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AN ALTERNATIVE METHOD FOR DETERMINING EROSION PARAMETERS RELATED TO NON-LINEAR MODEL; BASED ON SUBMERGED JET EROSION TEST

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ABSTRACT: Erosion parameters are important factors in riverbank erosion and retreat analysis. The submerged jet test is a widely used technique for measuring erosion resistance of soil based on impinging jet theory. This study aimed to develop a method for determining erosion parameters related to a non-linear erosion model based on JET data, namely, the Linearization method (LM). To verify the LM, the erosion resistance of the U-Tapao riverbank soil was determined using a JET device. Test results show that the exponential rise equation fit well with the scour depth JET data (R^2 = 0.931-0.999), and the critical shear stress of bank soils was in the range 1.306-24.33 Pa. The LM can be successfully applied to JET data, with R^2 in range 0.800-0.985. The erodibility coefficients from LM ranged within 0.02-2.53 $cm^3/N{\cdot}s$. The LM improved the determination of erosion parameters, being superior to three existing methods (Blaisdell, Iterative, and Scour depth solution method). The LM proved more reliable than linear models in erosion rate prediction. This model approach also suggests that the JET testing processes should be revised.

Keywords: Submerged jet test, Erodibility coefficient, Critical shear stress, Non-linear model

1. INTRODUCTION

Riverbank erosion is the main process in river morphodynamics, affecting a wide range of physical issues related to alluvial channels [1]. Riverbank erosion is defined as the direct removal of aggregates from the bank surface by flowing water that generates boundary shear stresses on the bank surface. If the boundary shear stresses are below the critical shear stress (shear stress at which erosion begins), the erosion rate is assumed to remain zero [2,3]. Erosion occurs when the boundary shear stress exceeds the critical shear stress. The erosion rate of a cohesive riverbank is commonly predicted using an excess shear stress model [3-5]:

$$\varepsilon_r = k_d\left(\tau_o - \tau_c\right)^a \tag{1}$$

where ε_r is the rate of erosion (m/s), k_d is the erodibility coefficient (m^3/N.s), τ_o is the developed boundary shear stress (Pa), τ_c is the critical shear stress (Pa), and a is a constant exponent. Often a is assumed to be unity and Eq. (1) becomes a linear excess shear stress equation [3,5,6].

The linear excess shear stress equation has been successfully used by numerous researchers [7,8]. In contrast, some studies claimed that the linear model was overly simplified and did not apply to a wide range of applied shear stresses [9], while other studies concluded that a non-linear detachment equation with a power relationship was better for describing ε_r [10].

Erosion parameters (τ_c and k_d) of riverbank soil are key factors in riverbank retreat analysis, but both of these are difficult to quantify [11]. There are various methods to determine the erosion parameters. Several methods have been developed to estimate τ_c and k_d, such as traditional flume tests [12], a hole erosion test [13], and erosion function apparatus [14]. Such determinations can be performed in a laboratory using disturbed soil samples. Alternatively, the in-situ submerged jet test (JET) developed by Hanson [5] is preferred over those methods, because it can be directly applied to the riverbank surface without sample preparation.

Many studies have used the JET test to determine the erosion parameters in the linear erosion model [15, 16]. Methods to identify parameters in a non-linear erosion model based on JET tests are still insufficient. Therefore, the objectives of this study were to develop alternative methods for determining erosion parameters in a non-linear erosion model based on JET erosion test, perform in-situ submerged jet tests for determining erosion parameters of U-Tapao riverbank soil and verify the proposed model predictions of erosion rate based on JET data.

2. BACKGROUND

2.1 Submerged jet test

Erosion parameters can be determined by conducting the submerged jet test [6], which is the most widely used type of in-situ test. A JET device (Fig. 1a) provides convenience and portability for testing in the field. This apparatus allows testing of varied soils under varied conditions. The JET device consists of three important parts, submergence tank, jet tube and point gage. The device distributes a circular jet through the nozzle at a uniform velocity. The jet water diffuses radially producing shear stress on the bank soil (Fig. 1b). Consequently, a scour hole is created and is measured at regular intervals throughout the test. Time series of shear stress and scour depth are recorded to determine the erosion parameters.

2.2 Determination of erosion parameters based on a linear model

Estimates of the erosion parameters have been developed by several researchers. Hanson and Cook [12] and Hanson et al. [17] developed an analytical procedure for the direct estimation of τ_c and k_d based on diffusion principles developed by Stein and Nett [18]. The critical shear stress can be determined as follows:

$$\tau_c = \tau_o \left(\frac{H_p}{H_e} \right)^2 \tag{2}$$

where τ_o is maximum stress at the nozzle ($\tau_o = C_f \rho U_o^2$), where C_f is the coefficient of friction, ρ is the density of water and U_o is velocity at the jet nozzle equal to $U_o = \sqrt{2gh}$, h is the differential head), H_p is potential core length ($H_p = C_d d_o$), C_d is diffusion constant ($C_d = 6.3$), d_o is jet nozzle diameter, and H_e is equilibrium scour depth. Three methods related to the linear erosion model (i.e. Eq. 1 with a = 1) have been proposed to determine both τ_c and k_d, and these approaches are briefly described below.

