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MODEL IDENTIFICATION FOR THE EVALUATION OF CRITICAL BUCKLING LOAD IN REINFORCED CONCRETE RECTANGULAR COLUMNS

*Oussama Jarachi, Moulay Larbi Abidi and Toufiq Cherradi

Mohammadia Engineering School, Rabat, Morocco

*Corresponding Author, Received: 19 Feb. 2019, Revised: 21 March 2019, Accepted: 15 April 2019

ABSTRACT: The stability analysis of columns is a complex problem that includes not only first order effects but also second order ones. The solution depends on a number of parameters and comes from non-polynomial equations which make the resolution tedious and needs the use of computer programs or abacuses. The objective of this paper is to establish a new model to determine the critical load for reinforced concrete rectangular columns subjected to compression in accordance with the Eurocode 2. The tools presented in this paper will enable engineers to design a broad spectrum of compressed columns or verify their stability without the use of a computer or design charts. It will also help them to design columns in an economic way as it evaluates the critical buckling load by taking into account the influence of each parameter on the stability of the element.

Keywords: Axial loading; Buckling; Reinforced concrete; Slender columns; Identification

1. INTRODUCTION

Civil engineers often opt for structures constituted of beams and columns. Therefore, due to the importance of compression in columns for the stability of the structure, the design codes require that second order effects be considered.

When these columns are subjected to axial compression, the buckling phenomenon happens earlier and it is necessary to perform a complete nonlinear analysis which requires the use of numerical methods.

In the previous article [1], we have computed the critical buckling load using the general method [2] and have proposed design charts to verify columns stability. In this article, we will write a model that allows designers to estimate the critical load of reinforced concrete rectangular columns subjected to compression without the use of computers or design charts.

2. PROBLEM PRESENTATION

The buckling phenomenon is characterized by a sudden sideways deflection of axially loaded structural members when they are slender. This phenomenon has been highlighted by Euler who determined the expression of the critical load $P_c = \frac{\pi^2 . E . I}{l^2}$, at which an ideal element will buckle.

In reinforced concrete, stability analysis consists of proving that there is a deflection of the element that equilibrates the design solicitations while taking into account second-order effects [3], [4].

The BAEL code and the Eurocode 2 admit that for usual structures, regardless of the end conditions, the study of a compressed column under an axial load $\mathbf{N_u}$ can be brought to the case of a double articulated column of length $\mathbf{l_f}$ well known as model column [4], [5], [6], [7].

The advantage of the model column is to rally the buckling problem into the study of one cross-section at the ultimate limit state. It is sufficient to verify, in the middle cross-section, that there is an equilibrium between internal and external loads.

The fundamental assumption of the model column is that the deformation is sinusoidal. The maximum deflection f and the curvature $\frac{1}{r}$ are therefore tied by the following equation:

$$\boldsymbol{f} = \frac{\mathbf{1}}{\mathbf{r}} \times \frac{\mathbf{l_f^2}}{\boldsymbol{\pi}^2} \tag{1}$$

The external eccentricity or the eccentricity of the axial load N_u in the middle cross section is, therefore:

$$e_{ext} = e_1 + \frac{1}{r} \times \frac{l_f^2}{\pi^2} \tag{2}$$

$e_1 = e_c + e_a$ where e_c is the structural eccentricity and e_a is the accidental eccentricity due to execution imperfections.

Furthermore, in the middle cross-section, each state of deformation defined by its curvature $\frac{1}{r}$ and the strain ε in a particular point of the cross-section (ε_{bc} for the most compressed fiber for example) leads to the equilibrium equations.

The stresses are functions of the strains, thus they depend on the curvature $\frac{1}{r}$ according to the compatibility relations. So, by setting the states of deformation of the cross-section by the couple$\left(\varepsilon_{bc}, \frac{1}{r}\right)$ the internal loads $N_i\left(\varepsilon_{bc}, \frac{1}{r}\right)$ and $M_i\left(\varepsilon_{bc}, \frac{1}{r}\right)$ can be calculated, and the internal eccentricity deduced:

$$e_{int} = \frac{M_i}{N_i} \tag{3}$$

In the $\left(\frac{1}{r}, e\right)$ plane (Fig.1), the geometrical equation (2) is represented by a straight line and the mechanical equation (3) is represented by blue network curves parameterized by N_i = constant.

