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LEANING PAGODA OF AYUTTHAYA ANALYSIS WITH THREE-DIMENSIONAL EFFECTS AND GROUND UNCERTAINTY

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ABSTRACT: The pagoda of Wat Krasai, built in the 17th century on the outskirts of the Ayutthaya district in the Kingdom of Thailand, tilts approximately 2° to the north and may continue to tilt further. The cause of this tilt is thought to be the uneven settlement of the foundation ground, but no supporting evidence has been presented. Three cohesive soil layers that may cause uneven settlement have been confirmed in the foundation ground around the pagoda, but soil tests are inadequate, leading to uncertainty in proving the cause of inclination. Hence, the cause of inclination was investigated in this study by calculating the inclination that can occur within the realistic ranges of the soil properties through numerical analysis using the finite element method. A method of correcting plane strain analysis, which has a low computational cost, with a three-dimensional effect coefficient, was utilized to assess over 500 cases considering ground material uncertainties, and the probability distribution of the possible inclinations was calculated. This method is useful as it considers fluctuations in the ground material parameters to address land settlement, which is largely influenced by the three-dimensional effects. This approach will also be applicable to future predictions aimed at pagoda preservation.

Keywords: Three-dimensional effects, Uneven settlement, Parameter uncertainty, FEM

1. INTRODUCTION

The pagoda of Wat Krasai, which is the focus of this study, is currently tilted approximately 2° to the north. It is located in the countryside on the outskirts of the Ayutthaya district of the Kingdom of Thailand and is believed to have been built in the 17th century.

There are many leaning structures in the world. Some of them have value because of their tilt, but there are others where such a tilt is a problem. The Leaning Tower of Pisa in Italy, which developed a tilt during its construction period, has attracted the attention of many researchers, and multiple numerical analyses have been conducted focusing on its foundation ground [1-7]. Burland et al. [2] used plane strain analysis and three-dimensional analysis to adjust the overturning moment of the tower to reproduce the tilt of the foundation. Klettke and Edgers [4] showed that correcting the weight of the tower in the plane strain analysis enabled the calculation of a tower tilt that matched the results of the three-dimensional analysis. These previous findings show that it is important to consider the three-dimensional effect and a reasonable tower overturning moment when reproducing the tower inclination through numerical analysis.

Furthermore, the validity of the numerical analysis of the Leaning Tower of Pisa is guaranteed by reproducing the ground input conditions (construction history and ground survey results) and past deformation history (measured values of inclination and settlement). However, many structures worldwide have historical significance, but no past record of alterations, and their investigations are restricted even today for the reason of cultural property protection. One such example is the pagoda of Wat Krasai, for which the history of tilt is yet unknown. It is difficult to investigate the foundation ground directly underneath this pagoda, and no soil test results are known to exist. There are many pagodas in the 'Historic City of Ayutthaya', which was registered as a UNESCO World Heritage Site in 1991[8], and some of these are tilted like that of Wat Krasai. It can be inferred that the inclination of these pagodas is due to the uneven settlement of the foundation ground, but no evidence supporting this inference is available. Therefore, in this study, an analysis was conducted based on the finite element method (FEM) using Wat Krasai as the subject to investigate the cause of inclination of this pagoda.

It was necessary for the analysis of this pagoda to consider the three-dimensional effect under the condition that there were uncertainties in the soil properties. Therefore, a method was adopted that combines plane strain analysis, which has a low computational cost, with three-dimensional analysis. The important aspect of this method is that it considers the correlation between the plane strain analysis results and the three-dimensional analysis results. This technique applies a method that the authors confirmed to be applicable to the deformation of the surrounding ground due to embankments [9].

The remainder of this paper is organized as follows. Section 2 describes the current state of the pagoda and the conditions of the foundation ground. Section 3 outlines the numerical analysis method adopted. Sections 4 and 5 presented specific modeling methods and the probability distribution of inclination of this pagoda estimated from the analysis results. Section 6 describes the necessity of predicting future deformation caused by the effects of restoration work carried out in recent years. Section 7 discusses the efficacy of the approach adopted. Finally, Section 8 presents concluding remarks.

