by analysis.

The results of which are plotted in Figs. 6 and 7 with the label Cal.

Furthermore, the coefficients ξ and η_1 are set by relating these coefficients to the stiffness ratio of the pile foundation to the spread foundation $|K_{v,PR}|/|K_{v,SF}|$ at a frequency of f = 0.1 Hz which is close to the static state.

The coefficient ξ is then calculated by substituting $|K_{v,PR}|$ and $|K_{v,PG}|/|K_{v,SF}|$ into Eq. (4).

Then, $|K_{v,PR}|$ is obtained by approximating the correlation between $|K_{v,PR}|/|K_{v,SF}|$ and $|K_{v,PG}|/|K_{v,SF}|$, as shown in Fig. 8. This approximation is given by the relationship proposed in [13] as:

$$\frac{|K_{v,PR}|}{|K_{v,SF}|} = \frac{1}{4}\left(\frac{|K_{v,PG}|}{|K_{v,SF}|}\right)^2 + 1 \quad (18)$$

Fig. 9 shows the correlation between $|K_{v,PG}|/|K_{v,SF}|$ and η_1. The coefficient η_1 is formulated in Eq. (17). This is the same equations as η in the horizontal and rotational components:

$$\eta_1 = \frac{1}{A\cdot\left(\dfrac{|K_{v,PG}|}{|K_{v,SF}|}\right)^2 + B\cdot\dfrac{|K_{v,PG}|}{|K_{v,SF}|} + C} \quad (19)$$

where $A = -0.12$, $B = 2.29$, and $C = -0.75$.

Fig. 10 shows the correlation between a_{i1} to the width of the foundation. So, a_{i1} is formulated by power function as in:

$$a_{i1} = 0.8 / B^{0.33} \quad (20)$$

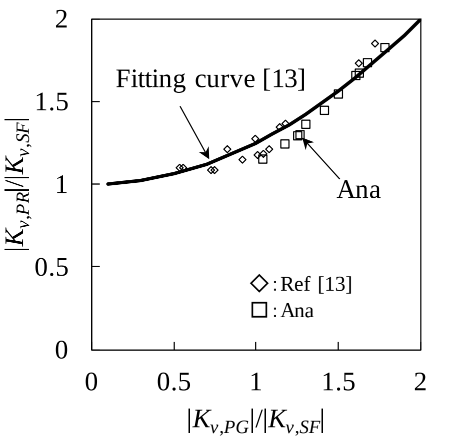


Fig. 8 Correlation between the stiffness ratio $|K_{v,PR}|/|K_{v,SF}|$ of the piled raft foundation to the spread foundation and the stiffness ratio $|K_{v,PG}|/|K_{v,SF}|$ of the pile foundation to the spread foundation

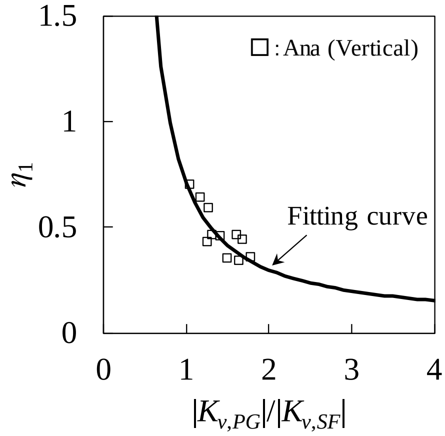


Fig. 9 Correlation between the stiffness ratio $|K_{v,PG}|/|K_{v,SF}|$ of the piled raft foundation to the spread foundation and the coefficient η_1

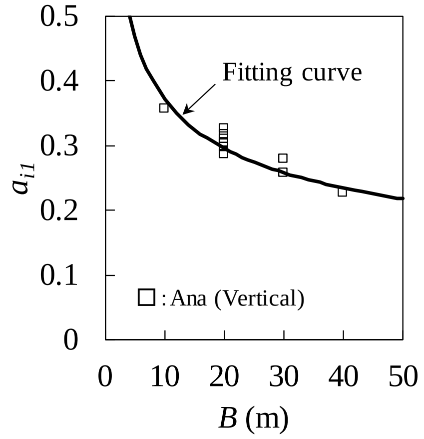
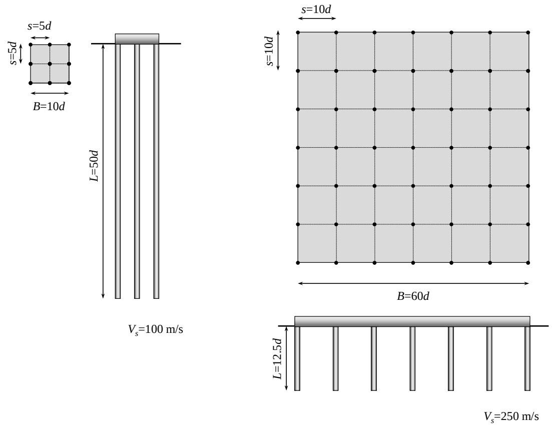