The critical load $\boldsymbol{N_{uc}}$corresponds to the curve N_i that is tangent to the straight line:

$$e_{ext} = e_1 + \frac{1}{r} \times \frac{l_f^2}{\pi^2} \tag{4}$$

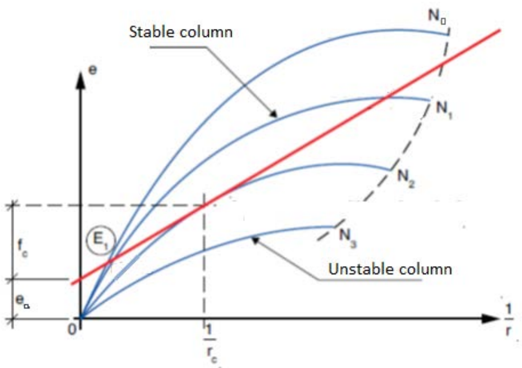

Fig. 1: Network curves for several values of N_i

3. GEOMETRIC IMPERFECTION EFFECT

According to Eurocode 2, the perfectly centered loading does not exist [2], there will always be an eccentricity due to geometric defects. For isolated columns, the geometrical imperfections may be taken into account as an additional eccentricity $\mathbf{e_a}$ given by:

$$\boldsymbol{e_a} = \frac{\boldsymbol{a_h a_m l_f}}{\mathbf{400}} \tag{5}$$

with $\begin{cases} \boldsymbol{a_h} = \frac{\mathbf{2}}{\sqrt{\boldsymbol{l}}} \quad \text{with} \quad \frac{\mathbf{2}}{\mathbf{3}} \leq \boldsymbol{a_h} \leq \\ \mathbf{1}\boldsymbol{a_m} = \sqrt{\mathbf{0.5}\left(\mathbf{1} + \frac{\mathbf{1}}{\boldsymbol{m}}\right)} \\ \text{m=1 for an isolated column} \\ \text{l= column's length} \end{cases}$

Note: For cross sections loaded by the compression force with symmetrical reinforcements, the Eurocode2 recommends to assume the minimum eccentricity $e_0 = \max\left(\frac{h}{30}, 20\text{mm}\right)$ where h is the depth of the cross-section.

4. NUMERICAL BUCKLING ANALYSIS

4.1 The nonlinear buckling analysis method

To assess most precisely the buckling resistance of columns, the general method analysis has been used. It is based on a nonlinear analysis including:

- Geometric nonlinearity (second order effects),
- Nonlinear mechanical behavior of materials.

4.2 Analysis of the middle cross section

According to the position of the neutral axis, there are two cases (Fig.2):

- The cross-section is entirely compressed,
- The cross-section is partially compressed.

Let's consider a linear deformation represented by a couple of parameters$(\varepsilon_{sup}, \varepsilon_{inf})$. For each value of this couple parameters one can evaluate:

- the resisting solicitations $\mathbf{N_i}$, $\mathbf{M_i}$
- the corresponding deformation of the section at the middle span.

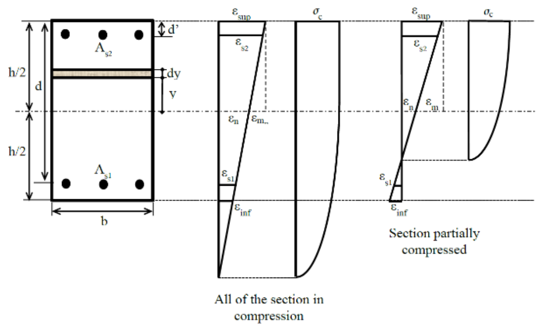

Fig. 2: Strains and stresses diagrams

The curvature of the middle cross section is $\frac{1}{r} = \frac{2\varepsilon_m}{h}$.

