EVALUATION OF PLASTICITY-BASED SOIL CONSTITUTIVE MODELS IN SIMULATION OF BRACED EXCAVATION

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ABSTRACT: Numerical modelling continues to play a unique and intrinsic role in the process of geotechnical design. Of greatest concern are soil constitutive models that are employed within finite element software to predict soil behaviour. The objective of this paper is to provide a numerical study of the Mohr-Coulomb and Hardening Soil constitutive models in simulation of a braced excavation. The Taipei National Enterprise Centre (TNEC) basement construction process was well documented and the commercial finite element code, Plaxis, was selected for this numerical comparative study. It was found that the Mohr-Coulomb soil model, a first order approximation, produced an underestimation of the diaphragm wall deflection, whilst the Hardening Soil model provided a good prediction of the observed in-situ diaphragm wall deflections.

Keywords: Constitutive Soil Models, Plasticity Theory, Finite Element Method, Numerical Modelling

1. INTRODUCTION

The behaviour of soil varies all over the world and continues to present a unique challenge to geotechnical engineers in professional practice on a daily basis. As a result, numerous soil theorists have emerged attempting to explain the exciting phenomenon observed under various in-situ site conditions. The fundamental knowledge an engineer employs when analysing an engineering system directly correlates to the success, safety, and precision of results.

On April 20th 2004, a 30m deep excavation adjacent to the Nicoll Highway in Singapore collapsed during construction. The transport authorities were constructing an underground railway tunnel when the braced excavation support failed, initiating rapid collapse of the diaphragm walls leaving a collapse zone which was 150m wide, 100m long and 30m deep. This tragedy took the lives of two workers, an engineer and a site foreman. Furthermore, economic losses were substantial with delays contributing to part of the US$4.14 billion subway project [1]. Upon post failure investigation, it was noted that certain errors in the input data of the original constitutive model used in design ultimately led to incorrect assumptions and underestimated calculations [1]; resulting in catastrophic failure.

In recent times, theoretical developments have significantly grown and with the advance of computer technology and software programming, Finite Element (FE) methods are readily available to assist geotechnical engineers in the interpretation, modelling and design of complex soil systems. Unfortunately, a minority of modern day geotechnical engineers have become accustomed to user-friendly computer software and consequently fail to comprehend the fundamental theories, principles and assumptions built within.

FE analysis is built upon the concept of continuum mechanics and constitutively models a soil system from its stress-strain characteristics. Various constitutive models have been proposed in literature to capture the properties and features of numerous soil types and model its responses to surface loading, displacements, excavation, slope stability and many other geotechnical actions.

This paper seeks to provide a brief overview into the fundamental principles, which directly influence the level of accuracy and prediction the MC, and Hardening Soil (HS) constitutive models offer.

The Mohr-Coulomb (MC) linear elastic soil model is an example of a commonly used constitutive relationship utilised in industry. This particular model provides a first order approximation helpful in providing a preliminary analysis to the problem. However, for an effective analysis to be carried out, it is imperative for the user to recognise the uncertainties and limitations each model has to offer, and consequently make a judgement between the reliability of the numerical results and the uncertainty therein.

A braced excavation simulation of the Taipei National Enterprise Centre (TNEC) has been modelled using the FE software package Plaxis.

Stage by stage construction procedures have been replicated in order to simulate in-situ conditions recorded on site. The diaphragm wall deformations obtained from field data history have been evaluated with respect to numerically computed wall deformations. Both the MC and HS constitutive models have been selected for this numerical comparative study

2. NUMERICAL APPROACH

2.1 Stress State and Equilibrium

Continuum theory considers solids, in this case soil particles, and fluids to behave as a continuous media. Equations of equilibrium and compatibility are independent of a material's physical properties, and thus form the basis of numerical modelling. There are three normal stress rates ($\dot{\sigma}$xx, $\dot{\sigma}$yy, $\dot{\sigma}$zz) and three shear stress rates ($\dot{\sigma}$xy, $\dot{\sigma}$yz, $\dot{\sigma}$xz). The spatial derivatives of the aforementioned stress rates are then assembled in a vector denoted σ and combined with the body force components, *x*, *y* and *z* assembled in a vector b. This presents the static equilibrium of a continuum [8].

