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PREDICTING THE STRESS-STRAIN BEHAVIOR OF MINE TAILING USING MODIFIED HYPERBOLIC MODEL

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ABSTRACT: The shear strength behavior of mine tailings was investigated through direct shear test to determine its applicability as embankment material. Dry tailings at very dense initial state and lower normal stresses exhibited peak shear strength and strain softening with dilative behavior, indicating that tailing samples failed in brittle manner. At higher normal stresses, the dry tailing samples attained the critical state shear stress and the volumetric strain is purely compressive. Tailings in dry condition manifested a strong particle interlock as indicated by high critical state friction angle ranging from 36.6º to 38.4º. Peak shear stress was not observed for saturated tailings even at very dense condition and low normal stresses indicating that saturated tailings have ductile behavior with contractive volumetric strain. Friction angles at failure of saturated tailings were lower at an average of 4º as compared to those obtained in dry condition. Modified hyperbolic model was formulated to predict the shear stress against shear strain and volumetric strain against shear strain responses of tailings to different stresses. The modified hyperbolic model provides a good approximation to the stress-strain and volumetric strain-shear strain responses measured during the tests of tailings that exhibited a ductile failure and compressive volumetric strain. However, the model does not give a good prediction of stress-strain response for specimens that exhibited brittle failure with dilative volumetric strain. The model cannot capture the strain softening phenomena, but it can be used to model the behavior leading to the strain softening as well as during the ductile stage.

Keywords: Mine Tailings, Friction Angle, Stress-strain Behavior, Modified Hyperbolic Model

1. INTRODUCTION

The abundance of solid wastes in the environment can pose risk to the health of the exposed population and damage to the environment. One way to respond to this environmental issue is to find useful application for these wastes materials. Numerous studies have been undertaken to determine the applicability of solid wastes as an alternative construction material. Ceramic waste and quarry dust aggregates were used as a possible replacement for conventional crushed stone coarse and fine aggregates [1]. Ashes from a wastewater treatment facility that incinerates its treatment sludge were evaluated for possible use as partial clay substitute for brick manufacture and as a soft soil stabilizing admixture [2].

In the Philippines, one of the most abundant solid wastes found in the environment are wastes from mining operations called mine tailings. The volume of tailings produced by mining operations often exceeds the volume of the recovered minerals by several orders of magnitude. For example, the production of 10 g of gold results in more than 5 tons of solid and liquid wastes generated by mining and milling processes [3]. Small-scale mining activity in the Philippines produced an estimated 38,230.63 kgs. of gold, 5.73 million metric tons of ore mined, and 14.32 million metric tons of tailings [4].

The most common environmental issue associated with mining activities is the disposal of the enormous amounts of tailings regularly produced from mining operations. The use of mine tailings as embankment materials is one possible option that alleviate disposal problem and reduce the negative environmental effects. To evaluate the applicability of mine tailings as embankment materials, thorough study of its geotechnical characteristics is needed. Geotechnical characterization includes the determination of its physical properties and hydraulic conductivity characteristics [5]. A study of its compaction behavior, compressibility and hydrocompression settlement describes its consolidation behavior [6]. This research study investigates the shear strength behavior of mine tailings through direct shear test. The determination of the shear strength parameters and understanding of the stress-strain response to different stresses are essential steps to completely evaluate the suitability of mine tailings for its geotechnical application.

Different methods have been developed to model the stress-strain behavior of soils prior to failure. The catastrophic failure of Merriesspruit gold tailings dam in South Africa led to the study of Fourie and Papageorgiou on the stress-strain behavior of Merriesspruit tailings under undrained loading to establish the steady state line using critical state soil mechanics [7]. Onur, M. et.al. determined the shear strength parameters of saturated clayey soils under unconsolidated undrained condition using the Mohr-Coulomb model with the aid of PLAXIS software program [8]. Hyperbolic stress-strain equations were developed by Duncan and Chang as a means for modeling the isotropic nonlinear elastic stress-strain behavior of soils prior to failure [9]. In this study, the critical state model and a modified version of the Duncan and Chang hyperbolic model were used to describe the stress-strain and volume change behavior of tailings. The shear strength behavior of tailings must be established because this is a determining factor for the stability of earth structures built using this waste material.

