

EFFICIENT REPAIR SCHEDULING STRATEGY OF A MULTI-SOURCE LIFELINE NETWORK USING CONSTRAINED SPANNING FOREST

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ABSTRACT: Pre-disaster programs, especially for seismic hazards, are necessary to quickly recover the services of a lifeline network. In the case of a multi-source (or multi-root) water lifeline network, an efficient repair schedule must be implemented immediately after an earthquake to assist in post-disaster activities as well as to minimize the subsequent health problems caused by the lack of potable water supply. As such water lifeline operators must establish restoration strategies especially if the supply of water comes from different sources and spatially distributed. For a single-source water network, Horn's algorithm can be used to determine an optimal restoration strategy. However, a variation of this algorithm is necessary in order to allow simultaneous repairs at any given time for a multiple-source lifeline water network. In this research, the authors employ a constrained spanning forest (CSF) algorithm to decompose the network into trees rooted at each source. After the decomposition, Horn's algorithm is used to determine the optimal restoration strategy for each tree in the network with the objective of minimizing a penalty value. Restoration of each node in the spanning forest is carried in sequence according to availability of the crew and allows simultaneous jobs to be done on consecutive arcs in the sequence.

Keywords: Water lifeline, Horn's algorithm, Constrained spanning forest

1. INTRODUCTION

Water distribution networks (WDNs) are vulnerable during strong ground motion events such as those experienced during the earthquakes in Kumamoto Japan [1], Christchurch, New Zealand [2] and Surigao Philippines [3]. Since modern society is very much dependent on this important lifeline, quick recovery is essential specifically for health and sanitation reasons and for post-seismic activities. It is also worth noting that a continuing inability to supply water to the affected area leads to the reduction of its habitability.

The estimation or modeling of the risk or damage of WDNs due to natural hazards has been studied extensively [4] – [6]. However, literature regarding optimal sequencing of the repair of a WDN after a seismic event [7], [8] are few. In these papers, Horn's algorithm was employed to determine an optimal restoration strategy for a single-source water network. However, if the WDN has multiple-sources a variation of this algorithm is necessary in order to allow simultaneous repairs at any given time.

The aim of this paper is to formulate and solve the problem of managing the repair of a network damaged due to a natural hazard such as an

earthquake. The network in consideration is a water pipeline system in a particular region in the Philippines. Here, we use a constrained spanning forest (CSF) algorithm to decompose the network into trees each of which contains exactly one water source. Assuming simultaneous repair jobs can take place at any given time, the problem of determining which node to be prioritized by the repair teams is solved using Horn's algorithm, which gives the optimal repair sequence.

The notion of a minimal CSF was used in order to extend the Christofides heuristic to a k -depot version of the Travelling Saleman Problem [9]. In their study of a resource allocation algorithm for multi-vehicle (i.e. unmanned aerial vehicles) systems, Rathinam et al. proved that the problem of finding a minimal CSF can be converted into a minimal spanning tree problem by introducing zero-weight edges between vertices that represent the vehicles/depots [10]. After the network has been decomposed into trees, the authors use Horn's algorithm [12] to determine an optimal repair sequencing of the damaged pipeline network.

2. MATHEMATICAL FRAMEWORK

In this study, the formulation requires that the network in consideration be represented as a

weighted and connected graph. A graph is a $G = (V, E)$ consists of a set V of vertices or nodes and a set E of edges uv where $u, v \in V$. We say that u and v are *adjacent vertices* in G if $uv \in E$. A graph G is connected if for any two vertices u and v , there is a sequence of consecutively adjacent vertices from u to v . A weighted graph is one where each edge is assigned a value, called its weight. A graph is called a *tree* if it contains no cycles and is called a *forest* if it is a union of trees. A rooted tree is a tree where one vertex is designated as a root. The reader is referred to [11] for a more exposition on these terminologies.

In the pipeline network being considered, five water sources were identified. The repair work necessarily involves restoring the links from these sources in order to initiate the delivery of water to other nodes. However, a major consideration is that each node x has a specific demand value $V(x)$ and pipes linking two nodes have a given length. An efficient way to determine which community to service first is of utmost importance.

The initial task involves finding a minimal constrained spanning forest (CSF) of the network with roots at the five identified water sources. The following algorithm for determining the minimal CSF is based on an algorithm in [10]. For this study, we assume that the sources are not adjacent; that is, there is no direct edge between any two source vertices.

2.1 Algorithm 1 (Prim's Algorithm for Minimal CSF).

1. Introduce a zero-weight edge between any pair of source vertices and denote by G' the resulting graph.
2. Apply Prim's algorithm to find the minimal spanning tree of G' : that is:
 - (a) Initialize a tree with a single vertex, chosen arbitrarily from the graph
 - (b) Grow the tree by one edge; of the edges that connect the tree to vertices not yet in the tree, find the minimum-weight edge, and transfer it to the tree
 - (c) Repeat step (b) until all vertices are in the tree
3. Remove the zero-weight edges from the constructed MST; the outcome is the minimal CSF.