$$\underline{L}^T \cdot \boldsymbol{\sigma} + \boldsymbol{b} = 0 \tag{1}$$

where $\underline{L}^T$ is the transpose matrix of a differential operator [8]. Similarly, the kinematic relationship between stress and strain can be formulated accordingly and is referred to as a constitutive relationship seen in Eq. 2 below.

$$\dot{\boldsymbol{\sigma}} = \underline{M}\dot{\boldsymbol{\varepsilon}} \tag{2}$$

where $\dot{\boldsymbol{\sigma}}$ and $\dot{\boldsymbol{\varepsilon}}$ is the stress and strain tensor respectively and $\underline{\boldsymbol{M}}$ is the material stiffness matrix.

3. CONSTITUTIVE THEORY

The constitutive relationship for a material depends on the homogeneity, isotropy and continuity of the body material, as well as its response to cyclic loading, and the rate and magnitude of the applied load [2]. General techniques have been developed to characterise soil as elastic, plastic, or viscous in nature to consequently conduct constitutive analyses.

3.1 Theory of Plasticity

Classical theory developed by Hill [4] seeks to explain the stress and strain behaviour of plastically deformed solids and is fundamentally analogous to Hooke's law which stipulates the relationship between stress and strain governed by

the material's modulus. It is important to note that a material's total strain rate is controlled by an elastic and plastic rate component.

$$\dot{\boldsymbol{\sigma}} = \underline{M}(\dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p) \tag{3}$$

The yield limit of an elastic soil material is defined by a yield function, denoted f, and is a function of the stress components, friction angle, φ and cohesion, c. The failure limit under all deviatoric loading combinations for a perfectly plastic material remains fixed and does not move in principal stress space. Hill [4] states that plastic strain rates are proportional to the derivative of the yield function with respect to the stresses. This notion provides the basis upon which plastic deformation can be determined once the stress point (p-q) reaches the yield surface.

$$\dot{\boldsymbol{\varepsilon}}^p = \lambda \frac{\partial f}{\partial \dot{\sigma}} \tag{4}$$

Experimental data indicates that plastic strain rates are not always orthogonal to the yield surface and hence cannot be accommodated under the coaxial assumption. Therefore, the plastic potential function has been derived to model this type of plastic strain rate behaviour and considers the dilatancy angle, ψ, where $\varphi \neq \psi$, to avoid the previous overestimation of dilatancy under the normality rule.

3.2 Mohr-Coulomb Soil Model

The MC model in Plaxis captures the linear elastic perfectly plastic stress-strain behaviour of a soil element when considered in its general stress state, and all deformations are fully recoverable upon unloading. Once the stress point (p-q) is loaded past the model's elastic limits, φ and cohesion, c, define a fixed shear failure surface upon which the stress point (p-q) is assumed to follow.

$$\tau = \sigma'_m tan(\varphi') + c' \tag{5}$$

The above failure criterion produces a linear failure envelope for a two-dimensional analysis. As the development of plastic strains occurs, the yield surface does not admit changes of expansion or contraction and hence is considered a fixed yield surface (perfectly plastic).

3.3 Hardening Soil Model

The HS model in Plaxis is a second order constitutive relationship that seeks to describe the non-linear behaviour of soil upon yielding and is

derived from the hyperbolic model of Duncan and Chang [3]. The term hardening within this context defines the various changes, in size, location or shape, of the yield surface and is directly related to the loading history measured by a form of plastic deformation [10]. Note, the Plaxis HS model considers isotropic hardening only.