2. EXPERIMENTAL PROGRAM

Mine tailing samples came from three (3) mining sites in the Philippines namely: wastes from concrete aggregate quarry in Cavite and designated as TS#1, gold mine tailings from gold processing plant in Davao del Norte and designated as TS#2, and gold mine tailings from mining site in Aroroy, Masbate and designated as TS#3.

In this study, direct shear test was carried out with samples in dry and saturated conditions following the procedures described in ASTM D3080. Samples are prepared at initial relative densities of 60% (medium dense), 80% (dense) and 90% (very dense) both in dry and fully saturated conditions.

For dry condition, test runs for each tailing samples used five (5) different vertical stresses of 13.63 KPa, 20.44 KPa, 27.25 KPa, 54.50 KPa and 87.50 KPa. The first two vertical stresses (13.63 KPa and 20.44 KPa) represent the low-stress conditions. The other three (3) vertical stresses of 27.25 KPa, 54.50 KPa and 87.50 KPa correspond to the overburden pressure in the field at the depth of 1.5m, 3.0m and 4.5m, respectively. The horizontal stress was applied at a fast rate of 1.25mm/min.

For the saturated condition, four (4) vertical stresses (13.63 kPa, 20.44 kPa, 27.25 kPa and 54.50 kPa) were used. The sample was allowed to come into drained equilibrium under the application of these vertical stresses before subjecting to horizontal load. A very slow strain rate of 0.12mm/min was used to simulate the drained condition.

3. PHYSICAL PROPERTIES OF MINE TAILINGS

The soil constants of mine tailings determined from laboratory tests are presented in Table 1. The fine contents of the three tailing samples are non-plastic. Tailings' specific gravity, *Gs* fall within the range of sand and silty sand. Tailings from concrete aggregate quarry (TS#1) consisted primarily of fine sands with very few silts. Based from Unified Soil Classification System (USCS), TS#1 is classified as poorly graded sand with silt and is given the symbol of SP-SM. Gold mine tailings from Davao (TS#2) and Masbate (TS#3) both exhibited an almost equal distribution of fine sands and silts. TS#2 and TS#3 both have USCS classification of silty sand with symbol of SM [5].

Table 1 Soil constants of mine tailings [5]

	TS#1	TS#2	TS#3
Specific gravity, *Gs*	2.57	2.72	2.71
Liquid Limit, LL %	27	24	23
Plasticity Index, PI %	0	0	0
Shrinkage Limit, %	21	20	20
Shrinkage Ratio	1.47	1.57	1.66
Min. Void Ratio, e_{min}	0.624	0.680	0.662
Max. Void Ratio, e_{max}	1.024	1.106	1.089
Max. dry unit weight, γ_{dmax} kN/m^3	15.56	17.12	17.72
Optimum moisture content, w_{opt} %	13.49	17.28	12.82

4. SHEAR STRENGTH PROPERTIES

4.1 Stress-strain and Volume Change Behavior of Tailings in Dry Condition

The typical stress-strain and volume change relationship of dry tailing samples is presented in Figs. 1. The stress-strain plot shows the typical response of soil to shearing forces at monotonic loading. The shear stress at failure is greater for specimens subjected to greater normal effective stress (σ') as compared to specimens with lower normal effective stress. The shear stress at failure, termed as the critical state shear stress is described as the shear strength at which continued shearing occurs without change in shear stress and volume for a given normal effective stress. The three (3) tailing samples exhibited similar stress-strain behavior. It can be noted that at very dense condition (Dr = 90%) and lower normal effective stresses (σ' ≤ 27.25 KPa), the specimen showed a rapid increase in shear stress reaching a peak value at low shear strains and then decreases with increasing shear strains indicating strain softening until the critical shear stress is attained. This indicates that specimen failed in brittle manner, typical for dense sands and over-consolidated clays. The determination of brittle failure is potentially important when dealing with embankment and slopes as this leads to progressive failure which may be sudden and catastrophic. At higher normal effective stresses and relative density of 90%, all the tailing samples behaved like loose sands or normally consolidated clays. There is a gradual increase in shear stresses as the shear strain increases until an approximately constant shear stress is attained. The stress-strain behavior suggests that tailings failed in ductile manner. For test specimen with relative densities of 60% and 80%, the peak shear stress was not observed even at lower stresses for all the tailing samples. The samples behaved like a ductile material at every normal effective stress.