The next task is to determine an optimal repair sequence per tree in the minimal CSF. We also assume that there is a repair crew for the tree. Furthermore, due to logistical constraints, a repair crew is designated to exactly one tree only. Finally, we assume that water flows from a source and from each node is uni-directional, hence we can assume a precedence relation among the nodes.

This means that if x and y are nodes of a tree, then x precedes y (written as xPy) if there is a directed path from x to y .

2.2 Algorithm 2 (Horn's Algorithm)

In [12], Horn provided an algorithm which gives an optimal job sequence in a tree-like structure with precedence relations. Given a vertex x in a rooted forest N , let $V(x)$ and $T(x)$ correspond to the value of the restoration job at vertex x and the time to complete the job at node x respectively.

Let $\mathcal{T}[x]$ denote the set of all trees rooted in a vertex x . For $S \in \mathcal{T}[x]$ let

$$V(S) = \sum_{u \in S} V(u) \quad (1)$$

$$T(S) = \sum_{u \in S} T(u) \quad (2)$$

From Eq. (1) and Eq. (2), for any tree rooted in a vertex x , The best ratio at x , denoted by $r(x)$, is defined as shown in Eq. (3) below.

$$r(x) = \max \left\{ \frac{V(S)}{T(S)} : S \in \mathcal{T}[x] \right\} \quad (3)$$

A maximal family tree of x denoted by F_x is an element of $\mathcal{T}[x]$ for which the best ratio is achieved.

A repair sequence of the forest N is a bijection $\sigma: V \rightarrow \{1, 2, 3, \dots, n\}$ assigning to each vertex $x \in V$, its position number $\sigma(x)$ in the repair sequence. An optimal repair sequence is one in which the value $f(\sigma) = \sum_{x \in X} V(x) (\sum \{T(y) : \sigma(x) \leq \sigma(y)\})$ (4) is minimum. This function in Eq. (4) is called a linear delay penalty function.

Horn asserts that an optimal repair sequence is achieved by a two-step process:

- (i) For each node x , calculate the best ratio $r(x)$ and the maximal family tree F_x .
- (ii) By comparing the best ratios, an optimal repair sequence of the network is determined.

3. WDN MODEL

The WDN network considered in this paper is managed and operated by SMWD or Surigao Metropolitan Water District. The network consists of transmission lines with a total length of 62 kilometers and 150 kilometers of distribution pipelines. At present, it serves 23 out of 33 mainland barangays of Surigao City (Figure 1). According to SMWD, as of August 2017, they supply water to more than 86% of Surigao City's population of 158,865 people. Unfortunately, on February 10, 2017, the city was hit by a 6.7 magnitude earthquake with the epicenter located off the coast of Surigao Del Norte. This event compromised the water delivery services to the residents.



Fig. 1 Water Distribution Network Model

To assess the vulnerability of the WDN to seismic hazard, the authors employed a probabilistic seismic hazard analysis (PSHA) within the WDN area. Figure 2 shows the results of the PSHA with the corresponding peak ground acceleration contours. The analysis reveals that the main city can experience peak ground acceleration (pga) from 0.6g to 0.8g.

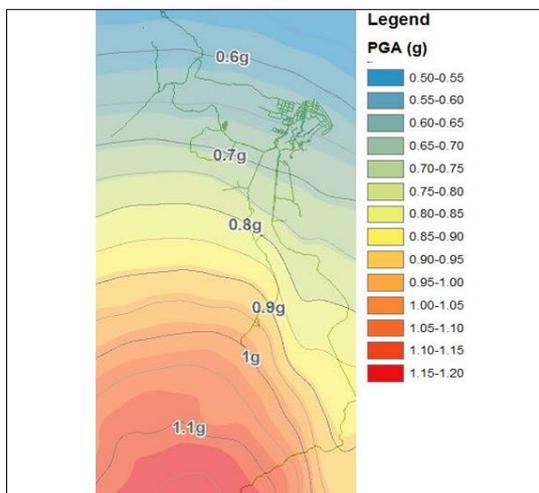


Fig. 2 PSHA PGA map of the WDN

Since the range of the PGA in the area is relatively high, the WDN is vulnerable to damage during a major seismic event. A thorough analysis of the network shows that the pipeline network consists of 416 nodes (junctions) and 415 links (pipes). Table 1 below shows sample nodes i and j with corresponding lengths. The weight of each edge is assumed to be the length of the edge.