Isotropic hardening is governed by the assumption that expansion or contraction about the centre of the yield surface is uniform, whilst the shape, centre and orientation of the yield surface remain unchanged. Fundamentally this type of hardening behaviour can be characterised into two components: shear hardening and compression hardening. When a soil body is subjected to primary deviatoric loading, plastic axial strains develop as observed in a triaxial test. These irreversible strains are consequently accounted for by the shear hardening component of the HS model. Furthermore, compression hardening is used to model the irreversible plastic volumetric strains due to primary compression in oedometer loading and isotropic loading. Careful consideration to stress dependent stiffness is a fundamental element of the HS model and is associated with a power law value, m. This power value equates to 1 when a linear analysis is assumed and is typically taken as 0.5 (recommended for hard soils) when attempting to model hyperbolic stress-strain behavior.

This model requires three input stiffness parameters, namely: E_{50}, E_{oed}, and E_{ur}. E_{50} is the confining stress dependent stiffness modulus for primary loading and is described by Eq. (6).

$$E_{50} = E_{50}^{ref} \left(\frac{c \cos\varphi - \sigma_3 \sin\varphi}{c \cos\varphi - p^{ref} \sin\varphi} \right)^m \tag{6}$$

For unloading and reloading elastic stiffness, Eur is defined as follows:

$$E_{ur} = E_{ur}^{ref} \left(\frac{c \cos\varphi - \sigma_3 \sin\varphi}{c \cos\varphi - p^{ref} \sin\varphi} \right)^m \tag{7}$$

The oedometer stiffness modulus, Eoed for primary compression is:

$$E_{oed} = E_{oed}^{ref} \left(\frac{c \cos\varphi - \sigma_3 \sin\varphi}{c \cos\varphi - p^{ref} \sin\varphi} \right)^m \tag{8}$$

Note, p^{ref} is defined as the reference pressure and is taken as 100 kN/m² within the Plaxis FEM software [8]. $E_{50}^{ref}, E_{ur}^{ref}$, and E_{oed}^{ref} are the respective stiffness moduli corresponding to p^{ref}.

The first type of hardening, shear hardening has a linear flow relationship and is characterised by the development of plastic strains when mobilising the soil's material strength, or

increasing the soil's preconsolidation stress, commonly referred to as compaction hardening. The fundamental shear hardening yield function is given by [8]:

$$f = \overline{f} - \gamma^p \tag{9}$$

where $\overline{f}$ is a function of stress and γ^p is the strain hardening parameter and is expressed as a function of plastic strains [8]:

$$\overline{f} = \frac{2}{2E_{50}} \frac{q}{1 - \frac{q}{qa}} - \frac{2q}{E_{ur}} \tag{10}$$

$$\gamma^p = -(2\varepsilon_1^p - \varepsilon_v^p) \tag{11}$$

For hard soils, plastic volume changes tend to be relatively small and hence ε_v^p can be assumed to equal 0 (hard soils only). Hence, the combination of Eq. 10 and Eq. 11 produces multiple yield surfaces with increasing values of γ^p as seen in Fig 1.

The second type of hardening mechanism is plastic volumetric strain. As an element of soil undergoes compressive loading, a stress point asymptotically follows the yield locus respective to its strain hardening parameter, thus the induction of a yield surface limits the elastic region upon which the stress point is asymptotically moving, based on a direct relationship between E_{50}^{ref} and E_{oed}^{ref}. Hence, the shear yield surface is controlled by the triaxial modulus and the cap yield surface by the oedometer modulus respectively [8]. The reader is suggested to make reference to Schanz et al. [9] published paper entitled, *"The Hardening Soil Model: Formulation and Verification"* for the mathematical derivations of the yield cap surface.
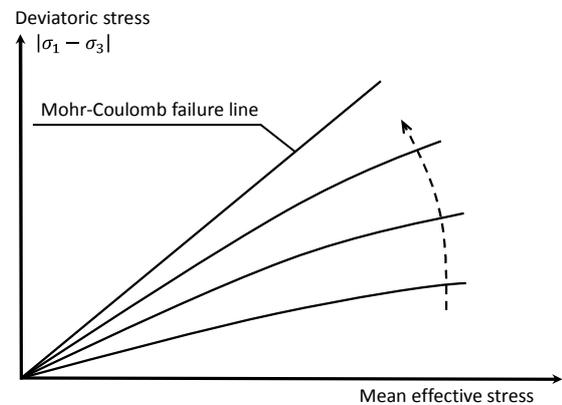