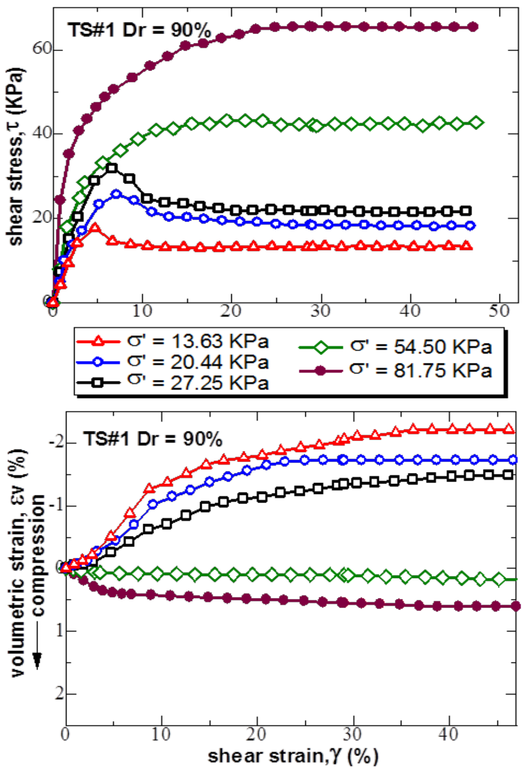

Fig. 1 Typical stress-strain and volumetric strain-shear strain plots of dry tailing

The volumetric strain plot shows that tailings followed a well-established behavior trends for granular materials. As the normal effective stress was increased, the tendency for the specimen to dilate was decreased. Specimens achieved greater compressive volume change as the normal effective stress is increased. Samples that exhibited peak shear stress showed an initial compressive behavior and then continued to increase in volume with shear strain manifesting a dilative behavior. This behavior arises because tailing, like soil, is essentially a particulate material. The particles must take up a suitable arrangement of packing before continued shearing can take place. If the particles are initially more densely packed, some loosening which corresponds to an increase in volume change, that is dilation, will have to occur before the critical shear can takes place.

4.2 Stress-strain and Volume Change Behavior of Saturated Tailings

The typical stress-strain and volume change behavior of saturated tailings are shown in Fig. 2.

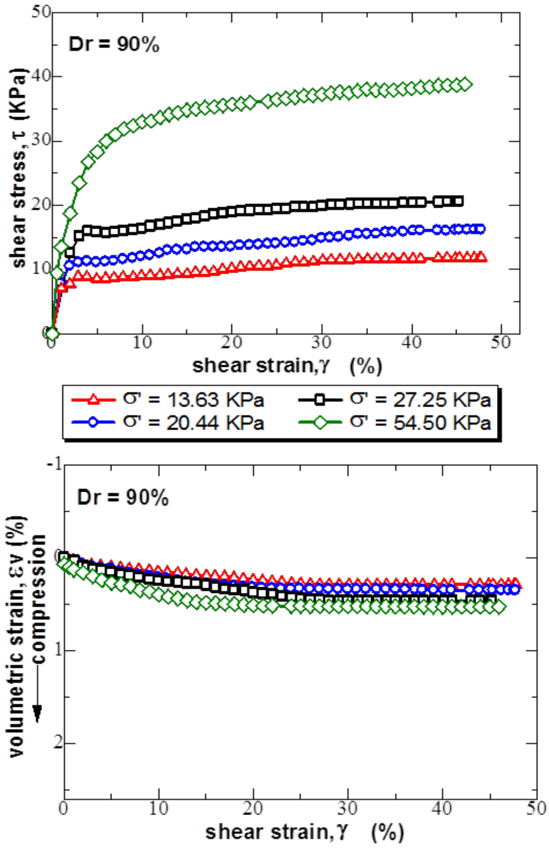

Fig. 2 Typical stress-strain and volumetric strain-shear strain plots of saturated tailings