2. STUDY LOCATION

Wat Krasai is located in the southwestern part of the historic city of Ayutthaya and is the ruins of a Buddhist temple consisting of a large pagoda and chapel, as well as walls surrounding the temple (Fig. 1). Although no clear evidence has been unearthed, there is a theory that this temple is the burial ground of the royal family who rebelled against King Narai (who ruled 1656–1688) during the late Ayutthaya period. Uneven settlement of the foundation ground is considered to have caused the inclination of this pagoda, although there is currently no way to measure the amount of settlement because the height of the surrounding ground surface has changed owing to the effects of several floods and repair work. Currently, the only information that indicates the deformation of this pagoda is its inclination of approximately 2° to the north.

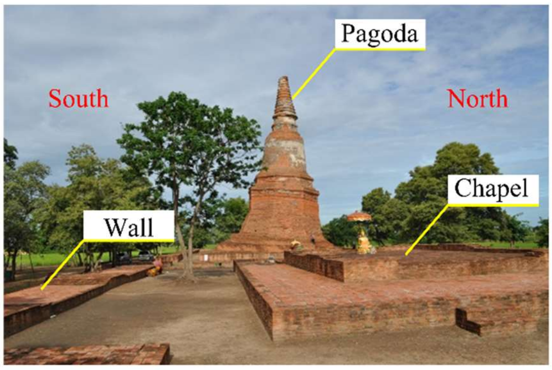

Fig. 1 Current status of Wat Krasai (view from the east side)

2.1 Inclination of Pagoda and Risk of Collapse

A pagoda is a structure in which bricks are glued together with mortar. Structures made of brick are generally prone to cracking along the joints. However, Wat Krasai is damaged because of the deterioration of the brick material (Fig. 2). This type of brick deterioration is concentrated in the northern base, which has become lower owing to the inclination of the pagoda. It also coincides with the part where rainwater flows on the pagoda surface are concentrated (Fig. 3). Wat Krasai is shaped like a cone overall, so there is little risk of the pagoda overturning, but the risk of partial collapse caused by tilting is present.

Fig. 2 Deterioration status of the northern base of the pagoda

Fig. 3 Rainwater traces on the northern base of the pagoda

2.2 Foundation Ground

Boring surveys were conducted at four locations around Wat Krasai to investigate the cause of its inclination (Fig. 4). Fig. 5 shows a cross-sectional view of the layers in the east-west and north-south directions estimated from the layer thickness confirmed in the boring surveys [10]. It is thought that the layer thickness directly under the pagoda is affected by the settlement, but the slope of the layer was estimated to be a straight slope because it could not be investigated owing to cultural property protection. The layer has a significant slope in the north-south direction, which is the inclination direction of the pagoda. However, assuming that the cause of inclination is the settlement of the three cohesive soil layers (Clays 1, 2, and 3), the Clay 1 and Clay 2 layers are thicker on the north side, and the Clay 3 layer is thicker on the south side. Hence, it is not clear at first glance in which direction the pagoda will tilt. Furthermore, soil parameters are extremely important information for calculating the inclination of the pagoda, but no consolidation tests for each layer were conducted, which constitutes a serious uncertainty factor.

