

FINITE DIFFERENCE METHOD FOR SOLVING HEAT CONDUCTION EQUATION OF THE GRANITE

*Dalal Adnan Maturi¹, Amal Ibrahim Aljedani² and Eman Salem Alaidarous³

¹Departement of Mathematics, Faculty of Science, King Abdulaziz University, Saudi Arabia

*Corresponding Author, Received: 10 Oct. 2018, Revised: 07 Nov. 2018, Accepted: 15 Dec. 2018

ABSTRACT: In this paper, Granite is an underground rock that is under high temperature. Granite is mainly formed of three minerals Marrow, Aluminum alkali silicate and plagioclase. These minerals make the granite white, crimson or light gray. The reasons for the presence of granite may be a lot of ease for the fusion of rock material the continents are largely granite buried under sedimentary rocks. Most of the granite appears at the height of the rocks buried deep into the surface of the earth as a result of mountain formation movements on the crust. This type of rock is widely used to sculpt statues and columns, and it is characterized by its tolerance to the factors of sculpture and erosion more than the types of sedimentary rocks. The granite is of great economic importance and is also used in homes such as stairs and kitchens Aldark and Najran Brown is located only in Najran South Saudi Arabia. So, the finite difference method has been presented to solve the heat conduction equation of the Granite. This method solves the heat conduction equation of the Granite with the help of Matlab. Also, five numerical examples are illustrated by this method. The results reveal that this method is very effective and prove the Granite is a material resistant to various high temperatures.

Keywords: Finite Difference Method, Heat Conduction Equation, Granite, Matlab.

1. INTRODUCTION

Finite differencing methods describe functions as discrete values across a grid, and approximate their derivatives as differences between points on the grid. Knowing a little about how difference methods are formulated and in what regimes they are stable can help save a lot of time, both in the design of finite differencing algorithms, and in the time that they take to run, the finite difference approximations for derivatives are one of the simplest and of the oldest methods to solve differential equations.

We need a fast, realistic and reliable method to solve the heat conduction equation of the Granite . It is one of the types of igneous rocks that form in the ground under the influence of high thermal pressure, usually consists of the main minerals in the form of alcohols and aluminum alkali silks. In addition, plagioclase, these three metals give the granite stone colors are characterized by crimson or light gray. According to biologists, it is formed by crystallization of magma by the slow cooling that came on the fringes of the fusion of buried rocks at a depth of 25-40 km below the surface

Land, and because of this common rock-proliferation must recognize the advantages and disadvantages of granite close.

Advantages and disadvantages of granite, it is lighter than the rest of the surrounding rocks in the

ground, the sizes of granite granules vary between medium and large size, Silica contains silica in abundance among its components, the most common types of rock, it is very beautifully unique due to its color overlap, and It is also characterized as a dielectric material, so the mica sheets are extracted from it and used in the production of insulation purposes.

2. ADVANTAGES AND DISADVANTAGES OF GRANITE

Advantages:

- 1-Its weight is lighter than the rest of the surrounding rocks in the ground.
- 2- The sizes of granite granules vary between medium and large size.
- 3- The most common types of rock.
- 4- It has very high hardness.
- 5- It can be relied upon in various construction works, because of its high ability to resist weathering factors; therefore, it is used in the construction of temples, palaces and museums.

Disadvantages:

- 1- The many cracks in the granite rocks are not polished, so it is filled when refined to be used in several uses.
- 2- The existence of some types of poor and unusable, especially those that do not match the ratios of metals.

3. FINITE DIFFERENCE METHOD FOR HEAT CONDUCTION EQUATION

The linear second order partial differential equation

$$Au_{xx} + 2Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu + G = 0$$

as a parabolic equation if $B^2 - AC = 0$. A parabolic equation holds in an open domain or in a semi-open domain.

Consider a thin homogeneous, insulated bar or a wire of length l . Let the bar be located on the x - axis on the interval $[0, l]$. See the rod have a source of heat. For example, the rod may be heated at one end or at the middle point or has some source of heat. Give $u(x, t)$ denote the temperature in the rod at any instant of time t . The problem is to study the flow of heat in the rod. The partial differential equation governing the flow of heat in the rod is given by the parabolic equation

$$u_t = c^2 u_{xx}, \quad 0 \leq x \leq l, \quad t > 0.$$

Where c^2 is a constant and depends on the material properties of the rod.

