Int. J. of GEOMATE, Dec, 2013, Vol. 5, No. 2 (Sl. No. 10), pp. 678-684
Geotec., Const. Mat. & Env., ISSN:2186-2982(P), 2186-2990(O), Japan

ULTIMATE BEARING CAPACITY ANALYSIS OF GROUND AGAINST INCLINED LOAD BY TAKING ACCOUNT OF NONLINEAR PROPERTY OF SHEAR STRENGTH

Du N L[1], Ohtsuka S[1], Hoshina T[1], Isobe K[1], and Kaneda K[2]
[1]Department of Civil and Environmental Engineering, Nagaoka University of Technology, Japan
[2]Takenaka corporation, Japan

ABSTRACT: In the assessment of bearing capacity of footing, the bearing capacity formula proposed by The Architectural Institute of Japan (AIJ) has been widely used in Japan for the design of building foundation. However, this formula is limited to simple conditions like simple footing shape, flat ground and uniform material property. Although the rigid plastic finite element method (RPFEM) can solve this complex problem, it has not taken into account the size effect in assessment of bearing capacity. This study newly develops the RPFEM by introducing the nonlinear shear strength property against the confining stress and proposes the rigid plastic constitutive equation of parabolic yield function, basing on the change in the angle of shear resistance of Toyoura sand. The developed method and the bearing capacity formula (established by empirical approach) provided identical results for various footing sizes showing good estimation for wider range of footing size.

Keywords: Rigid plastic finite element method, Bearing capacity, Inclined load effect, Size effect

1. INTRODUCTION

Assessment of bearing capacity is necessary to ensure the stability of a building. However, the bearing capacity of footing could be considerably influenced by many factors, including inclination load, eccentric load, shape, depth, size effect, etc.

Many researches have investigated these effects on bearing capacity. Terzaghi & Peck (1948) developed the classical bearing capacity equation as follow:

$$q_u = \gamma . D_f . N_q + c . N_c + 0.5\gamma . B . N_\gamma$$

and defined bearing capacity factors N_c, N_q, N_γ in terms of a linear strength envelope $\tau_{max} = \sigma \tan\phi + c$.

Meyerhof (1951, 1963), and Brich Hansen (1970) based on the limit equilibrium method and the slip line method which were extended by the original proposal of Terzaghi (1948) to apply that fomurla with more general factors and also derived expressions for the inclination factors.

In Japan, bearing capacity formula proposed by Architectural Institute of Japan (AIJ) has been also widely used. This formula uses bearing capacity factors N_c, N_q given by Prandtl, N_γ and inclination load factors i_c, i_γ, i_q described by Meyerhof, and other extended factors: shape coefficient α,β defined on the basic of footing shape and size effect factor. In recent years, finite element method has envolved as a well known method in structural analysis, a revival of interest in finite element, mainly focusing on soil mechanics problems and exploiting the kinematic.

On the other hand, rigid plastic finite element method uses only strength parameters of the ground. RPFEM is an analysis method which has been developed in the field of plastic working of metal. It is the effective analysis method to calculate the ultimate bearing capacity of the ground. Initially, RPFEM was established on the basic of the upper bound limit analysis, then it developed the rigid plastic constitutive equation to provide the similar result as in limit analysis. RPFEM with several constitutive equations is quite often utilized to predict the critical behavior.

However, RPFEM does not take into account the size effect in assessment of bearing capacity. This study newly develops the RPFEM by introducing the nonlinear shear strength property against the confining stress and proposes the rigid plastic constitutive equation of parabolic yield function against the confining stress.

This paper describes a simple method of calculating bearing capacity by using rigid plastic finite element method which is derived from the rigid plastic constitutive equation, and then compares the result which those of other methods (limit equilibrium method (Meyerhof, AIJ) and, Slip line method (Hansen) with the width range of foundations from 1m to 100m.

Results obtained from this study shows that although Meyerhof, Hansen or AIJ methods are widely used in design, they are not sufficiently clarified. As we have known, although many experiments have been carried out with model tests, a conclusion for the size effect of model scale is difficult to obtain. Moreover, these formulas use the simple linear strength model. This study develops RPFEM to solve the boundary problem with kinematical properties and provide a summary of good estimation for wider range of footing size.