Therefore:

$$\boldsymbol{e_{ext}} = \frac{\boldsymbol{a_h \cdot l_f}}{\mathbf{400}} + \frac{\mathbf{2}\boldsymbol{\varepsilon_m}}{\boldsymbol{h}} \times \frac{\boldsymbol{l_f^2}}{\boldsymbol{\pi^2}} \tag{6}$$

The concrete resistant solicitations can be computed by the following formula:

$N_c = \int dN_c \text{with} dN_c = \text{b}.\sigma.dy$
$M_c = \int dM_c \;\text{ with }\; dM_c = \text{b}.\sigma.y.dy$

The internal solicitations are, then, given by:

$N_i = N_c + A_{s1}\sigma_{s1} + A_{s2}\sigma_{s2}$
$M_i = M_c + A_{s2}\sigma_{s2}\left(\frac{h}{2} - d'\right) - A_{s1}\sigma_{s1}(d - \frac{h}{2})$

with $\sigma_s < 0$ if tension

Hence: $$e_{int} = \frac{M_i}{N_i} \quad (7)$$

If the equation $e_{int} = e_{ext}$ has a solution then the column will be stable, else it will buckle. To determine the critical normal load, the initialization of the normal effort should be at a suitable N_0 value, then the stability of the column is checked. Next to the value of N_u is incremented until the column is no longer stable. The maximum normal load supported without instability is the sought critical normal load, named N_{max}.

In the previous article [1], the critical normal load N_{max} was computed with an application created in the mathematical program Matlab simulink.

5. HYPOTHESIS AND MATERIALS PROPRIETIES

5.1 Hypothesis

- The column is isolated and simply supported at both ends.

- The cross-section of the columns is rectangular.

  b = cross section width,

  h = cross-section height in the buckling plane(in most cases $h < b$)

  l_f The = effective length of the column,

- The section contains bars put symmetrically: $A_{s1} = A_{s2} = \frac{A}{2}$

  A is the total area of the steel reinforcements

5.2 Properties of steel bars

The design diagram used is the horizontal top branch one presented in the article 3.2.7 of EC2 [1], [2].

$E_s = 200\text{GPa}$: modulus of elasticity

$f_{yk} = 500\text{MPa}$: characteristic yield strength

5.3 Concrete stress-strain relationship

To compute a design value of the ultimate load, the stress-strain relation used is shown in the article 5.8.6(3) of the EC2 [1] [2].

5.4 Effect of the load's duration

The creep may be taken into account by multiplying all strain values in the concrete stress-strain diagram with a factor $(1 + \varphi_{ef})$ where φ_{ef} is the effective creep ratio given by:

$$\varphi_{ef} = \varphi(\infty, t_0)\frac{M_{0Eqp}}{M_{0Ed}} \quad (8)$$

$\varphi(\infty, t_0)$ is the final value of creep coefficient,
M_{0Eqp} is the first order bending moment in quasi-permanent load combination (SLS),
M_{0Ed} is the first order bending moment in design load combination (ULS).

5.5 Notations

$p = \frac{A.f_{yd}}{b.h.f_{cd}}$: mechanical percentage of reinforcement

$\nu = \frac{N_u}{b.h.f_{cd}}$: relative normal load

ν_{max} is the critical relative normal load.

f_{yd} is the design yield strength of reinforcement

f_{cd} is the design value of concrete compressive strength.

$\boldsymbol{f_{cd} = f_{ck}/\gamma_c}$ with $\boldsymbol{\gamma_c = 1.5}$
and $\boldsymbol{f_{ck} = 25\ MPa}$

6. INFLUENCE ON ν_{max} OF DIFFERENT PARAMETERS

6.1 Influence of the slenderness l_f/h

The plot of ν_{max} versus the slenderness l_f/h for $\varphi_{ef} = 0; 1$ and 2 and for different values of p shows that the curves are exponential. Fig.3 is an example of these curves for $\varphi_{ef} = 1$.