The heat conduction equation of the Granite

$$u_t = (0.011)u_{xx}$$

$$u(0, t) = u(1, t) = 0, \forall t \in (0, t_F)$$

$$u(x, 0) = u_0(x), \quad \forall x \in [0, 1],$$

Where t_F Denotes the terminal time for the model. Here without loss of generality, we assume that the spatial domain is $[0, 1]$.

At first divide the physical domain $(0, t_F) \times (0, 1)$ by $N \times J$ uniform grid points

$$t_n = n\Delta t, \Delta t = \frac{t_F}{N}, n = 0, 1, \dots, N,$$

$$x_j = j\Delta x, \Delta x = \frac{1}{J}, j = 0, 1, \dots, J.$$

Then, we denote the approximate solution $u_j^n \approx u(x_j, t_n)$. At an arbitrary point (x_j, t_n) . To obtain a finite difference scheme, we need to approximate the derivatives in (1) by some finite differences. (Explicit scheme)

Substituting

$$u_t(x_j, t_n) \approx (u_j^n - u_j^{n-1})/\Delta t,$$

$$u_{xx}(x_j, t_n) \approx (u_{j+1}^n - 2u_j^n + u_{j-1}^n)/(\Delta x)^2,$$

Into (1), another difference scheme for (1) can be constructed as:

$$\frac{u_j^n - u_j^{n-1}}{\Delta t} = \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{(\Delta x)^2}, \quad 1 \leq j \leq J - 1, 1 \leq n \leq N.$$

4. FINITE DIFFERENCE FORMULATION FOR A ONE-DIMENSIONAL PROBLEM

We consider a bounded domain $\Omega =]0, 1[\subset \mathbb{R}$ and $u: \bar{\Omega} \rightarrow \mathbb{R}$ solving the non-homogeneous Dirichlet problem:

$$D \begin{cases} -u''(x) + c(x)u(x) = f(x), & x \in]0, 1[, \\ u(0) = \alpha, \quad u(1) = \beta, \end{cases} \quad (1)$$

where c and f are two given functions, defined on $\bar{\Omega}, c \geq 0$.

5. A FINITE DIFFERENCE SCHEME

Suppose functions c and f are at least such that $c \in C^0(\bar{\Omega})$ and $f \in C^0(\bar{\Omega})$. The problem is then to find $u_h \in \mathbb{R}^N$, such that $u_i \approx u(x_i)$ for all $i \in \{1, \dots, N\}$,

where u is the solution of problem (1).

Introducing the approximation of the second order derivative by a differential quotient, we consider the following discrete problem:

$$D \begin{cases} -\frac{u_{j+1} - 2u_j + u_{j-1}}{h^2} + c(x_j)u_j = f(x_j), & j \in \{1, \dots, N\} \\ u_0 = \alpha, \quad u_{N+1} = \beta, \end{cases} \quad (2)$$

The problem D has been discretized by a finite difference method based on a three-points centered scheme for the second-order derivative. The problem (2) can be written in the matrix form as:

$$A_h u_h = b_h,$$

Where, A_h is the tridiagonal matrix defined as:

$$A_h = A_h^{(0)} + \begin{pmatrix} c(x_1) & 0 & \dots & \dots & 0 \\ 0 & c(x_2) & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & c(x_{N-1}) & 0 \\ 0 & \dots & \dots & 0 & c(x_N) \end{pmatrix}$$

With

$$A_h^{(0)} = \frac{1}{h^2} \begin{pmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & -1 & 2 & -1 \\ 0 & \dots & 0 & -1 & 2 \end{pmatrix}$$

and

$$b_h = \begin{pmatrix} f(x_1) + \frac{\alpha}{h^2} \\ f(x_2) \\ \vdots \\ f(x_{N-1}) \\ f(x_N) + \frac{\beta}{h^2} \end{pmatrix}$$

The question raised by this formulation is related to the existence of a solution. In other words, we have to determine if the matrix A_h is invertible or not. The answer is given by the following proposition.

Proposition 1 Suppose $c \geq 0$. Then, the matrix A_h is symmetric positive definite.