2. RIGID PLASTIC FINITE ELEMENT METHOD (RPFEM)

2.1 Limit analysis and its application to FEM

Hill (1951) and Drucker (1951, 1952) published their ground breaking lower and upper bound theorems of plasticity theory, on which limit analysis is based. It is apparent that limit analysis would be an effective tool to provide important insights into the bearing capacity problem.

The upper bound theorem of classical plasticity theory which assumption of a perfect plastic soil model with an associated flow rule, is an useful tool to predict the stability of problems in soil mechanics. It states that the power dissipated by any kinematically admissible velocity field could be equated to the power dissipated by the external loads, and so enables a strict upper bound on the true limit load to be deduced. A kinematically admissible velocity field satisfies the conditions of compatibility, flow rule and velocity boundary. In order to provide practical solutions, the upper bound theorem is often used in parallel with the lower bound theorem.

The formula of the Rigid Plastic finite element method is built on the basic of the upper bound theorem. The arbitrary general strain rate element $\dot{\varepsilon}_{ij}$ is not always allowed for the strain rate $D(\dot{\varepsilon}_{ij})$ since some kind of constraint is more specific. It is assumed that no volumetric plastic strain rate occurs under the limit state. This is naturally accepted for the soil structure. Define further the subset $K_0(\subset K_p)$

The upper bound theorem previously proved is found to be valid when we restrict $\left(\dot{u}_i, \dot{\varepsilon}_{ij}\right)$ to K_0 since $\left(\dot{u}_i^*, \dot{\varepsilon}_{ij}^*\right)$ at the limit state also stays in K_0 .

Minimize $\int_{\dot{\varepsilon}_{ij}\in K_0} D(\dot{\varepsilon}_{ij})dV$ (1)

It should be noted that this problem falls into the category of the convex programming problem, i.e., the functional to be minimum is always identical to the global one.

Where:

$\dot{\varepsilon}$: Vector of strain rates of all elements

$\dot{u}$: Vector of all nodal velocities

$\dot{v}$: Vector of rates of volume change of all elements.

2.2 Rigid Plastic constitutive equation

Tamura (1991) developed the rigid plastic constitutive equation for frictional material. The Drucker-Prager's type yield function is expressed as follow:

$$f(\sigma) = aI_1 + \sqrt{J_2} - b = 0 \qquad (2)$$

Where:

I_1 is the first invariant value of stress, J_2 is the second invariant value of shear stress, the constants of a and b express the angles of shear resistance and dilatancy.

$$I_1 = tr\boldsymbol{\sigma} \qquad (3)$$

$$J_2 = \frac{1}{2}\mathbf{s}:\mathbf{s} \qquad (4)$$

Following the non-associated flow rule, the strain rate $\dot{\varepsilon} = \dot{\varepsilon}^p$ could be written as follow:

$$\dot{\boldsymbol{\varepsilon}} = \lambda\frac{\partial g}{\partial \boldsymbol{\sigma}} = \lambda\left(\alpha\delta_{ij} + \frac{\mathbf{s}}{2\sqrt{J_2}}\right) \qquad (5)$$

Where:

λ is an indeterminate multiplier and δ_{ij} is the Kronecker's delta symbol, $\dot{e} = \sqrt{\dot{\boldsymbol{\varepsilon}}:\dot{\boldsymbol{\varepsilon}}}$ is the norm of strain rate and **I** is a unit tensor. However, the strain rate $\dot{\varepsilon}$ should satisfy the volumetric constraint condition as follow:

$$h(\dot{\boldsymbol{\varepsilon}}) = \dot{\varepsilon}_v - \frac{3\alpha}{\sqrt{3\alpha^2 + 1/2}}\dot{e} = 0 \qquad (6)$$

In which $\dot{\varepsilon}_v$ and $\dot{e}$ indicate the volumetric strain rate and norm of the strain rate, respectively. The rigid plastic constitutive equation was expressed by Tamura (1991) as follow:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)} = \gamma\frac{\partial f}{\partial \boldsymbol{\sigma}} + \beta\frac{\partial h}{\partial \dot{\boldsymbol{\varepsilon}}} \qquad (7)$$

The variable of γ is determined by inserting Eq. (4) into the plastic potential of Eq. (2). On the other hand, the indeterminate stress parameter β still remains unknown until the whole problem with the kinematical constraint conditions of Eq. (6) is solved. The stress-strain rate relation for the Drucker-Prager's type by means of the non-associated flow rule is finally expressed in the following form:

$$\boldsymbol{\sigma} = \frac{\psi - 3\alpha\beta}{\sqrt{3\alpha^2 + \frac{1}{2}}}\frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}} + \beta\mathbf{I} \qquad (8)$$

Where:

The coefficient ψ is a variable, which is determined by stress σ located on the yield function, α as cohesion.