All specimens exhibited the same trend in stress-strain behavior. Peak shear stress was not observed even at very dense condition and low normal stresses. The samples behaved like loose sands and normally consolidated clays where strain hardening was observed indicating a ductile failure. The increase in shear strength was coupled with decreased in volume. This compressive behavior increased as the normal effective stress is also increased. The rearrangement of soil particles into a denser configuration was easily facilitated because of the presence of moisture, thus resulted to ductile and compressive behavior of the saturated tailings.

It is expected that all soils reach an approximately constant shear stress irrespective of their initial state. In the case of saturated tailings in this study, the samples continued to gain strength until the end of the test. The ductile behavior manifested by tailings is preferable as this will not lead to progressive collapse. Many of the design procedure in geotechnical engineering assume that the soil can be relied on to behave in a ductile manner, where it will undergo continued deformation at constant load. This is in contrast to a brittle material, which at failure breaks and loses it load carrying capacity entirely.

4.3 Friction Angles

Tailings in dry condition reached a critical state in which unlimited shear strain could be applied without further changes in volume, normal effective stress or shear stress. Test results revealed that the critical state reached by the specimen depends on the normal effective stress at which it is sheared. The initial void ratio, as expressed by relative densities, has no significant effect on the critical shear strength. The critical state shear stress when plotted on a graph of τ against σ' lies in a straight line with slope tan (ϕ_{cs}) and can be described by equation $\tau = \sigma' \tan\phi_{cs}$. This line represents the failure line or termed as critical state line (CSL) and ϕ_{cs} is called the critical state friction angle. Soil states below the CSL will cause the soil to behave in a ductile manner and is desirable in engineering design. Soil states above the CSL and bounded by the peak shear strength envelope will cause the soil to behave in brittle manner which can cause sudden failure or collapse. Although high shear strengths (peak) are observed in this region, there is no guarantee that the high shear strengths will be uniformly mobilized at `the same time and this will present high safety risk which should be avoided. High values of friction angles are obtained from the tests indicating that dry tailings have strong particle interlock resulting to high shear strength.

For tailings in saturated condition, the shear stress corresponding to 15% horizontal displacement or approximately equal to 29% horizontal strain was considered as the shear stress at failure and friction angle obtained using this shear strength is referred to as friction angle at failure (ϕ_f). Similar to the test results in dry condition, the values of shear strength at failure of each tailings is unaffected by its initial void ratio. The friction angles at failure for saturated tailings were lower at an average of 4^o as compared to those obtained in dry condition. The moisture serves as lubricant that reduces frictional resistance between particles, therefore, it resulted to lower values of friction angles. Cohesion values are almost zero. The saturated tailings did not exhibit cohesive behavior since samples are classified as non-plastic.

The friction angles in both dry and saturated condition are summarized in Table 2 together with some values of friction angles from literature. The values of friction angles derived from this study are within the range of values obtained from studies of other researchers.