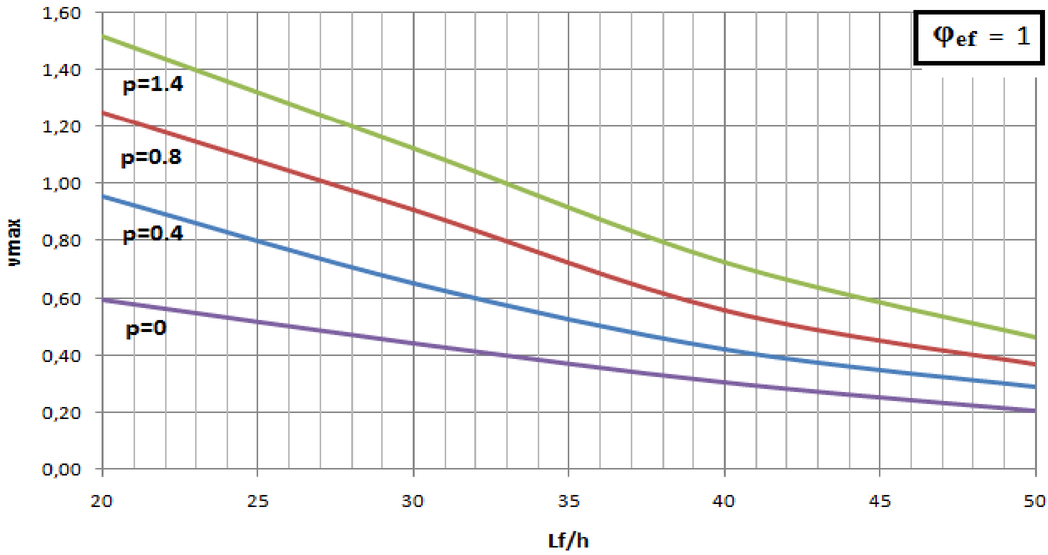

Fig. 3: ν_{max} versus the slenderness l_f/h for $\varphi_{ef} = 1$

6.2 Influence of the column length

We try, then, to see if there is an influence of the length of the column (with equal slenderness) on the results.

We, therefore, vary the length of the column while keeping the slenderness constant; we obtain hence the results represented in fig.4.

By drawing the line minimizing the deviations, we obtain the most representative line of the experimental point cloud. It has for the equation:

$$\frac{\nu_{max}(l_f)}{\nu_{max}(20)} = -2.74 \times 10^{-3} l_f + 1.04$$

6.3 Effect of the mechanical percentage of reinforcements

The plot of ν_{max} versus p for $\varphi_{ef} = 0; 1$ and 2 and for different values of the slenderness l_f/h shows that the curves ν_{max} over p are practically linear. Fig.5 is an example of these curves for $\varphi_{ef} = 2$.