Table 1. Link/length of each pipe

Link	Node i	Node j	Length (m)
448	352	354	1432
564	354	355	425
562	348	347	1531
444	348	349	581

4. DATA AND RESULTS

4.1 Optimal Restoration Sequence

In the WDN, the nodes represent the water demand in a specific area and the links represent the supply pipes. Each node x has a value $V(x)$ corresponding to the base demand in liters per second. Furthermore, we define $T(x)$ to be the time (proportional to the length of the pipe in meters) to supply water to node x . Sample data are given in Table 2.

Table 2. Sample nodes with corresponding values of $V(x)$ and $T(x)$

Node (x)	$V(x)$ (in lps)	$T(x)$ (proportional to length in m)
393	0	5691
360	0.89	313
358	0.081	95

Using Prim's algorithm, a minimal spanning forest for the entire network is obtained with the root at junctions 206, 207, 376, 174, and 160. The entire spanning forest consists of 416 nodes and 415 links, removing 127 links from a total of 542 pipes as shown in Figure 3. The minimal spanning forest consists of 5 minimal spanning trees, each corresponding to roots (water supply sources) at junctions 206, 207, 376, 174, and 160, respectively.

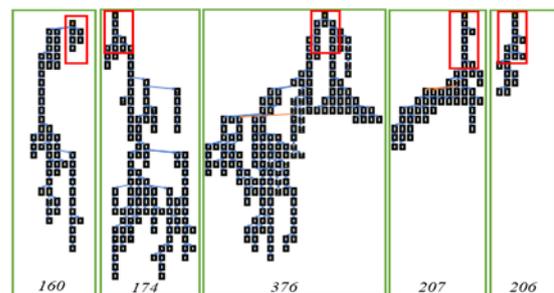


Fig.3 Overview of the minimum spanning forest

Due to the huge data, only the schematic overview of the minimal spanning forest and portions of the 5 minimum-weight spanning trees are shown in Figure 4.

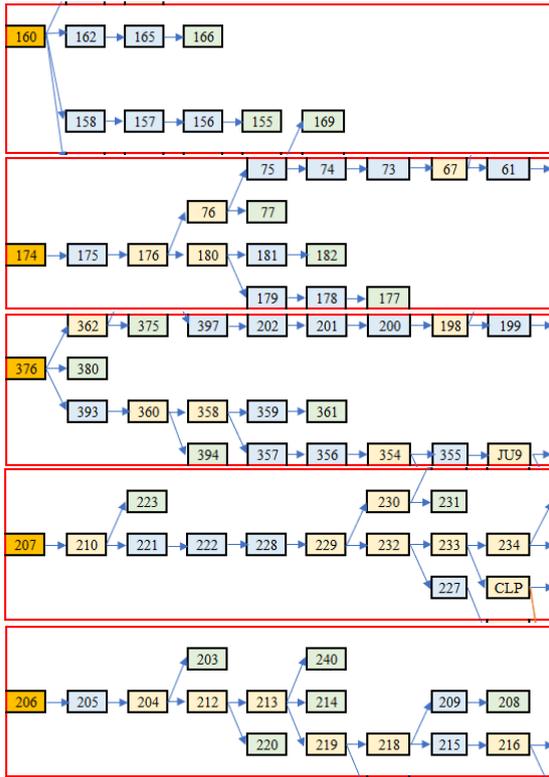


Fig 4. Portions of the minimum weight spanning trees (rooted at nodes 160, 174, 376, 207, and 206)

The spanning trees rooted at 206, 207, 376, 174, and 160 have 18 nodes and 17 links, 53 nodes and 52 links, 167 nodes and 166 links, 116 nodes and 115 links, and 62 nodes and 61 links, respectively.

Applying Horn’s algorithm to the rooted trees obtained above yield an optimal repair sequence for the network. The sequencing of repairs of the first 10 nodes of each tree in the network is shown in Table 3. The value of the least penalty functions $[f(\sigma)]$ for the network rooted at 206, 207, 376, 174, and 160 are 5888.94, 30875.10, 2670469.26, 378509.29, and 103484.26, respectively.

Table 3. First 10 restoration jobs in an optimal sequencing for the 5 minimum spanning trees

Root 160			
order $\sigma(x)$	node x	order $\sigma(x)$	node x
1	158	6	154
2	157	7	164
3	156	8	167
4	162	9	364
5	159	10	128
Root 174			
order $\sigma(x)$	node x	order $\sigma(x)$	node x
1	175	6	75
2	176	7	179
3	180	8	178

4	181	9	177
5	76	10	77
Root 376			
order $\sigma(x)$	node x	order $\sigma(x)$	node x
1	393	6	356
2	360	7	354
3	358	8	352
4	359	9	353
5	357	10	348
Root 207			
order $\sigma(x)$	node x	order $\sigma(x)$	node x
1	210	6	232
2	221	7	233
3	222	8	234
4	228	9	235
5	229	10	241
Root 206			
order $\sigma(x)$	node x	order $\sigma(x)$	node x
1	205	6	203
2	204	7	219
3	212	8	218
4	213	9	215
5	214	10	216

4.2 Restoration Curves

Figures 5 to 9 show the restoration process of the water pipeline of SMWD. These restoration curves are non-decreasing functions [7] and show the repair on a node-to-node basis until the entire network is in full operation.