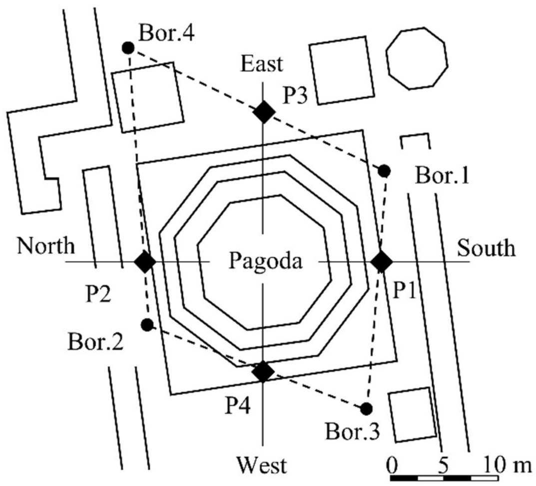

Fig. 4 Boring survey positions

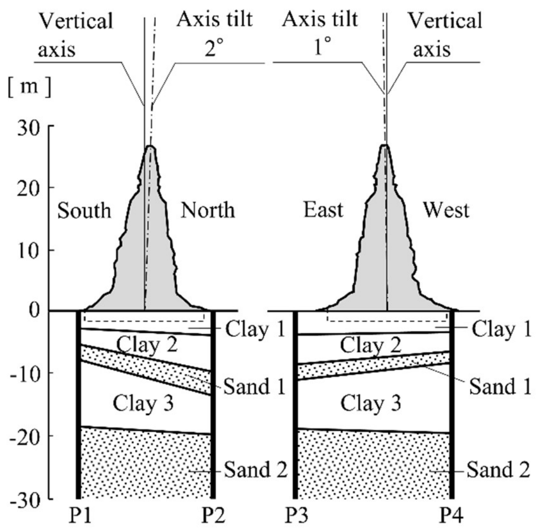

Fig. 5 Cross-sectional view of layers [10]

3. METHOD OF COMBINING PLANE STRAIN ANALYSIS WITH THREE-DIMENSIONAL ANALYSIS

It is important to consider the three-dimensional effect and overturning moment of a tower when using numerical analysis to simulate the tower inclination due to uneven settlement of the foundation ground. In other words, it is possible to calculate the tower inclination by performing geometrical and material nonlinear analysis using a three-dimensional model for the foundation ground and tower. However, numerous cases must be analyzed for Wat Krasai, where the soil parameters of the three cohesive layers have uncertainty factors. This assessment is considered difficult because of the general software and hardware environment. Therefore, in this study, the cause of the inclination of the pagoda was investigated by using a method that combines plane strain analysis, which has a low computational cost, with three-dimensional analysis. Fig. 6 shows the procedure of this method. Multiple cases considering the fluctuations in soil parameters (all cases) and a representative case (test case) were utilized, as will be discussed in greater detail later.

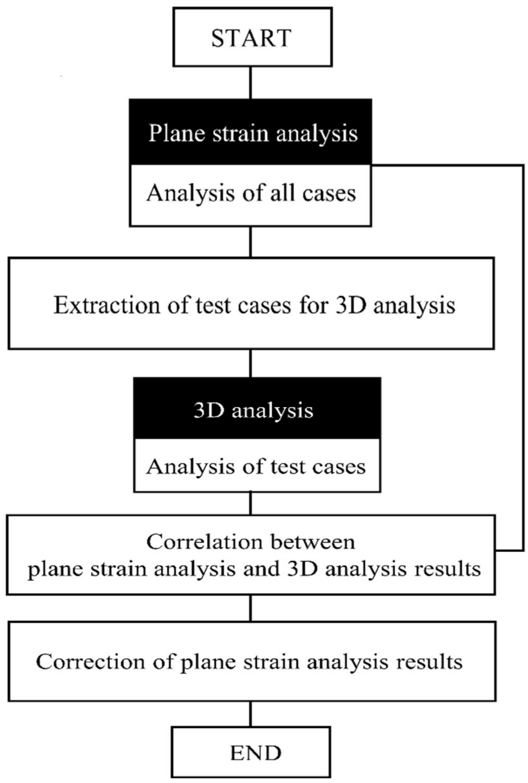

Fig. 6 Analysis procedure that combines plane strain and three-dimensional analysis

4. PLANE STRAIN ANALYSIS

4.1 Plane Strain Model

The slope of the layers was simply modeled in the north-south direction, which is the inclination direction of the pagoda. This approach was used because the north-south slope is greater than the east-west slope. Furthermore, the layers other than those directly below the pagoda were assumed to be horizontal because no survey results are available. Fig. 7 shows the FEM model utilized for the plane strain analysis. A model with unit depth width using solid elements was constructed to facilitate comparison with the three-dimensional analysis.

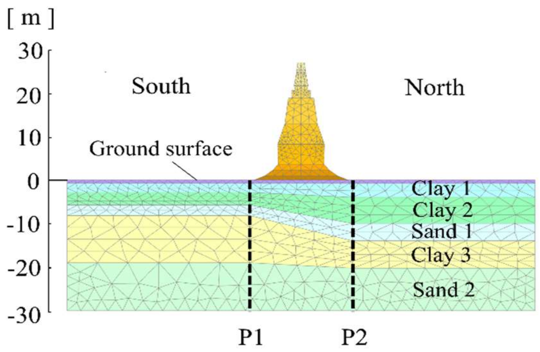

Fig. 7 Plane strain model

4.2 Modelling of Pagoda

The overturning moment, which increases with the tower inclination, was considered by weighting the tower elements and using the updated Lagrangian method. However, Wat Krasai is cone-shaped and hollow. Therefore, the pagoda was divided into ten blocks (Fig. 8), and the unit volume weight of each block was reduced considering the cavity (Table 1).