Definition 1 A finite difference scheme is said to be consistent with the partial differential equation

it represents, if for any sufficiently smooth solution \mathbf{u} of this equation, the truncation error of the scheme, corresponding to the vector $\boldsymbol{\varepsilon}_h \in \mathbb{R}^N$ whose components are defined as:

$$(\boldsymbol{\varepsilon}_h)_j = (\mathbf{L}_h \mathbf{u})(\mathbf{x}_j) - f(\mathbf{x}_j), \quad \text{for all } j \in \{1, \dots, N\}$$

tends uniformly towards zero with respect to \mathbf{x} , when \mathbf{u} tends to zero, i.e. if:

$$\lim_{h \rightarrow 0} \|\boldsymbol{\varepsilon}_h\|_\infty = 0.$$

Moreover, if there exists a constant $\mathbf{C} > 0$, independent of \mathbf{u} and of its derivatives, such that, for all $h \in]0, h_0]$ ($h_0 > 0$ given) we have:

$$\|\boldsymbol{\varepsilon}_h\| \leq \mathbf{C}h^p,$$

with $p > 0$, then the scheme is said to be accurate at the order p for the norm $\|\cdot\|$.

The definition states that the truncation error is defined by applying the difference operator \mathbf{L}_h to the exact solution \mathbf{u} . This means that a consistent scheme implies that the exact solution almost solves the discrete problem.

Lemma 1 Suppose $\mathbf{u} \in \mathbf{C}^4(\Omega)$. Then, the numerical scheme (2) is consistent and second-order accurate in space for the norm $\|\cdot\|_\infty$

6. SEVERALS EXAMPLES

(i) **Example1.** Find the solution of the heat conduction equation of the Granite

$$\begin{aligned} u_t &= (0.011)u_{xx} & 0 < x < 1, & t > 0; \\ u(0, t) &= u(1, t) = 0, & t > 0; \\ u(x, 0) &= \sin \pi x, & 0 \leq x \leq 1. \end{aligned}$$

Applying the finite difference method using Matlab, then the result show as follows.

(ii) **Example2.** Find the solution of the heat conduction equation of the Granite

$$\begin{aligned} u_t &= (0.011)u_{xx} & 0 < x < 1, & t > 0; \\ u(0, t) &= u(1, t) = 0, & t > 0; \\ u(x, 0) &= \sin 2\pi x - \sin 5\pi x, & 0 \leq x \leq 1. \end{aligned}$$

Applying the finite difference method using Matlab, then the result show as follows.

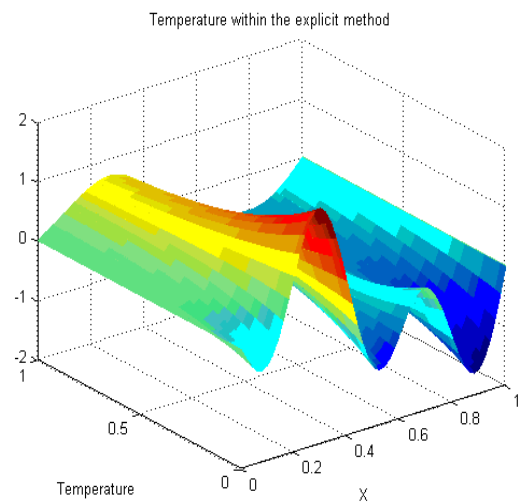
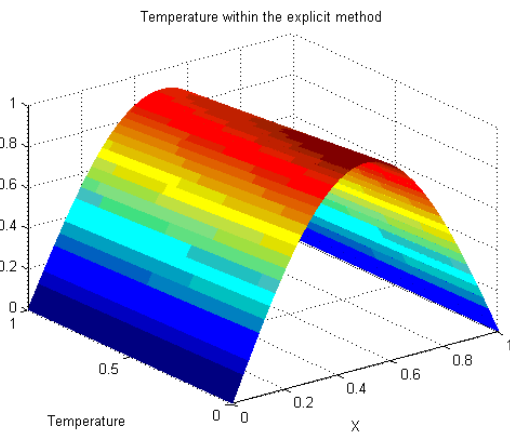
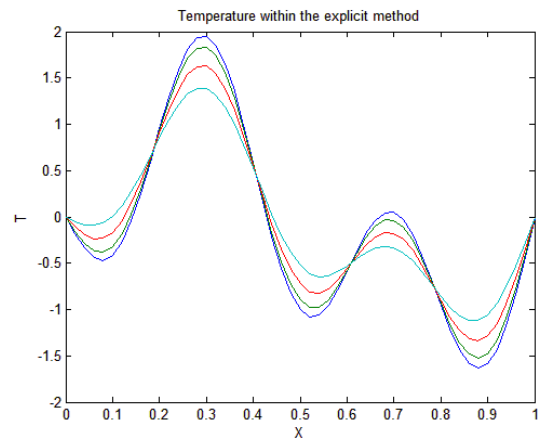
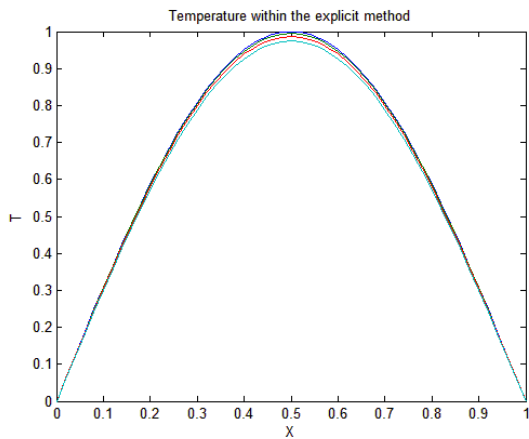