3. BEARING CAPACITY OF FOOTING UNDER PLANE STRAIN CONDITION

3.1 Bearing capacity formulas

Meyerhof (1951, 1953, 1963, 1965 and 1976) developed the bearing capacity equations by extending the Terzaghi's mechanism to the soil above the base of the footing. The bearing capacity factors N_c and N_q are given by Prandtl (1921). Meyerhof extended the proposal of Terzaghi (1948) and incorporated those factors as follow:

$$q = i_c c N_c + 0.5 i_\gamma \gamma_1 B N_\gamma + i_q \gamma_2 D_f N_q \qquad (9)$$

In 1953, Meyerhof extended his theory for ultimate bearing capacity under vertical load to the case with inclined load. The assumed failure mechanism is confined to one side of the footing for all values of the inclination angle, and composed of three zones whose geometry is changed to account for the load inclination. Furthermore, two slightly different mechanisms were considered, one for small inclinations and another for large inclinations.

Brich Hansen (1957, 1970) used the slip – line method and provided a new expression for the semi - empirical N_γ factor, with other bearing capacity factors kept unchanged. Hansen also derived expressions for the empirical inclination factors. This expression has assumption on one sided mechanism and accounts for the adhesion between the soil and the footing base.

Bearing capacity formula of AIJ using bearing capacity factors N_c, N_q given by Prandtl and N_γ and inclination load factors i_c, i_γ, i_q described by Meyerhof

$$q = i_c \alpha c N_c + i_\gamma \gamma_1 \beta B \eta N_\gamma + i_q \gamma_2 D_f N_q \qquad (10)$$

and some extended factors: shape coefficient defined based on the shape of foundations. α,β with the strip foundations are 1 and 0.5 respectively; Size effect factor (B is a width of footing and B_0 is a reference width):

$$\eta = \left(\frac{B}{B_o}\right)^n \qquad (11)$$

3.2 Bearing capacity of footing for vertical load

For under concentrated vertical load, the numerical solution is used with a rigid plastic constitutive equation under plain strain condition. This is the linear shear strength properties.

This study uses the angle of shear resistance ϕ=30 deg. The result presented from fig.1 shows that the bearing capacity of footing increases with the increase of size footing. As can be seen the value obtained from RPFEM and other methods have large difference. This difference is due to the significant effect of confining stress when calculating the bearing capacity of footing. This would be improved in the next section when using RPFEM considers confining stress.

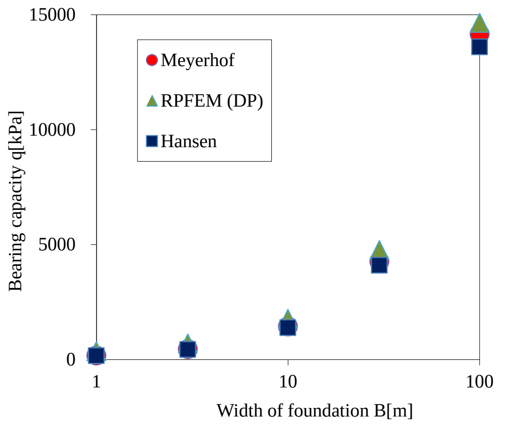

Fig. 1 Effect of footing width on bearing capacity for concentric vertical load application (ϕ = 30^0)

3.3 Bearing capacity of footing for inclined load

We calculate the bearing capacity regardless of size effect. The bearing capacity under vertical load is calculated for the case with inclined load and, angle of shear resistance ϕ = 30 deg. Load inclination angle θ is inclination of load with respect to vertical. The load inclination angle changes from 10 deg to 30 deg.

As presented above, inclination load only effect when inclined load angle θ < 30 deg with the Meyerhof's formula.