Fig. 1   Yield loci for varying constant values of the γ^p parameter

4. TNEC CASE STUDY

The Taipei National Enterprise Centre (TNEC) is the chosen case study for this paper due to readily available documentation of construction procedures and in-situ conditions released by Ou et al. [6]. The TNEC building was built in 1991 and comprises of 18 storeys and 5 basement levels. The TNEC was excavated to a depth of 19.7m using the top-down construction method and a retaining wall measuring 90cm thick and 35m deep was elected to be supported by a series of temporary steel struts and concrete floor slabs. According to site investigation data, the soil located within the Taipei basin consisted of six layers of alternating silty clay and silty sand deposits overlying a thick gravel formation. The groundwater level was observed at a depth of 2m below the ground surface, with incremental dewatering corresponding to excavation depth. General phreatic level conditions were assumed, generating hydrostatic pore pressure distribution. Soil-water coupling was not considered since the CL soil layers were analysed as undrained. Fig. 2 depicts the excavation cross-section. Numerical parameters are showed in Tables 1-4.
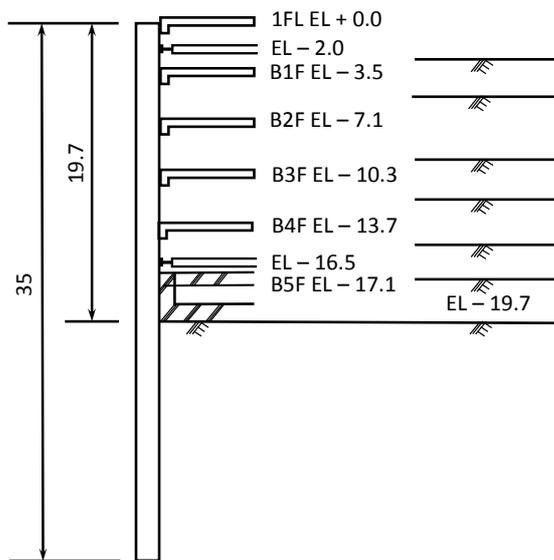

Fig. 2 TNEC excavation cross-section with relevant dimensions in metres [6].

Table 1 Input parameters of diaphragm wall [7]

Parameter	Abbr.	Value	Unit
Axial Stiffness	EA	22140000	kN/m
Flexural Rigidity	EI	1494450	kN/m^2/m
Wall Thickness	d	0.9	m
Weight	w	21.6	kN/m/m
Poisson's ratio	v	0.15	-

Table 2 Input parameters of struts and slabs [7]

Strut	Behaviour	EA (kN)	L_s (m)
H300	Elastic	1954810	8
H400	Elastic	3569021	3
B1F	Elastic	3690000	1
B2F	Elastic	3690000	1
B3F	Elastic	3690000	1
B4F	Elastic	3690000	1
B5F	Elastic	3690000	1

Table 3 Input parameters for HS model

Type	GL -m	m	v	E^{ref}_{50} kN/m^2	E^{ref}_{ur} kN/m^2	E^{ref}_{oed} kN/m^2
CL	5.6	0.5	0.2	3531	10594	4414
SM	8	-	0.3	-	-	-
CL	33	0.5	0.2	36022	108067	45028
SM	35	-	0.3	-	-	-
CL	37.5	0.5	0.2	65236	195709	81545
SM	46	-	0.3	-	-	-