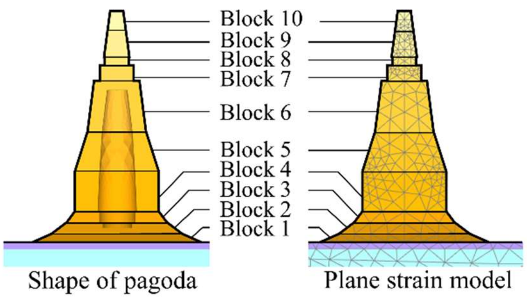

Fig. 8 Divided blocks to reduce unit weight of plane strain model

The pagoda was set as a linear elastic body, and material parameters were assumed to be typical brick values (E = 10,000,000 kN/m^2, ν = 0.18). However, the base of the pagoda (Blocks 1 and 2 in Table 1) was significantly damaged during the restoration work conducted in 2013, and it was shown that the damaged bricks were excavated from the surrounding ground. Consequently, an elastic constant similar to that of the ground on the ground surface was assumed for these blocks (E = 5,000 kN/m^2).

Table 1 Unit weights of the pagoda in-plane strain model

Block	Volume m^3	Area of plane strain model m^2	Unit weight of plane strain model kN/m^3
10	5.3	3.9	1.0
9	14.9	7.5	1.4
8	6.2	2.8	1.5
7	22.6	7.2	2.2
6	147.1	37.2	2.8
5	208.7	39.1	3.7
4	284.1	45.5	4.4
3	103.7	14.2	5.1
2	187.2	17.8	7.4
1	313.3	17.7	12.4

Note: The original unit weight of brick is 15kN/m^3.

4.3 Constitutive Model of Soil

The Sekiguchi-Ohta model was used as the constitutive model for the three cohesive soil layers (Clays 1, 2, and 3), which are thought to be the cause of the uneven settlement of the pagoda. This model is well known for being able to account for the anisotropy of stress due to natural deposition. The input parameters of the soil used for this model should be determined from the results of soil tests, but no consolidation test results are available for this site. Thus, the soil parameters of the three layers were changed such that they were within the range of possible fluctuations, and all of them were combined to perform a plane strain analysis of 512 cases to incorporate the uncertainty factors into the calculation of the pagoda inclination.

For each parameter, an approximate value was calculated using the flowchart in Fig. 9, after which the width of the fluctuations was considered. The flowchart in Fig. 9 was proposed by Iizuka and Ohta [11], and the parameters of cohesive soil required for consolidation analysis could be analyzed from the plasticity index. Furthermore, Iizuka and Ohta [11] showed variations in the estimation formulas for each parameter and many actual soil test results. The variance of this variation (±1σ) was used to set the fluctuation width of each parameter (Figs. 10–12). Table 2 lists the material parameters for each layer. In addition, the unit weight and coefficient of permeability were based on the soil test.

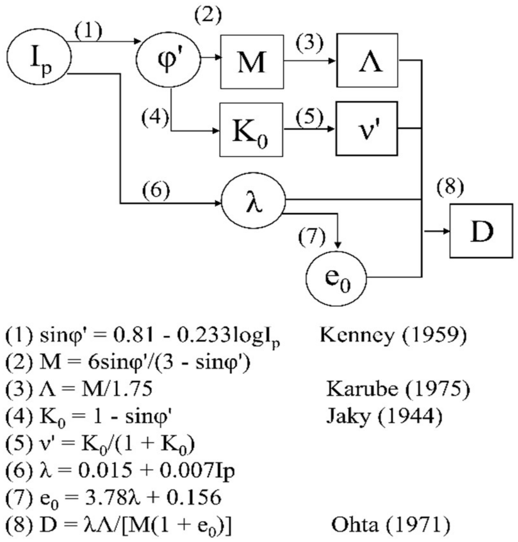

Fig. 9 Flowchart for estimating parameters from plasticity index [11]