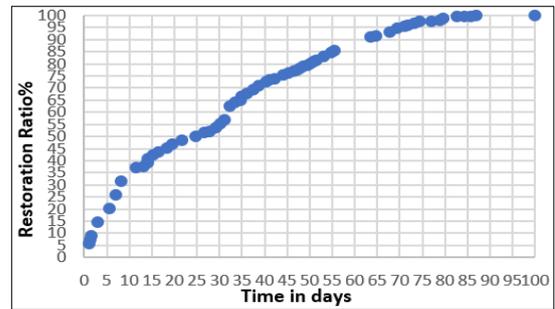


Fig. 5. Restoration Curve (rooted at 160)

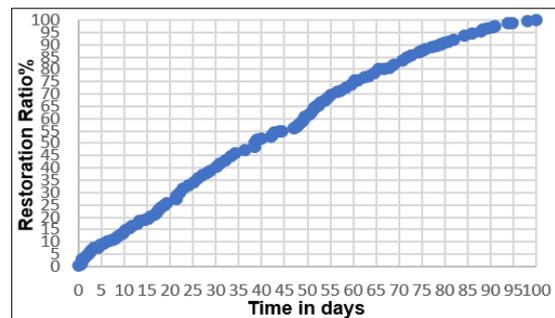


Fig. 6. Restoration Curve (rooted at 174)

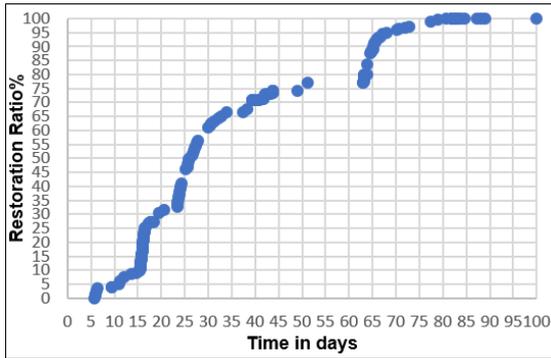


Fig. 7. Restoration Curve (rooted at 376)

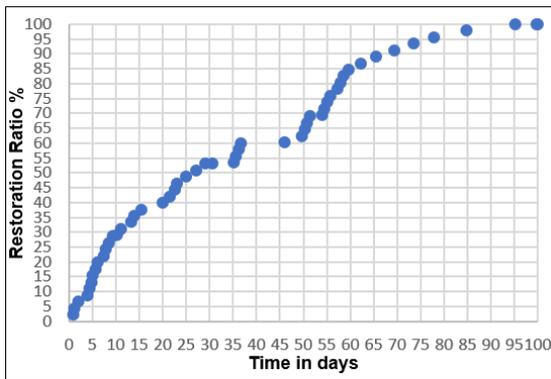


Fig. 8. Restoration Curve (rooted at 207)

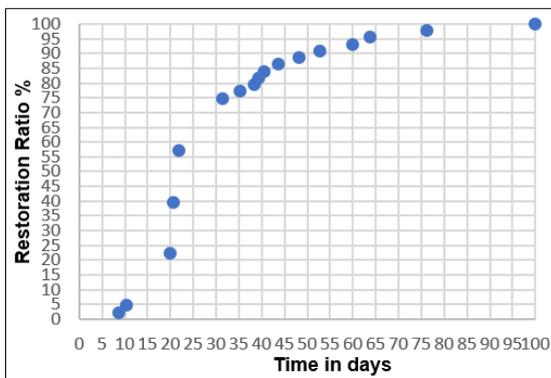


Fig. 9 Restoration Curve (rooted at 206)

5. CONCLUSION

In this paper, the authors employed a variation of Horn's algorithm to allow simultaneous repairs at any given time for a multiple-source lifeline water network. The use of a minimal spanning forest from a network with multiple roots (supply sources) is essential in order to remove the redundancy of pipelines and minimize total repair time.

A constrained spanning forest (CSF) algorithm was used to decompose the WDN into trees rooted at each water supply source. After the

decomposition, Horn's algorithm is used to determine the optimal restoration strategy for each tree in the network with the objective of minimizing a penalty value. Restoration of each node in the spanning forest is done in sequence according to availability of the crew and allows simultaneous jobs to be done on consecutive arcs in the sequence.

For future work, the direction of study is to find a framework whereby a repair crew maybe assigned to an incomplete restoration job to reduce further the repair time and corresponding penalties.

6. ACKNOWLEDGMENTS

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