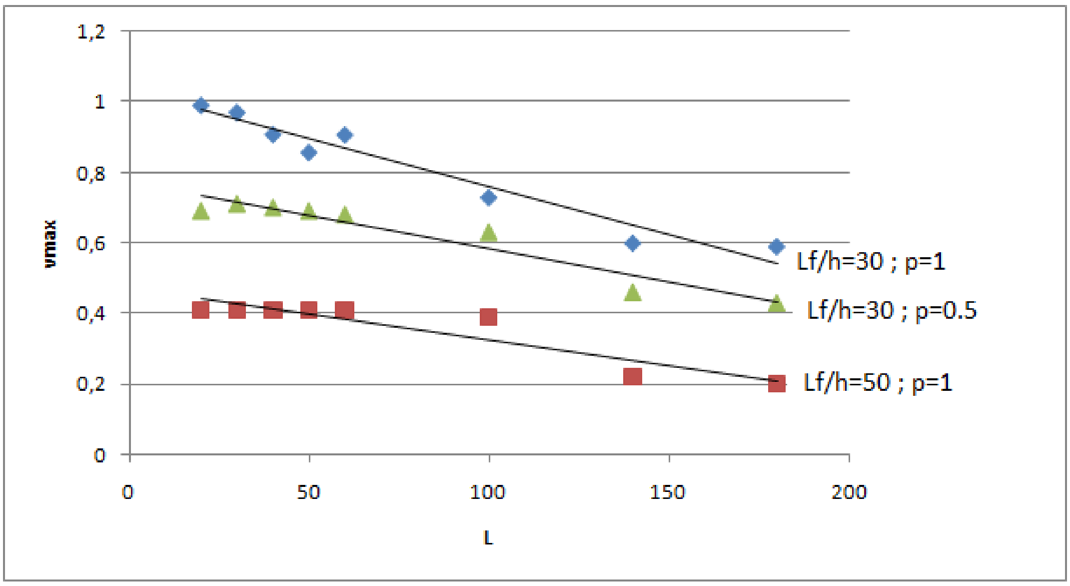

Fig. 4: Influence of the length of the column.

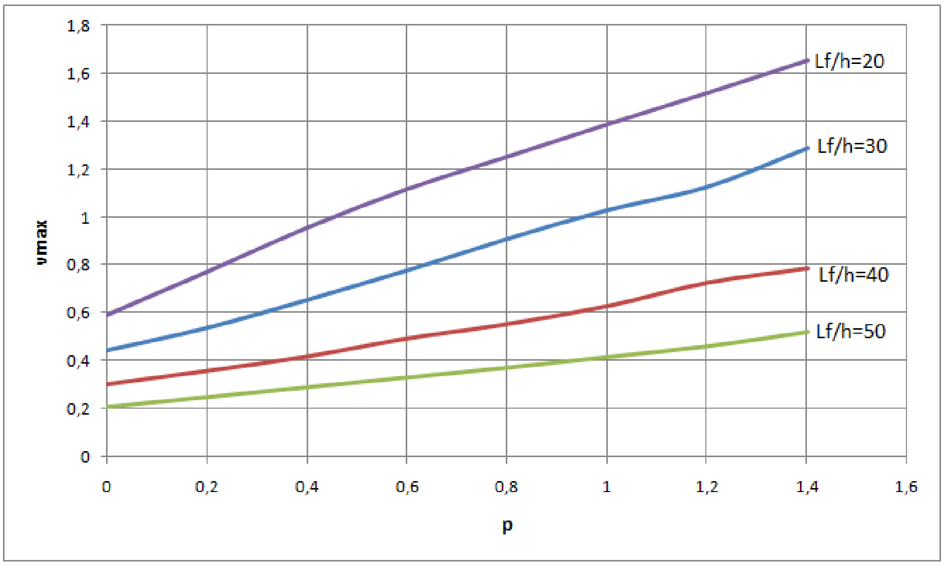

Fig. 5: ν_{max} versus p $\varphi_{ef} = 2$

6.4 Influence of the concrete compressive strength

We search then the influence of f_{ck} thus we plot on ν_{max} versus f_{ck} (Fig.6)

The most representative line of the experimental points cloud has for the equation:

$$\frac{\nu_{max}(f_{ck})}{\nu_{max}(25)} = -1.98 \times 10^{-3} f_{ck} + 1.08$$

The influence of the concrete strength on ν_{max} is lower when it is less than 80 MPa. Thus we can neglect it.

7. SEARCH FOR AN ADEQUATE MATHEMATICAL MODEL

We propose, then, to look for a mathematical model giving the buckling critical load. So we will first proceed to an identification routine on curves obtained in fig.3.

We choose a theoretical curve that closes to the real curve (fig.7) and then we calculate the correlation coefficient R^2.