Table 2 Friction angle at failure of tailings

Type of Tailings	Friction angle at failure		Source
	Dry condition ϕ_{cs} (deg.)	Saturated condition ϕ_f (deg.)	
TS#1	38.4	34.4	This study
TS#2	36.8	32.5	This study
TS#3	36.6	33.3	This study
Gold Slimes	20 – 40.5		Vick [10]
Fine Coal Refuse	22 – 39		Vick [10]
Copper Slimes	24 – 37		Shamsai [11]

5. MODIFIED HYPERBOLIC MODEL

Strain hardening manifested in all test runs of saturated tailings. The stress-strain curves reflect an asymptotic behavior, as such, hyperbolic fitting techniques were applied to characterize the stress-strain and volume-change behavior of tailings in saturated condition determined from direct shear tests. The hyperbolic model is one of the most frequently used models for predicting the behavior of soils especially if one wishes to apply the finite element method to describe the non-linear movement within soil masses. The Duncan and Chang [9] hyperbolic model was developed based on data obtained from the triaxial tests. In this study, the stress-strain data are obtained from direct shear test; hence, there is a need to modify the method in the hyperbolic model to obtain the hyperbolic parameters from the direct shear test.

The modified model approximates the stress-strain behavior from direct shear tests by the following hyperbolic relation:

$$\tau = \frac{\gamma}{a + b\gamma} \qquad (1)$$

where τ is the shear stress, γ is the shear strain, *a* and *b* are parameters evaluated from the tests data.

The stress-strain data is represented in a transformed plot where the value of shear strain, γ measured during the test is divided by the corresponding value of shear stress, τ and plotted against the shear strain (γ/τ vs. γ). If the stress-strain relationship measured during the test is hyperbolic, the transformed diagram is a straight line. A typical transformed plot is presented in Fig. 3.

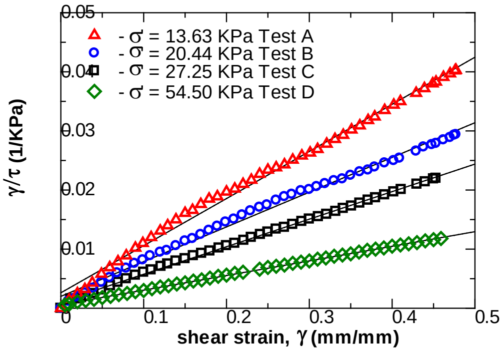

Fig. 3 Transformed stress-strain plot from direct shear test on aggregate tailings (TS#1) with relative density Dr = 90%

The transformed plot exhibited a straight line indicating that the stress-train relationship measured during the tests was hyperbolic. The hyperbolic parameters *a* and *b* of Eq. (1) is the intercept and slope of this straight line, respectively. The intercept *a* of this straight line on the γ/τ axis is the reciprocal of initial shear modulus, *Gi* of the tailing sample while the slope of the line, *b* is the reciprocal of the asymptotic shear stress, τ_{ult}.

The variation of the initial shear modulus, *Gi* in response to change in normal effective pressure can be represented using the power law approach as suggested by Janbu:

$$G_i = K \cdot P_a \left(\frac{\sigma'}{P_a} \right)^n \qquad (2)$$

The parameters *K* (shear modulus number) and *n* (shear modulus exponent) describing initial shear modulus (*Gi*) are obtained from a best-fit straight line drawn through data points of the logarithmic diagram showing the values of normalized shear modulus (*Gi*/*Pa*) against the values of normalized normal effective stress (σ'/*Pa*), where the normalizing parameter *Pa* is the atmospheric pressure equal to 101.325 KPa.

The failure ratio R_f that relates the asymptotic shear stress with shear stress at failure is defined as the ratio of the shear stress at failure, τ_f to the asymptotic shear stress, τ_{ult}. Since the stress-strain plot exhibited an asymptotic behavior, the value of shear stress at failure, τ_f from stress-strain plot of the test is taken as the shear stress at 15% horizontal displacement.

The variation of angle of internal friction ϕ' with respect to normal stress, σ' is described in terms of hyperbolic parameters ϕ_o and Δϕ. The angle of internal friction, ϕ' is given by the expression,

$$\phi' = \phi_o - \Delta\phi \log\left(\frac{\sigma'}{P_a} \right) \qquad (3)$$

The parameter ϕ_o is the value of angle of internal friction, ϕ' at σ' equal to *Pa* and Δϕ is the reduction in ϕ' for a ten-fold increase in σ' which are obtained from best-fit line through data points on the plot of ϕ' vs. logarithm of normalized σ'/*Pa*.