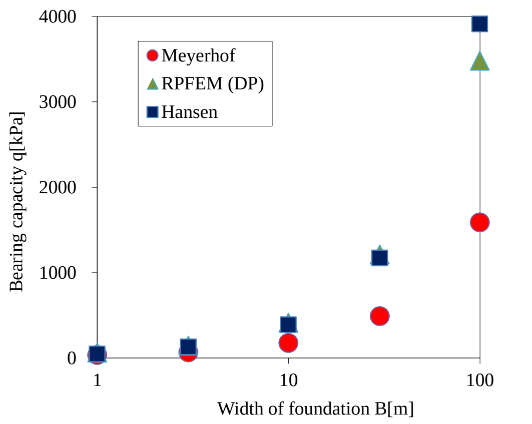

Fig. 2 Bearing capacity with angle of inclined load θ = 20^0

Fig. 2 describes the relationship between bearing capacity and width of footing. The different values also consistently confirmed the effect of confining stress dual on bearing capacity of footing. Furthermore, results presented in the Fig. 2 indicate the influence of size effect. The larger width of footing, the larger difference between RPFEM with other methods.

The empirical results shows that the values of bearing capacity of footing decrease as the load inclination angle increase, even for small inclination angles.

4. RIGID PLASTIC CONSTITUTIVE EQUATION OF TOYOURA SAND

4.1 Strength tests of Toyoura sand by Tatsuoka

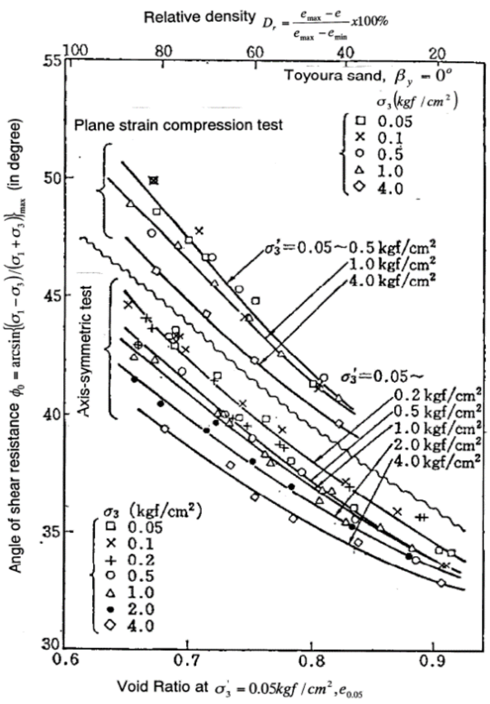

Fig. 3 Experimental result of Toyoura sand (Tatsuoka, 1986)

Applying Drucker-Prager criteria and Mohr-Coulomb criterion are based on the frictional characteristics to represent the strength of the ground. When angle of shear resistance is increased, the effect of the ultimate bearing capacity of the ground changes constantly on the extremely large ultimate bearing capacity of the ground. Therefore, when determining the angle of shear resistance and adhesion by soil test, in the compression zone, low pressure area always overestimates the strength. It is pointed out that the development of yield function by the non-linear shear strength is needed to improve the accuracy when calculating the bearing capacity of footing.

As mentioned above confining stress and size effect have significant effects on the shear strength using shear strength linear properties. It is more than evident through experiments of Tatsuoka.

Tatsuoka surveyed the shear strength of Toyoura sand and clarified that the angle of shear resistance varied with the confining stress as Fig. 3 above. So, the development of yield function by the non-linear shear strength is needed. Higher yield function and parameters of the yield function are proposed on the basic of experimental results of Tatsuoka. The ultimate bearing capacity results gained through the application of high yield function in RPFEM in this study shows more consistent with the actual ground than the values obtained in the Drucker-Prager yield function. The study's results shown that it is possible to evaluate the ultimate bearing capacity values, taking into account the nonlinearity's influence of the soil strength due to confining pressure of the ground.

Based on the layout of Toyoura sand from Fig.3, the results shown that the confining stress effect on the behavior of ground in Fig. 4. So we can establish the relationship between the value of the first stress invariant I_1 and the second invariant value of shear stress J_2. This allows us to design parameters when referring to nonlinear shear strength properties of soil for different types of soil. In this case, we defined parameters of yield function for Toyoura sand is: a=0.24, b=2.4 and n=0.56.