Table 4 Input parameters for MC model

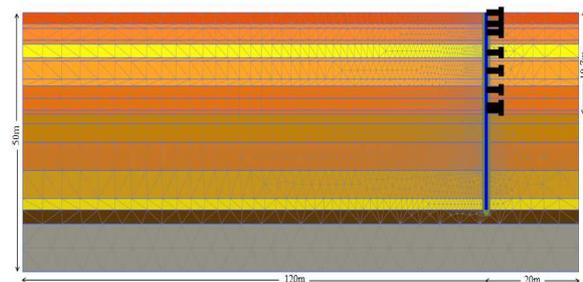| Type	GL -m	γ_t kN/m^3	ϕ' °	v	s_u kPa	E kN/m^2
CL	2	18.25	34	0.5	5	2500
CL	5.6	18.25	34	0.5	30	15000
SM	8	18.93	31	0.3	-	56471
CL	13	18.15	29	0.5	45	22500
CL	18	18.15	29	0.5	35	17500
CL	23	18.15	29	0.5	50	25000
CL	28	18.15	29	0.5	75	37500
CL	33	18.15	29	0.5	100	50000
SM	35	19.62	31	0.3	-	173620
CL	37.5	19.13	0	0.5	125	62500
SM	46	19.62	32	0.3	-	192571

Fig. 3 Finite element mesh of the TNEC

5. RESULTS AND DISCUSSION

Figure 3 shows the FE mesh of the TNEC model used for Plaxis modelling. Lateral boundaries have been assumed free-rollers, while the bottom boundary is fixed. In-situ soil parameters were obtained from Ou et al. [6] and Hui & Hie [5]. In the analysis, construction stages were performed by deactivating FE elements in accordance with construction procedures. With consideration to consistency between MC and HS simulations, the drained layers, represented as SM were modelled using MC parameters during the HS simulation. The drained layers consisted of silty sand and hence the MC model provided an accurate representation for those respective layers. It can be seen in Fig. 4 that the MC model underestimated the observed deflections of the diaphragm wall quite significantly. The maximum deflection value predicted by this model was approximately 7.0cm in comparison to the maximum observed deflection of 9.95cm. In contrast, the HS constitutive model provided a much better prediction of the excavation process and slightly overestimated the maximum deflection of the wall by approximately 0.35cm.

Firstly, the MC model provides a first order prediction of the excavation procedure and consequently assumes linear elastic behaviour during the excavation process. Within the strain range of 10^{-3}-10^{-6}, it is common for soils to exhibit non-linear stress-strain behaviour where the variability of stiffness is readily observed. This does not indicate the soil has fully yielded, because
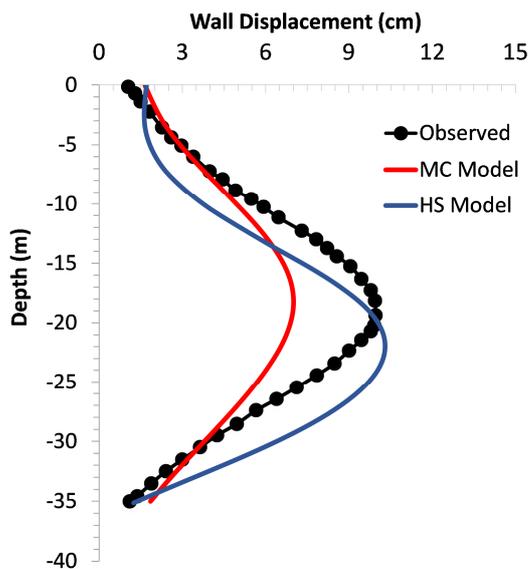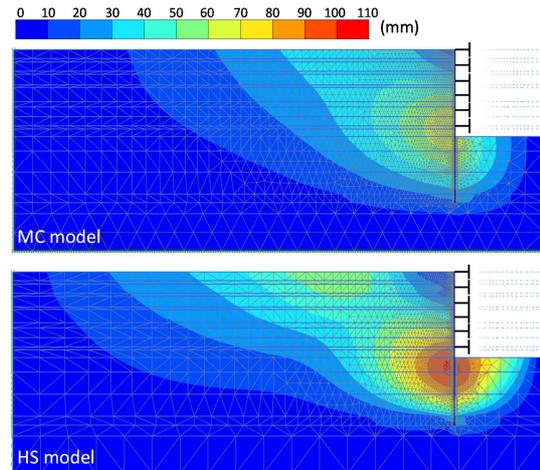