The volumetric strain vs. shear strain behavior can also be approximated by hyperbolic equation of the form,

$$\varepsilon_v = \frac{\gamma}{\alpha + \beta\gamma} \qquad (4)$$

where ε_v is the volumetric strain, α and β are volumetric strain parameters that can be determined from test data presented in transformed plot of γ/ε_v vs. γ. The value of α is the intercept of the best fit line of the data points while the value of β is the slope of the plot and represents the asymptotic value of ε_v. The parameters α and β vary with normal effective stress and can be represented by the following expressions:

$$\alpha = K_a P_a \left(\frac{\sigma'}{P_a} \right)^m \qquad (5)$$

$$\beta = K_b P_a \left(\frac{\sigma'}{P_a} \right)^r \qquad (6)$$

The values of K_a and *m* are determined in a logarithmic diagram of normalized α/Pa versus normalized σ'/Pa while the values of K_b and *r* are determined in a logarithmic diagram of normalized β/Pa versus normalized σ'/Pa. The parameters K_a and K_b (volumetric strain numbers) are the values of normalized α and β, respectively, for a confining stress of 1 atm. The slope of the best fit line is the value of parameters *m* and *r* (volumetric strain exponents).

After the hyperbolic parameter values are determined, the stress-strain and volumetric strain-shear strain responses of tailings as a function of normal effective stress, σ' can be predicted using the following expressions:

The shear stress is given by equation,

$$\tau = \frac{\gamma}{\dfrac{1}{K \cdot P_a \left(\dfrac{\sigma'}{P_a}\right)^n} + R_f \left(\dfrac{\gamma}{\tau_f}\right)} \tag{7}$$

The volumetric strain is calculated using the equation,

$$\varepsilon_v = \frac{\gamma}{K_a P_a \left(\dfrac{\sigma'}{P_a}\right)^m + \gamma K_b P_a \left(\dfrac{\sigma'}{P_a}\right)^r} \tag{8}$$

The hyperbolic parameters to describe the shear modulus (*Gi*) and volumetric strain parameters (α and β) of tailings in saturated condition are presented in Table 3 and Table 4, respectively.

Table 3 Hyperbolic parameters describing stress-strain response from test data of tailings in saturated condition

Type of tailing	Dr (%)	Shear modulus hyperbolic parameters				
		K	*n*	Øo (deg.)	ΔØ (deg.)	R_f
TS#1	90	36.11	1.20	31.61	8.09	0.897
	80	28.39	1.04	31.75	5.85	0.888
	60	23.84	1.02	32.13	4.70	0.902
TS#2	90	17.84	0.894	30.89	4.42	0.882
	80	15.95	0.842	31.22	3.29	0.868
	60	13.91	0.831	30.55	4.18	0.856
TS#3	90	34.69	1.115	30.97	6.85	0.899
	80	18.11	0.917	31.81	2.86	0.892
	60	15.39	0.604	31.68	2.67	0.907

The modified hyperbolic model was also applied to tailings in dry condition. This is to verify if the proposed model is applicable to samples with stress-strain and volumetric strain-shear strain curves that do not exhibit an asymptotic trend. It can be noted that it was not possible to obtain the hyperbolic parameters (*Ka, m, Kb, r*) to describe the volumetric strain parameters α and β for specimens that exhibited dilation. The volumetric strain against shear strain curve of dilatant samples is not hyperbolic and the transformed plot produced negative values of α and β, thus, the determination of *Ka, m, Kb* and *r* using the normalized α and β in logarithmic diagram was not possible. It was observed that the modified hyperbolic model does not give a good prediction of the stress-strain response for dry tailings. This is because the stress-strain curves do not exhibit an asymptotic trend.