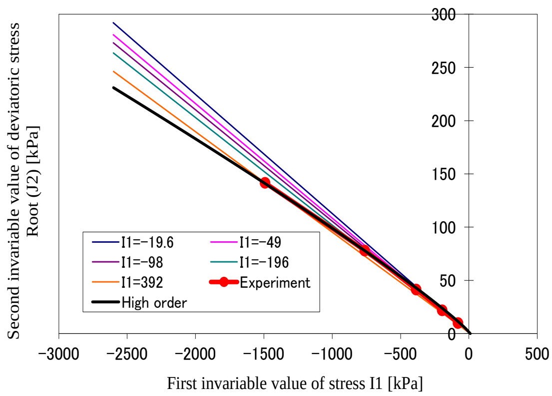

Fig. 4 Effect of confining stress to soil behavior

4.2 Non – linear rigid plastic constitutive equation for confining stress

From yield function:

$$f(\boldsymbol{\sigma}) = aI_1 + (J_2)^n - b = 0 \quad (12)$$

The first invariant of the stress is decided by modifying the yield function, the first invariant of the stress using the second invariant of the deviatoric stress is updated by the following equation:

$$I_1 = \frac{1}{a}\left(b - \left(J_2\right)^n\right) \quad (13)$$

The strain rate for nonlinear case:

$$\dot{\boldsymbol{\varepsilon}} = \lambda \frac{\partial f(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}} = \frac{a\mathbf{I} + nJ_2^{n-1}\mathbf{s}}{\sqrt{3a^2 + 2n^2(b - aI_1)^{2-1/n}}}\dot{e} \quad (14)$$

and the volumetric strain rate $\dot{\varepsilon}_v$:

$$\dot{\varepsilon}_v = tr\dot{\boldsymbol{\varepsilon}} = \frac{3a}{\sqrt{3a^2 + 2n^2(b - aI_1)^{2-1/n}}}\dot{e} \tag{15}$$

The first invariant and stress is expressed by strain rate so this relationship can be obtained as follows:

$$\boldsymbol{\sigma} = \left[\frac{1}{n}\left(\frac{9a^2}{2n^2}\left(\frac{\dot{e}}{\dot{\varepsilon}_v}\right)^2 - \frac{3a^2}{2n^2} \right)^{\frac{1-n}{2n-1}} \left[3a\frac{\dot{\boldsymbol{\varepsilon}}}{\dot{\varepsilon}_v} - a\mathbf{I} \right] \right. \tag{16}$$
$$\left. + \frac{1}{3}\left[\frac{b}{a} - \frac{1}{a}\left(\frac{9a^2}{2n^2}\left(\frac{\dot{e}}{\dot{\varepsilon}_v}\right)^2 - \frac{3a^2}{2n^2} \right)^{\frac{n}{2n-1}} \right]\mathbf{I} \right]$$

In this study, we developed a rigid plastic constitutive equation incorporating a confining pressure dependence of the ground strength which is evaluated with highly accurate the ultimate bearing capacity. The first term of the above equation is determined by the stress constitutive relation and the second term is the stress component along the yield surface. So the stress σ can be shown as follows:

$$\boldsymbol{\sigma} = \frac{\left(aI_1 + 2n\left(b - aI_1\right)\right)}{\sqrt{3a^2 + 2n^2\left(b - aI_1\right)^{(2-1/n)}}}\frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}} \tag{17}$$
$$+\kappa\left(\dot{\varepsilon}_v - \beta\dot{e}\right)\left(\mathbf{I} - \frac{3a}{\sqrt{3a^2 + 2n^2\left(b - aI_1\right)^{(2-1/n)}}}\frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}}\right)$$

Where κ : is the variable determined by inserting Eq. (8) into the plastic potential of Eq. (3).

Through analysis of the explicitly dilatancy characteristics, computation of stable displacement velocity field can be implemented by using above nonlinear equation. Further, the advantage of the numerical analysis on the rigidity matrix is symmetrical.

5. APPLICABILITY OF PROPOSED CONSTITUTIVE EQUATION TO BEARING CAPACITY ASSESSMENT

5.1 Bearing capacity of footing for vertical load

As mentioned above, high pressure and low pressure area of the confining pressure of the ground is a problem which leads to the overestimate of Drucker-Prager's yield function. This is primarily due to the intensity nonlinear characteristic of confining pressure by soil. Therefore, it is reasonable to evaluate the strength by applying the intensity equation linear shear strength of the soil. In practice, non-linear strength of the ground may dominate the stability, hence it is important to incorporate the stability evaluation of the nonlinearity strength in engineering. The rigid-plastic constitutive equation, that is non-linear, should be used for higher-order yield function. In this case, the effect of compression stress on estimated values is smaller than those obtained from the linear intensity formula. In this section, we perform a numerical analysis with respect to the horizontal ground and the comparision with theoretical solution of Meyerhof.