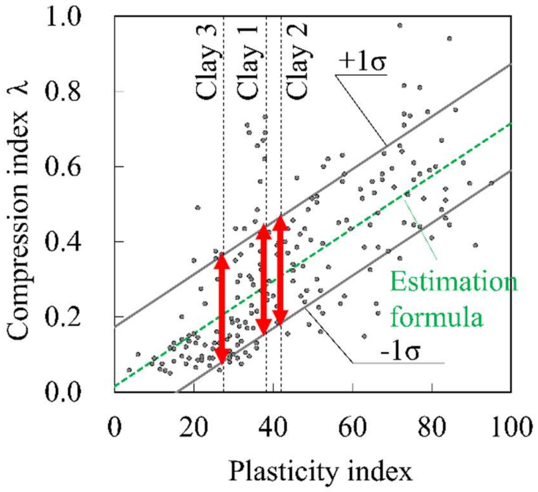

Fig. 10 Parameter fluctuation (Compression index λ)

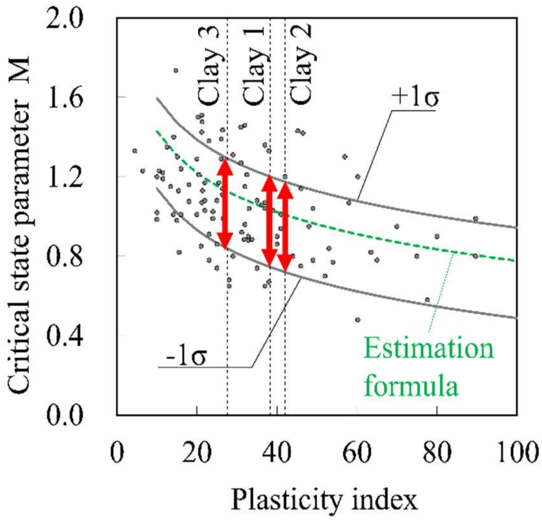

Fig. 11 Parameter fluctuation (Critical state parameter M)

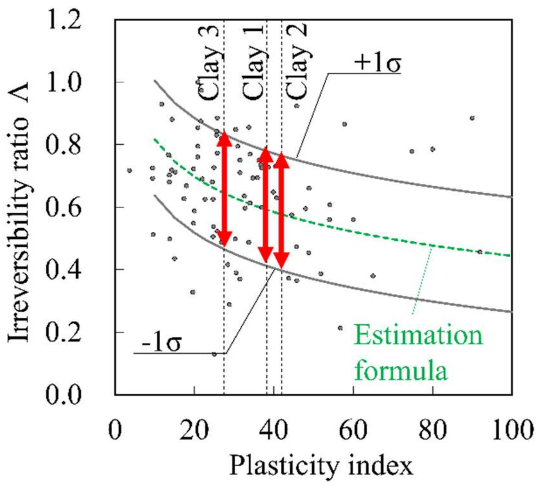

Fig. 12 Parameter fluctuation (Irreversibility ratio Λ)

Table 2 Material parameters considering fluctuations (Clays 1, 2, and 3)

	I_P	Unit weight kN/m³	Parameters of Sekiguchi-Ohta model				k m/day
				λ	Λ	M	
Clay 1	38.2	19.2	+1σ	0.440	0.779	1.201	2.4E-06
			−1σ	0.157	0.412	0.747	
Clay 2	42.0	17.1	+1σ	0.467	0.764	1.175	3.6E-06
			−1σ	0.184	0.397	0.720	
Clay 3	27.5	19.2	+1σ	0.365	0.832	1.294	3.6E-06
			−1σ	0.082	0.465	0.839	

Table 3 Material parameters (Sand 1, Sand 2, and ground surface)

	Material model	Unit weight	N	E	ν’	c’	φ’	k
		kN/m^3	-	kN/m^2	-	kN/m^2	°	m/day
Ground surface	MC	19.2	12.2	8450	0.2	80.0	6.0	2.4E-06
Sand 1	Linear elastic	20.0	21.5	15050	0.3	-	-	2.0E-04
Sand 2	Linear elastic	20.0	40.8	28560	0.3	-	-	1.2E-03

An elastic–perfectly plastic model (Mohr-Coulomb model) was used because the ground surface is cohesive soil that has been dried and solidified by sunlight, and the sand layers (Sand 1 and Sand 2) were assumed to be linear elastic bodies because they were dense silty sand. Young's modulus, c’, and φ’ were estimated from the N value as general values with reference to the Japanese design guidelines. In addition, the unit weight and coefficient of permeability were based on the soil test. The soil parameters of these layers are listed in Table 3. The bottom surface of the model was set as a non-drainage boundary condition because it is known that a cohesive soil layer is distributed in the Ayutthaya area at depths of 30 m and deeper.