Fig. 5   Comparison of MC and HS soil models for horizontal displacement.

the soil is able to almost recover initial stiffness upon unloading, it does however behave differently to the MC assumption of linear elasticity. This results in the underestimation of horizontal displacements occurring behind the retaining wall since the soil stress path remains in the linear elastic domain and the development of plastic shear strains are erroneously overlooked. A key feature of the HS model is its capability to capture the non-linear elastic stress-strain relationship that is typically observed in soils before yielding. This type of hardening mechanism behaviour is referred to as deviatoric hardening, and has been introduced earlier in this paper. When a triaxial test is performed on a soil sample, the shear hardening phenomenon is observed due to the development of shear strains under confining stress conditions. This type of confining stress state is generally experienced by a soil element located behind a retaining wall, where the lateral resistance of the diaphragm wall, coupled with lateral soil pressure induces a deviatoric stress. As the development of plastic deformation increases, the degradation of soil stiffness occurs, thus resulting in considerable diaphragm wall deflection and surface settlement. The MC model is unable to explain this type of behaviour due to its elastic framework which restricts the modelling of plastic shear strain influence.

In addition, the HS model takes into account the stress dependent stiffness of soil. Upon loading, a soil's stiffness is expected to increase due to the densification of soil particles, resulting in a lower volume of voids. This phenomenon is observed when plastic deformation begins to occur, i.e. the permanent rearrangement of the soil's lattice structure. Similarly, when a soil is subject to unloading, exemplary of an excavation

Fig. 4   Comparison of observed diaphragm wall deflections with MC and HS soil models.

scenario, the increasing levels of strain significantly reduce the soil's stiffness attributed by the decrease in confining stress. This highlights the co-dependent relationship between stress and stiffness which the MC model fails to account for. Due to this fundamental behavioural characteristic of soil, constant stiffness cannot be assumed, especially when considering a soil body in undrained conditions. Due to the limitations of the MC model, primary compression stiffness, E_{oed}, is automatically assumed to equal the soil's initial modulus, E_o, and constant elastic stiffness is assumed as a result. This inevitably overestimates the soil body's ability to remain rigid upon unloading as it is unable to account for the softening effect of the soil, thus resulting in the significant underestimation of wall deflection. By alternatively incorporating the three input stiffness parameters into the HS model, E_{50}, E_{ur}, and E_{oed}, this permits the scale of soil deformations to be modelled much more precisely as it takes into account the stiffness variation of soil with loading. Furthermore, this model is able to express a reduction of mean effective stress observed in soft soils when operating in undrained conditions.

The secondary hardening mechanism of the HS model, volumetric hardening, provides an elastic deviatoric hardening limit and when reached plastic volumetric strains are seen to develop. This is another key difference between the MC and HS models, since the MC model is unable to distinguish volumetric strain from shear strain. Thus, when a soil body is loaded past its preconsolidation pressure, p_o the soil's modulus is typically overestimated. The HS model however derives a cap yield surface based on p_o, which delineates the beginning of plastic volumetric failure, thus avoiding the overestimation of a normally consolidated soil's stiffness. Since the yield surface of the MC model is based upon φ and c, the development of plastic strains do not expand or contract the failure surface accordingly. Hence, the yield surface of the MC is fixed. The HS model accommodates the expansion or contraction of the shear yield surface and introduces an additional volumetric cap yield surface which induces the reduction of soil stiffness as a result of high amplitude strain.

6. CONCLUSION

The TNEC braced excavation case study is one example of the reliability and realistic prediction of soil deformation associated with the application of non-linear elastic constitutive models. The HS model provided a competent result in comparison to observed diaphragm deflections. This was directly due to the model's incorporation of deviatoric and volumetric hardening mechanisms, stress-path dependent stiffness, soil dilatancy and the expansion or contraction of the yield surface with respect to plastic straining. In contrast, the MC model assumed linear elastic perfectly plastic behaviour which significantly underestimated the diaphragm wall deflections.

7. REFERENCES

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