Table 4 Hyperbolic parameters describing change in volumetric strain of tailings in saturated condition

Type of Tailing	Dr (%)	Volumetric strain parameter, α		Volumetric strain parameter, β	
		K_a	*m*	K_b	*r*
TS#1	90	0.0357	-1.130	1.369	-0.307
	80	0.0212	-0.911	0.822	-0.350
	60	0.0220	-0.997	0.625	-0.342
TS#2	90	0.0094	-1.080	0.281	-0.188
	80	0.0149	-0.759	0.247	-0.280
	60	0.0120	-0.639	0.223	-0.280
TS#3	90	0.0139	-1.024	0.342	-0.191
	80	0.0107	-1.013	0.221	-0.357
	60	0.0071	-0.868	0.220	-0.372

Using the determined hyperbolic parameters, the model's response to the test data was compared with experimental data using Eqs. (7) and (8). The comparison of the test data and the calculated hyperbolic response is shown in Fig. 4.

The comparison showed that the modified hyperbolic model provides a good approximation to the stress-strain response measured during the tests for tailings with ductile behavior since the stress-strain curve shows a hyperbolic trend. However, the model does not give a good prediction of stress-strain response for specimens that exhibited brittle failure. The model cannot capture the strain softening phenomena, but it can be used to model the behavior leading to the strain softening as well as during the ductile stage. The volumetric strain-shear strain response using the modified hyperbolic model also compares fairly well with the test data for specimens with compressive volumetric strain. However, the model does not apply to dilatant samples because the present model cannot account for the change in sign of the volumetric strain.

6. CONCLUSION

The shear strength behavior of tailings from aggregate quarry and gold mining sites in the

Philippines were investigated through direct shear tests. Experimental results showed that:

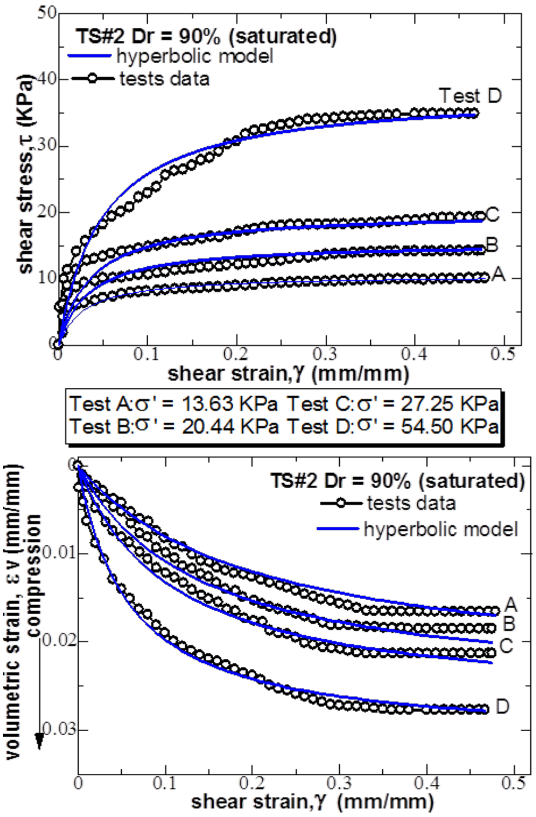

Fig. 4 Comparison of hyperbolic model and test data for saturated tailing.

Dry tailings at very dense initial state and lower normal stresses exhibited peak shear strength and strain softening with dilative behavior, indicating that tailing samples failed in brittle manner. At relative densities lower than 90%, samples reached the critical state even at lower normal stresses indicating that tailings have ductile and contractive behavior.

Saturated tailings exhibited strain hardening indicating a ductile failure with contractive volumetric strain. Friction angles at failure were lower at an average of 4^{o} as compared to those obtained in dry condition.

The modified hyperbolic model provides a good approximation to the stress-strain response measured during the tests of tailings that exhibited a ductile failure. However, the model cannot capture the strain softening phenomena, but it can be used to model the behavior leading to the strain softening as well as during the ductile stage.

The volumetric strain-shear strain response using the modified hyperbolic model also compares fairly well with the test data specifically for specimens which have compressive volumetric strain. However, the model does not apply to dilatant samples.

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