4.4 Time History Settings

The time history shown in Table 4 was set considering the period from the beginning of the reign of King Narai (1656) to the restoration work performed in 2013.

Table 4 Time history used for analysis

		Time
		days
Construction (1656–1658)	Block 1	177
	Block 2	106
	Block 3	59
	Block 4	160
	Block 5	118
	Block 6	83
	Block 7	13
	Block 8	3
	Block 9	8
	Block 10	3
Consolidation (1658–2013)		129,661

The construction period of the pagoda was set to two years, and the construction period of each block was estimated from the ratio of the volume of each block to the volume of the entire pagoda.

4.5 Plane Strain Analysis Results

Fig. 13 presents the results of the plane strain analysis of 512 cases with different parameter combinations. The vertical axis shows the settlement of the ground surface in the center of the pagoda, and the horizontal axis shows the inclination of the pagoda. The inclination varies between 0.1° and 1.1° (northward inclination) when fluctuations of ±1σ are considered for the parameters of each layer, and reproducing the current inclination of the pagoda (2°) is difficult with plane strain analysis that ignores the three-dimensional effect. Furthermore, the settlement amount and inclination do not have a linear proportional relationship, and the inclination of the pagoda does not reach 2° even when assuming soft ground where settlement is likely to occur.

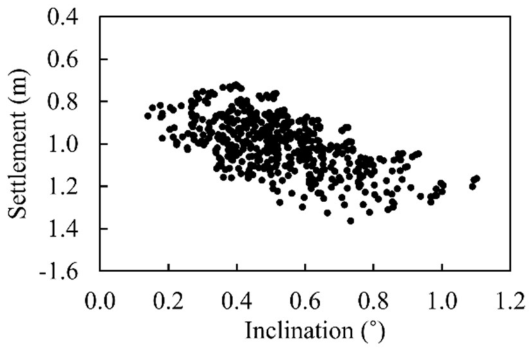

Fig. 13 Relationship between the settlement amount and inclination (plane strain analysis)

5. THREE-DIMENSIONAL ANALYSIS TO CORRECT PLANE STRAIN ANALYSIS

5.1 Extraction of Test Cases for Three-Dimensional Analysis

Several cases of three-dimensional analysis must be selected to correct plane strain analysis. As shown in Fig. 14, the pagoda inclination varies between 0.1° and 1.1° depending on the combination of parameters according to the plane strain analysis. Therefore, the combination of the parameters indicated by the circles in Fig. 14 was used as a parameter in the three-dimensional analysis. Here, this combination is referred to as the test case (approximately 4% of all cases).

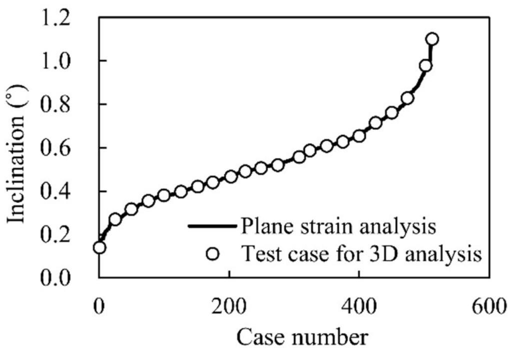

Fig. 14 Extraction of test cases for three-dimensional analysis (case numbers are arranged in ascending order of inclination

5.2 Three-Dimensional Model

Fig. 15 shows the FEM model used for the three-dimensional analysis. The three-dimensional model was set as a half model with the center of the pagoda as a symmetrical plane, and the layer shapes were extended to the west from the cross-section of the plane strain analysis. The pagoda shape was modelled with the same hollow cone-shaped structure described previously.