Fig.1 Temperature distributions at several times for the heat conduction of the Granite for example 1.

Fig.2 Temperature distributions at several times for the heat conduction of the Granite for example 2.

(iii) **Example3.** Find the solution of the heat conduction equation of the Granite

$$\begin{aligned}
 u_t &= (0.011)u_{xx} \quad 0 < x < 1, \quad t > 0; \\
 u(0, t) &= u(1, t) = 0, \quad t > 0; \\
 u(x, 0) &= 2 \sin\left(\frac{\pi x}{2}\right) - \sin \pi x + 4 \sin 2\pi x, \quad 0 \leq x \leq 1.
 \end{aligned}$$

Applying the finite difference method using Matlab, then the result show as follows.

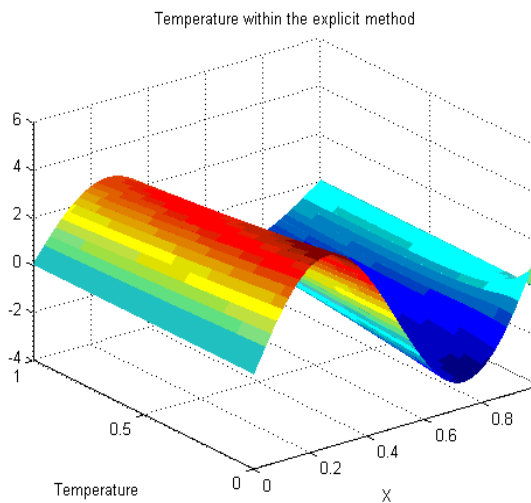
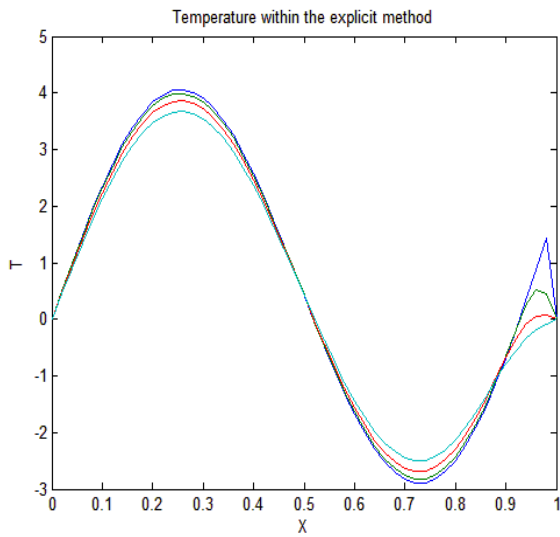


Fig.3 Temperature distributions at several times for the heat conduction of the Granite for example 3.

(iv) **Example4.** Find the solution of the heat conduction equation of the Granite

$$u_t = (0.011)u_{xx} \quad 0 < x < 1, \quad t > 0;$$

$$\begin{aligned}
 u(0, t) &= u(1, t) = 0, \quad t > 0; \\
 u(x, 0) &= x(1-x), \quad 0 \leq x \leq 1.
 \end{aligned}$$

Applying the finite difference method using Matlab, then the result show as follows.