Fig. 10 Correlation between a_{i1} to the width of the foundation



(a) Case-I (b) Case-II

Fig. 11 Extended analytical model; (a) Case-I (n = 9, s/d = 5, L/d = 50, E_p/E_s = 4.40×10^2); (b) Case-II (n = 49, s/d = 10, L/d = 12.5, E_p/E_s = 7.04×10^1)

	|K|	ϕ
Ana	◇	×
Cal	——	- - - -

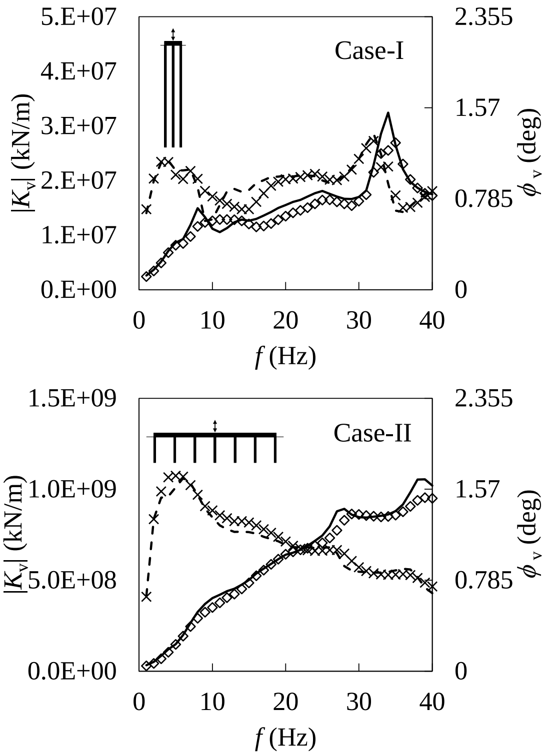


Fig. 12 Validation of the simplified method of evaluating dynamic impedance by comparison with the analytical results in case of the extended model

4. VALIDATION OF SIMPLIFIED METHOD

The parametric study is carried out under limited conditions with only one factor being changed. The dynamic interaction factor is investigated under the condition of the foundation shape combining multiple factors and the calculation precision of the dynamic impedance.

Fig. 11 shows the foundation shape of the analytical model under extended conditions. Fig. 11 (a) shows the case having a small foundation with a long pile (n = 9, s/d = 5, L/d = 50, V_s = 100 m/s), and Fig. 11 (b) shows the case having a large foundation with a short pile (n = 49, s/d = 10, L/d = 12.5, V_s = 250 m/s).

Fig. 12 shows the dynamic impedances for the analytical method (Ana) and the proposed simplified method (Cal) for the extended model. The dynamic impedance values for the spread and pile foundations required by the simplified method are calculated in advance using the analytical model. It is confirmed that the simplified method generally matches the trends of the analytical results within the frequency range examined in both cases.

5. CONCLUSION

The vertical dynamic interaction factor between the raft and the pile group that constituted the piled raft foundation is investigated, based on the dynamic impedances obtained by numerical analysis. The amplitude and the initial phase angle of the dynamic interaction factor are expressed as functions. The coefficients in the function used to express the dynamic interaction factor are related to the width of the foundation or the stiffness ratio of the spread and pile foundations for the same dimensions as in the case of the piled raft foundation.

The results of the simplified calculation method are confirmed to correspond well to the results of numerical analysis, even in cases in which the shape of the foundation is similar to spread foundation or to that of the pile foundation.

Future research will examine extending this simplified method to calculate the dynamic impedance in the case of multi-layered ground.

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