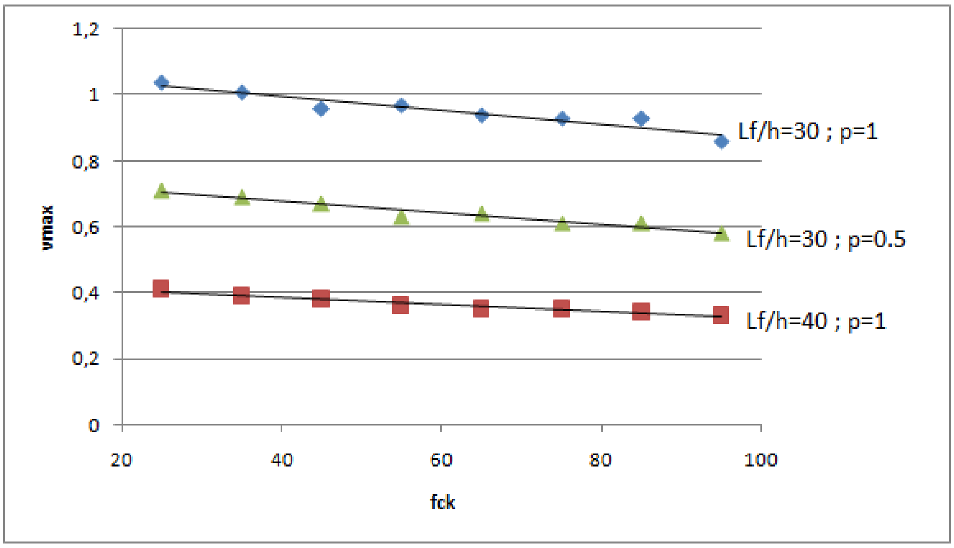

Fig. 6: Influence of f_{ck}.

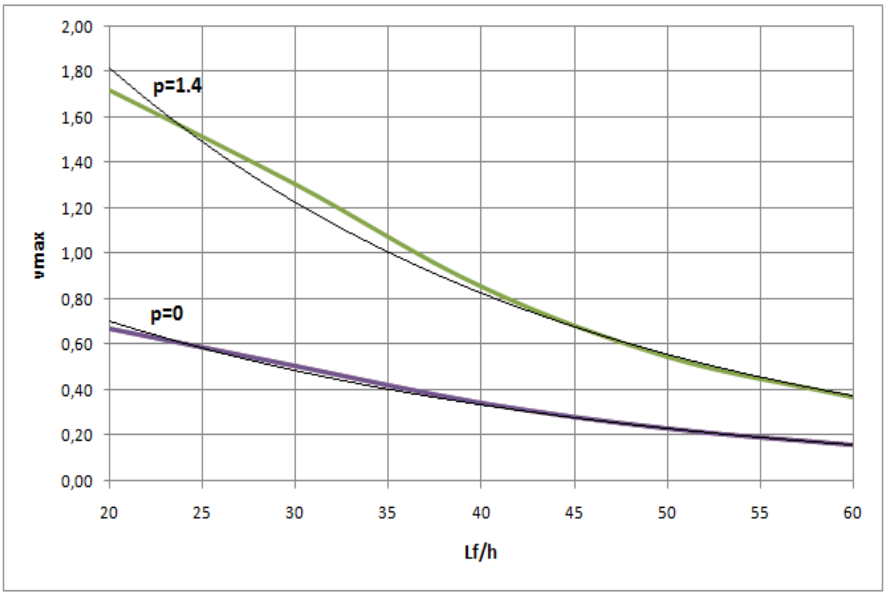

Fig. 7: Identification curves

When $\varphi_{ef} = 0$ and p=1.4 we find that the equation $\nu_{max} = 4.00e^{-3.94\times10^{-2}\times l_f/h}$ best approximates the experimental curve with a high correlation coefficient $R^2 = 0.99$

When $\varphi_{ef} = 0$ and p = 0 we find that the equation $\nu_{max} = 1.47e^{-3.94\times10^{-2}\times l_f/h}$ best approximates the experimental curve with a high correlation coefficient $R^2 = 0.99$

Furthermore, we have noted (fig.4) that the curves ν_{max} over p are practically linear. We can then make a linear interpolation with respect to p.