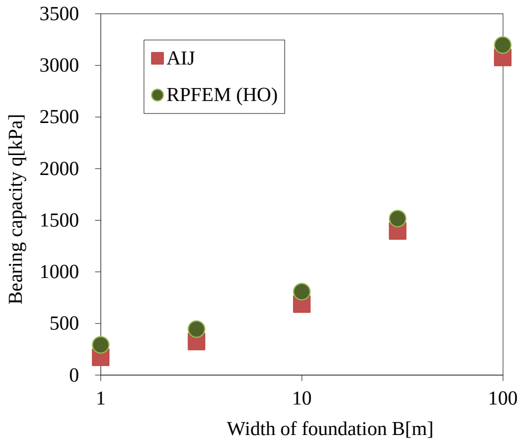

Fig. 5 Bearing capacity with higher order yield function

Fig. 5 shows the development of Rigid Plastic Finite Element Method. RPFEM results coincide with the results of Meyerhof, which mean that the results obtained from non-linear shear strength properties is much better than the linear function described earlier.

5.2 Bearing capacity of footing for inclined load

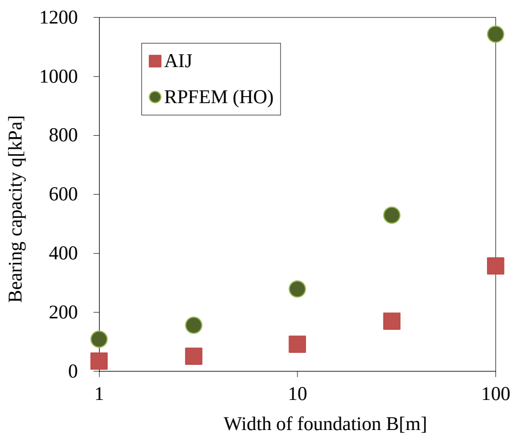

Fig.6 Bearing capacity with angle of Inclined load θ = 20 deg

Results from Fig. 6 indicated that the differences between RPFEM and AIJ methods are significantly improved when comparing with the results calculated from linear shear strength properties.

As can be seen from Fig. 7, with width of footing B=30m when considering the influence of inclined load, the effect of confining stress is very significant and the use of nonlinear properties of shear stress to calculate the bearing capacity of footing is perfectly reasonable.

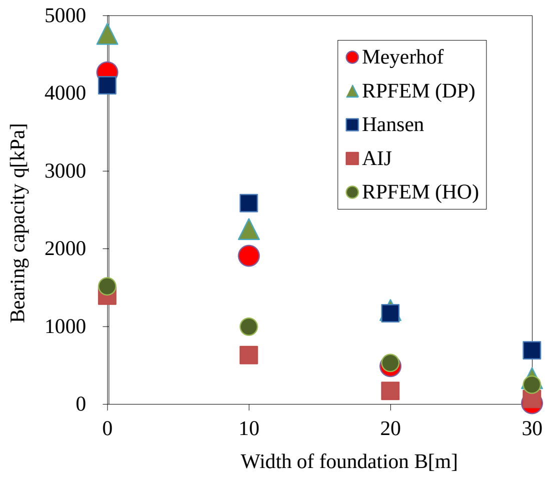

Fig. 7 Bearing capacity with Inclined load effect in case width of footing B = 30m

In this case, the values of bearing capacity decrease quickly under effect of inclination load. Impact of size footing is considerable. The value of bearing capacity reduces from 3 to 4 times when considering size effect. This finding is very important to insure stability of building.

The use of rigid-plastic constitutive equation with higher-order yield function proposed, which can be expressed nonlinearity strength by confining stress of the ground and checked the applicability of Druker-Prager method to the ultimate bearing capacity problems.

6. CONCLUSION

This study has developed the ultimate bearing capacity analysis method using a rigid-plastic constitutive equation with higher-order yield function by confining pressure. The empirical results indicated that, ultimate bearing capacity value decreases in accordance with the increase of the load inclination angle in both cases linear and nonlinear shear strength properties. However, the values of ultimate bearing capacity from developed RPFEM with high order yield function is less than those values obtained from Drucker-Prager's yield function.