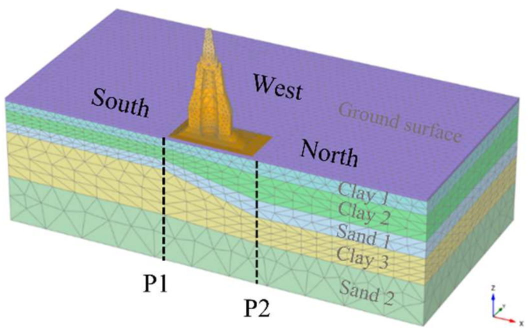

Fig. 15 Three-dimensional model

5.3 Plane Strain Analysis Correction Using Three-Dimensional Analysis Results

Fig. 16 presents the results of the three-dimensional analysis and plane strain analysis of the test case. Fig. 16(a) depicts the pagoda inclination, and Fig. 16(b) shows the settlement of the ground surface in the center of the pagoda. A linear relationship can be seen between the three-dimensional analysis results and plane strain analysis results. The slopes of the straight lines are 3.4 and 2.0 for the inclination and settlement, respectively, and these values are referred to as the three-dimensional effect factors.

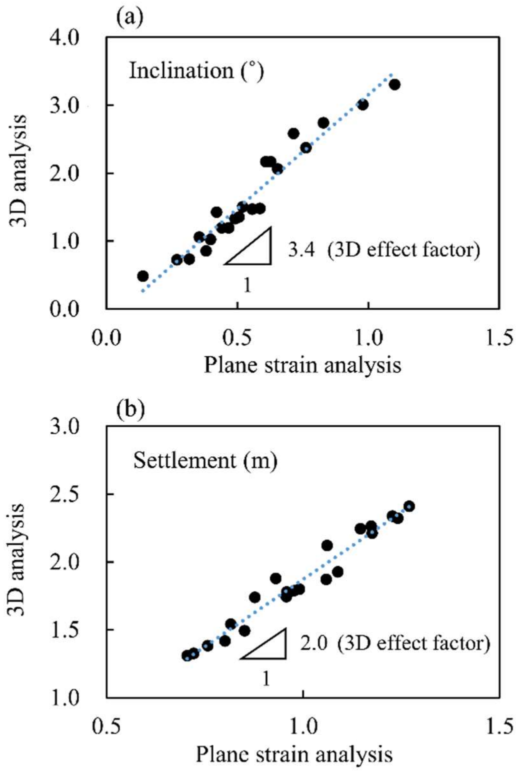

Fig. 16 Correlation between the three-dimensional analysis results and plane strain analysis results (test case)

Fig. 17 shows the results of the plane strain analysis (black points) and the correction results obtained by multiplying times the three-dimensional effect factors (blue points).

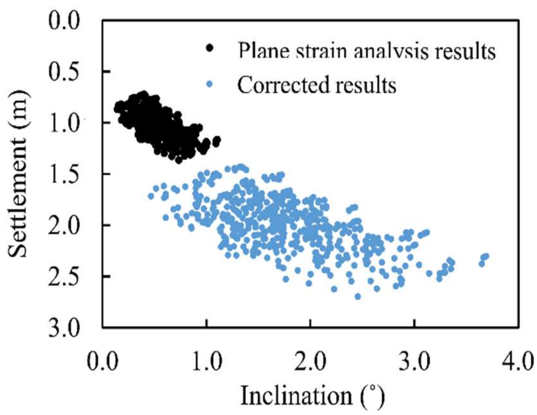

Fig. 17 Results of correcting the plane strain analysis

Even when using the same combination of parameters, it can be seen that correcting the plane strain analysis results using the three-dimensional effect factor greatly changed the settlement and inclination to 1.5–2.7 m and 0.5–3.7°, respectively.

Fig. 18 depicts the probability density functions of the plane strain analysis results and the results corrected using the three-dimensional effect factor. The plane strain analysis cannot reproduce the actual inclination angle of the pagoda (2°), but an inclination of over 2° can be generated within a realistic parameter range by correcting the results using the three-dimensional effect.