For $\varphi_{ef} = 0$ we obtain:

$$\nu_{max} = (1.806p + 1.468)e^{-3.94\times10^{-2}\times l_f/h}$$

Then we want to find a valid model for all values of φ_{ef}. Thus we will exploit the following 8 points (Table 1), taken from the previous abacuses.

Table 1

φ_{ef}	p	l_f/h	ν_{max}
0	0	20	0.67
		50	0.23
	1.4	20	1.72
		50	0.55
2	0	20	0.56
		50	0.20
	1.4	20	1.62
		50	0.51

We deduce that:

for p=0 and $l_f/h = 20$ then $\frac{\nu_{max}(2)}{\nu_{max}(0)} = 0{,}84$

for p=0 and $l_f/h = 50$ then $\frac{\nu_{max}(2)}{\nu_{max}(0)} = 0{,}87$

for p=1.4 and $l_f/h = 20$ then $\frac{\nu_{max}(2)}{\nu_{max}(0)} = 0{,}94$

for p=1.4 and $l_f/h = 50$ then $\frac{\nu_{max}(2)}{\nu_{max}(0)} = 0{,}94$

By security we take:

$$\text{and} \quad \begin{cases} \frac{\nu_{max}(2)}{\nu_{max}(0)} = 0{,}84 \text{for} \quad p=0 \\ \frac{\nu_{max}(2)}{\nu_{max}(0)} = 0{,}94 \text{for} \quad p=1.4 \end{cases}$$

This gives the following equation by linear interpolation:

$$\frac{\nu_{max}(2)}{\nu_{max}(0)} = 0{,}84 + \frac{p}{14}$$

By linear interpolation in the function of φ_{ef} we obtain:

$$\frac{\nu_{max}(\varphi_{ef})}{\nu_{max}(0)} = 1 - \left(0.08 - \frac{p}{28}\right)\varphi_{ef} = \gamma_{\varphi}$$

The influence of creep on the buckling resistance will, therefore, be quantified by γ_{φ}. The model becomes then:

$$\nu_{max} = \gamma_{\varphi}(1.806p + 1.468)e^{-\frac{0.04 l_f}{h}}$$

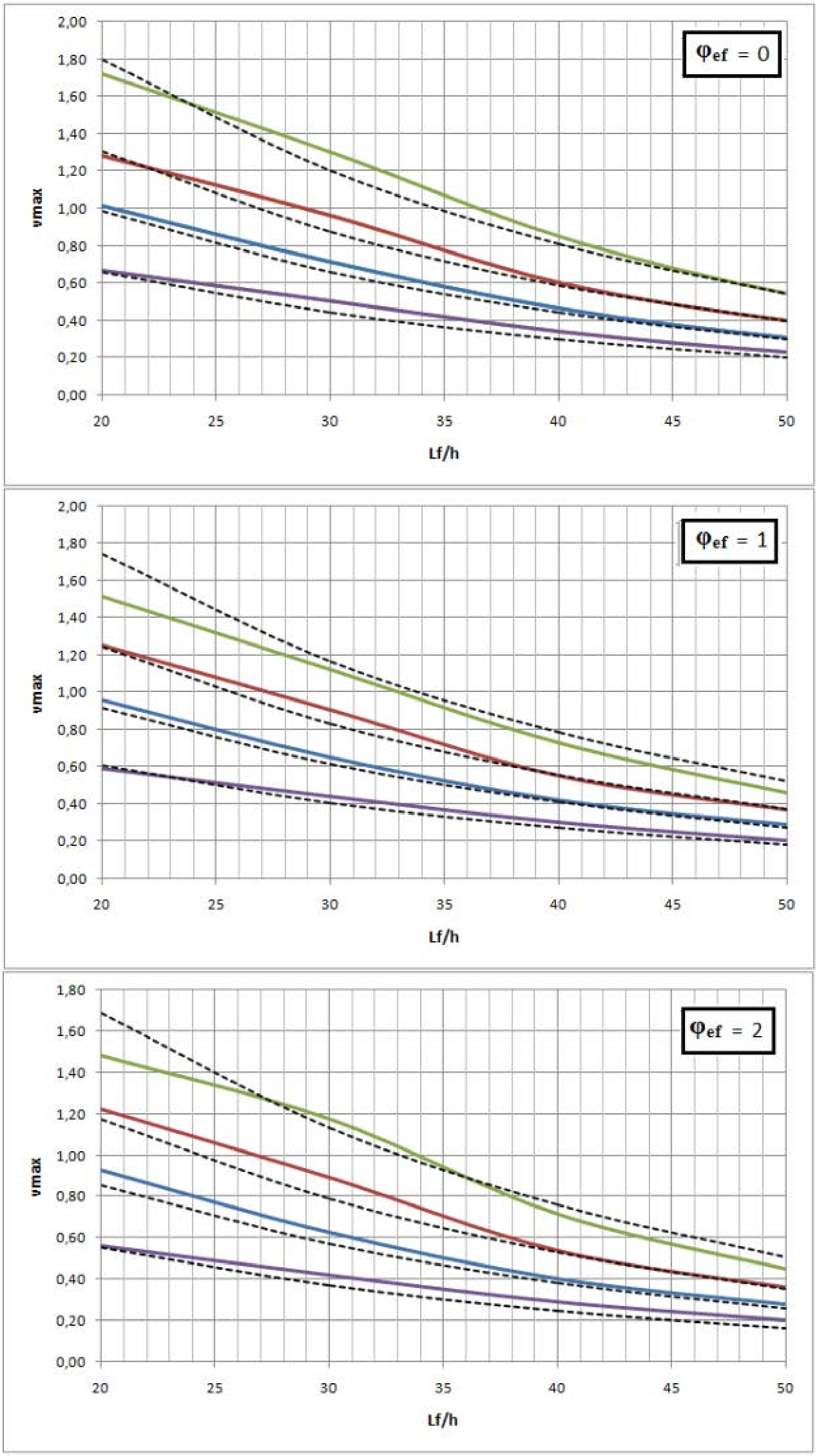

Fig. 8: Theoretical curves versus real curves

$$\gamma_{\varphi} = 1 - \left(0.08 - \frac{p}{28}\right)\varphi_{ef}$$

Finally, the identification shows that the following equation is the best model to calculate the critical load:

$$\nu_{max} = \gamma_{\varphi}\gamma_{l}(1.806p + 1.468)e^{-\frac{0.04 l_f}{h}}$$

$$\gamma_{\varphi} = 1 - \left(0.08 - \frac{p}{28}\right)\varphi_{ef}$$

$$\gamma_{l} = 1.04 - 2.74 \times 10^{-3} l_f$$

8. VALIDATION OF THE MODEL

In order to validate the model, we have plotted (Fig.8) the real curves (in color) superimposed on the curves given by the theoretical model (dashed line). We note that the theoretical and real curves are very close to each other, which validates the previous model.

9. CONCLUSION

A new model has been produced in this article for the determination of buckling critical load of rectangular columns under uniaxial loading. It is congruent to the prescriptions of Eurocode 2 and applies to rectangular columns with symmetrical reinforcements and slenderness up to 50. In other cases, conventional methods must be used.

This model can be used to determine the buckling resistance of these columns and verify their stability without having to use computer programs or abacuses. It can also be used to design economically the column so that it doesn't face instability.

10. ACKNOWLEDGMENTS

Please note that this will be double-blind submission. The author’s identities will be removed before sending to reviewers. The authors will not know who the reviewers are. Try to follow the reference style below. Some references from International Journal of GEOMATE would be appreciated. The benefit will be returned to all authors by increasing the impact factor of the journal.

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