In addition, be evaluated properly to yield function from the change in values of the first invariant I1 of the stress and, the effects of confining pressure of the ground can be evaluated reasonably.

However, the results from empirical investigation are still limited by the experimental method in case of size of footing. RPFEM coincided with the calculated results from the experimental method. This persuasively demonstrates the accuracy of this method. We proposed to use numerical method with large width of footing. The applicability of RPFEM is to solve more complicated problems in case of layered ground and improved ground. The developed rigid plastic finite element method can provide the good estimation for wider range of footing size.

7. REFERENCES

[1] Aysen A, "chapter 10", Soil Mechanics – Basic concepts and engineering applications, Balkema A A publisher, 2002, pp. 413 – 419.

[2] Drucker D C, Greenberg H J, Lee E H, Prager W, "On plastic rigid solutions and limit design theorems for elastic plastic bodies", 1st US NCAM, 1951, pp. 533 – 538

[3] Hill R, "On the limits set by plastic yielding to the intensities of singularities of stress", J. Mech. Phys. Solids, Vol. 2, 1954, pp. 278-285.

[4] Okamura M, Takemura J, Kimura T, "Centrifuge model tests on bearing capacity and deformation of sand layer overlying clay", Soil and Foundations, Vol. 37, No.1, 1997, pp.73–87.

[5] Fumio Tatstuoka et al, "Model tests and FEM simulation of some factors affecting the bearing capacity of footing on sand", Soil and Foundations, Vol. 41, No.2, 2001, pp. 53 – 74.

[6] Tatsuoka F, Sakamoto M, Kawamura T and Fukushima S, "Strength and deformation characteristics of sand in plane strain compression at extremely low pressures", Soil and Foundations, Vol.26, No.1, 1986, pp.65–84

[7] Takeshi Hoshina, Satoru Ohtsuka and Koichi Isobe, "Ultimate bearing capacity of ground by Rigid plastic finite element method taking account of stress dependent non-linear strength property", Journal of Applied Mechanics, Vol.6, 2011, pp.191 – 200

[8] Du N L, Ohtsuka S, Hoshina T, Isobe K, and Kaneda K, "Ultimate bearing capacity analysis of ground against inclined load by taking account of non-linear properties of shear strength", Third International Conference on Geotechnique, Construction Materials and Environment,Vol.3(1), 2013, pp. 286–291.

[9] Takeshi Tamura, "Rigid-Plastic Finite Element Method in Geotechnical Engineering", The Society of Materials Science, Japan, Vol.7, pp. 135–16.

[10] Mohammed Hijaj, Andrei V. Lyamin, Scott W. Sloan, "Bearing capacity of cohension-

frictional soil under non-eccentric inclined loading", Vol 31, 2004, pp. 491 – 516.

[11] Hjiaj M, Lyamin A V, Sloan S W, "Numerical limit analysis solutions for the bearing capacity factor N_γ", International Journal of Solids and Structures, Vol. 42, 2005, pp. 1681 – 1704.

[12] Lyamin A V and Sloan S W, "Upper bound limit analysis using linear finite elements and non-linear programming", International Journal for numerical and analytical methods in geomechanics, Vol. 26, 2002, pp. 181 – 216.

[13] Bolton M D and Lau C K, "Vertical bearing capacity factors for circular and strip footings on Mohr – Coulomb soil", Canada Geotechnical Journal, Vol 30, 1993, pp. 1024 – 1033.

[14] Siddiquee M S A, Fumio Tatsuoka, Tadatsugu Tanaka, Kazuo Tani, Kenji Yoshida and Tsutomu Morimoto, "Model tests and fem simulation of some factors affecting the bearing capacity of a footing on sand", Soil and Foundations, Vol. 41, No.2, 2001, pp. 53 – 76.

Int. J. of GEOMATE, Dec, 2013, Vol. 5, No. 2 (Sl. No. 10), pp. 678-684.
MS No. 3323 received on June 15, 2013 and reviewed under GEOMATE publication policies.

Pertinent discussion including authors' closure, if any, will be published in the Dec. 2014 if the discussion is received by June, 2014.
Corresponding Author: Nguyen Le Du