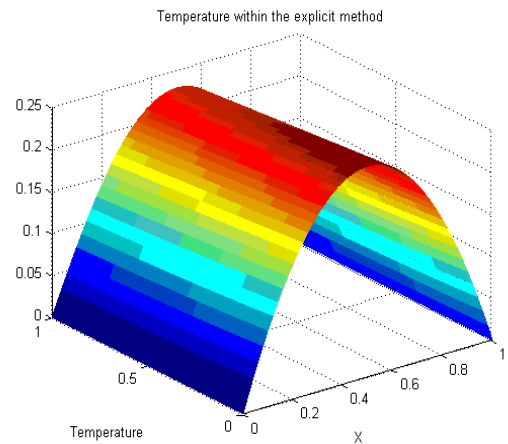
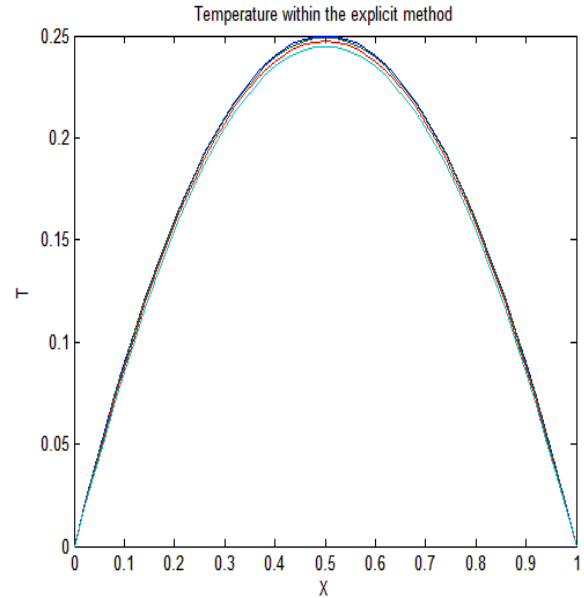


Fig.4 Temperature distributions at several times for the heat conduction of the Granite for example 4.

(v) **Example5.** Find the solution of the heat conduction equation of the Granite

$$\begin{aligned}
 u_t &= (0.011)u_{xx} \quad 0 < x < 1, \quad t > 0; \\
 u(0, t) &= u(1, t) = 0, \quad t > 0; \\
 u(x, 0) &= \sin(x) - 3 \cos 4x, \quad 0 \leq x \leq 1.
 \end{aligned}$$

Applying the finite difference method using Matlab, then the result show as follows.

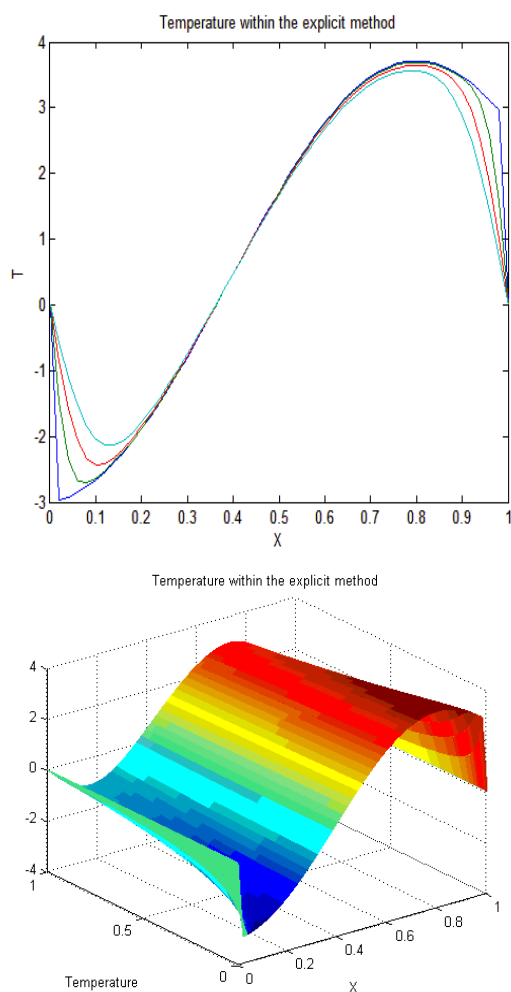


Fig.5 Temperature distributions at several times for the heat conduction of the Granite for example 5.

7. CONCLUSIONS

In this paper, the Finite difference method for solving heat conduction equation of the Granite.

All the examples show that the finite difference method is a powerful mathematical tool for solving heat conduction equation of the Granite. Graphical figures we presented to determine the higher accuracy and simplicity of the proposed method.

Moreover, it should be mentioned that the propounded method can be easily generalized for more heat conduction for another materials different. This work is focused on rocks especially Granite because it is very high hardness. It can be relied upon in various construction works, due to its high resistance to weathering factors; therefore it is used in the construction of temples, palaces and museums.

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