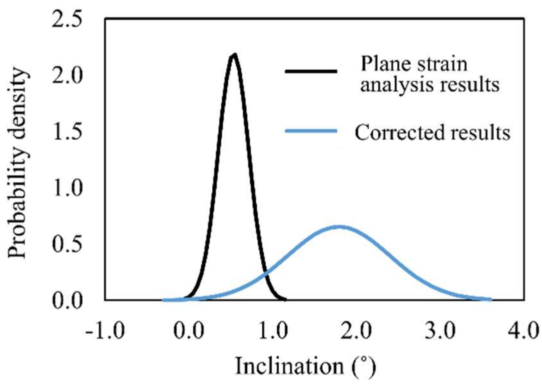

Fig. 18 Results of correcting the plane strain analysis

6. FUTURE CONSERVATION OF THE PAGODA

It was quantitatively shown in this study that the inclination of Wat Krasai could be caused by the uneven settlement of the three cohesive soil layers. However, the focus of this analysis was the inclination that has occurred in the 350 years since the construction of the pagoda. Therefore, one may consider what the inclination of the pagoda will be in the future, which is of interest for pagoda conservation. Wat Krasai underwent a major restoration in 2013, with a new foundation and brick restoration installed in the damaged base to cover the perimeter of the pagoda. As shown in Table 5, the total volume of the pagoda after the restoration was estimated to be 1.36 times that before restoration. The increased load will cause new deformations if the cause of the pagoda inclination is an uneven settlement.

Fig. 19 shows the gap between the reconstructed base and pagoda on the side opposite to the direction of the pagoda inclination (south side of the pagoda). Measurements of the changes in gap length during the six months from August 2019 to February 2020 showed that this length increased from 30 mm to 45 mm on average. The increase in gap length indicates that the pagoda may continue to tilt, and the new loads introduced by the restoration work will need to be considered for the future conservation of the pagoda.

Table 5 Volume increases of Wat Krasai due to restoration in 2013

Block	Volume: Before Restoration	Volume: After Restoration
	m^3	m^3
10	5.3	5.3
9	14.9	14.9
8	6.2	6.2
7	22.6	22.6
6	147.1	153.2
5	208.7	227.8
4	284.1	384.0
3	103.7	171.7
2	187.2	305.3
1	313.3	466.6
Total	1293.0	1757.5

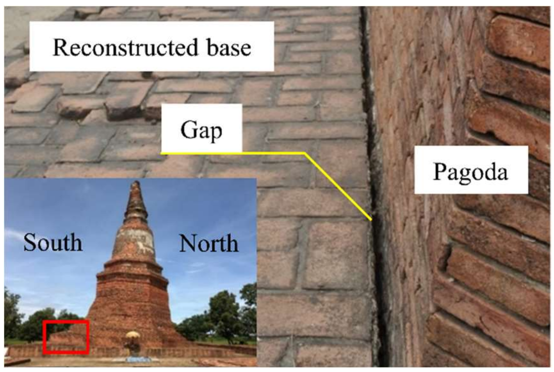

Fig. 19 Gap between the reconstructed base and pagoda (February 2020)

7. DISCUSSION

In this study, a method of correcting plane strain analysis with a three-dimensional effect coefficient was utilized to assess over 500 cases considering ground material uncertainties, and the probability distribution of the possible inclinations was calculated. The efficacy of this approach can be summarised as follows:

(1) The efficacy of the method of using plane strain analysis, which considers the fluctuation of the soil material, together with three-dimensional analysis, was demonstrated for the problem of ground subsidence, which is thought to have a large three-dimensional effect. This technique can be regarded as one of the methods of applying V & V to numerical analysis using FEM.
(2) Results that were almost identical to those of the three-dimensional analysis could be obtained from the plane strain analysis results by using the method described in this paper without requiring extensive three-dimensional analysis. However, three-dimensional effect factors may differ for each boundary value problem, so care must be taken when calculating the three-dimensional effect factor.

8. CONCLUSION

This study concludes that the current inclination of Wat Krasai can occur within the realistic ranges of the soil properties. Furthermore, in terms of the conservation of Wat Krasai, the following statements can be made:

(1) The future inclination must be predicted by combining the effects of the weight increases caused by the restoration work conducted in 2013.
(2) The risk that the deformation of the northern base, which is thought to be caused by the inclination, could lead to the collapse of the entire pagoda must be investigated. In addition, climate change increases the risk of flooding in Ayutthaya. It is important to investigate the impact of rainfall and floods on the pagoda.
(3) This study targeted foundation ground that does not include artificial countermeasure construction methods. In future research, we will examine evaluation methods, including countermeasures, that can contribute to the conservation of the authenticity of the leaning structures and foundations.

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