2.2.1 Blaisdell solution method

Blaisdell solution method (BS) was proposed early on to determine the τ_c and k_d. Blaisdell et al. [19] developed a hyperbolic logarithm equation for determining the τ_c as shown in Eq. (3) from the equilibrium scour depth, which was assumed to be the long-term asymptotic limit. In this case, the erosion rate was equal to zero.

$$x = [(f - f_o)^2 - A^2]^{0.5} \tag{3}$$

where $x = \log[(U_o t)/d_o]$ t is time step, $f = \log[H/d_o] - \log[(U_o t)/d_o]$, H is scour depth at each time step, $f_o = \log(H_e/d_o)$, and A is the value of semi-transverse on the semi-conjugate axis of a hyperbola. Least-squares fit was used to find suitable f_o and A, by fitting Eq. (3) to JET data.

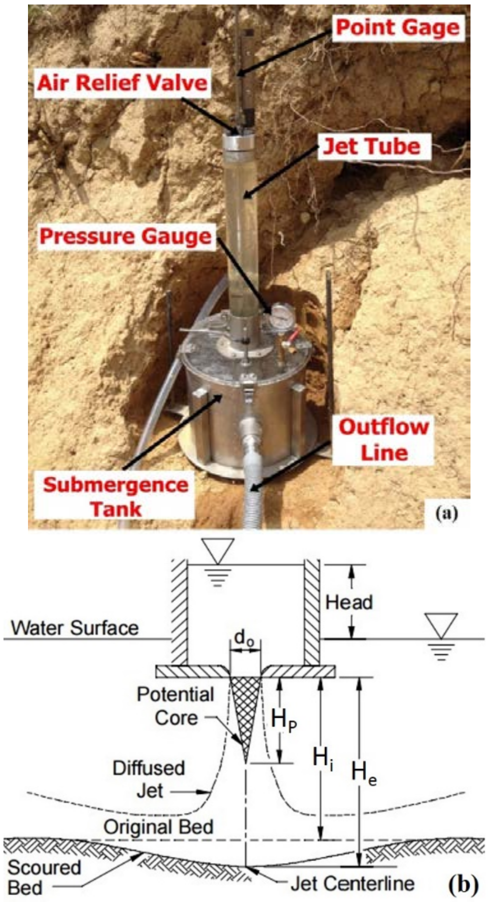

Fig. 1 Schematic representation of (a) submerged jet device and (b) diffuse jet producing boundary shear stress on the bank soil (adept from Hanson and Cook [6])

The erodibility coefficient is then determined by fitting the scour data into the excess shear stress equation (Eq. 1), based on the measured scour depth, time, and pre-determined value of τ_c. Eq. (4) was obtained by combining Eq. (2) and Eq. (1) for a dimensionless relationship. Then, integrating Eq. (4) from the nozzle to the scour depth gives the theoretical time function in Eq. (5).

$$\frac{dH^*}{dT^*} = \frac{(1 - H^{*2})}{H^{*2}} \tag{4}$$

$$t_m = \frac{H_e^3}{k_d \tau_o H_p^2}[0.5\ln\left(\frac{1+H^*}{1-H^*}\right) - H^* - 0.5\ln\left(\frac{1+H_i^*}{1-H_i^*}\right) + H_i^*] \quad (5)$$

Here $H^* = H / H_e$ and t_m is theoretical time. The least-squares method is employed to fit the predicted t_m and the measured time for finding the best solution of k_d. The analytical estimates can be calculated in an Excel spreadsheet, the template for which was early developed by Hanson and Cook [6], and further modified by Daly et al. [20].

2.2.2 Iterative solution method

Simon et al. [21] reported that the erosion parameter estimates based on the equilibrium scour depth from BS do not always converge to a reasonable solution. Therefore, they developed an alternative approach, the so-called Iterative solution method (IS). This approach minimizes the root mean square error between t_m from Eq. (5) and the measurement t_m from JET test to determine τ_c and k_d simultaneously. The iterations start by using initial τ_c and k_d from the BS method. In calculations, a lower limit was used to preclude τ_c from reaching zero and to avoid H_e values smaller than the maximum scour depth form JET test.

2.2.3 Scour depth solution method

Daly et al. [20] stated that the BS method provided a too conservative solution. The BS method has become the default approach for determining τ_c and k_d form JET data. Daly et al. [20] also developed an alternative method called the Scour depth solution method (SS). The SS method is an iterative approach similar to IS. In this method, τ_c and k_d are calculated using Eq. (1) by minimizing the sum of squared errors between measured scour depth and calculated scour depth. Initial erosion parameters come from BS or from other empirical equations [20,22], and also a relationship between τ_c and k_d were used.

3. PROPOSED ALTERNATIVE METHOD FOR DETERMINING EROSION PARAMETERS

The excess shear stress equation (Eq. 1) is non-linear when the exponent a is not unity. In this case, the three methods of determination described previously cannot be used for interpreting the JET results. This study intends to find a simple way to determine τ_c and k_d based on the non-linear erosion model. According to the original Blaisdell solution, critical shear stress was initially determined by considering the maximum or equilibrium scour depth (H_e) from which the scour during the JET test cannot be increased. However, the equilibrium scours depth can take a long time to approach [19,23]. The relationship between scour depth and testing time is likely either an exponential or a hyperbolic function. Thus, using these functions can be an alternative allowing estimates of H_e mathematically. The exponential rise to maximum (Eq. 6) was used to fit the scour depth data from JET erosion test, and then used to predict H_e

$$H(t) = H_i + H_f\left(1 - e^{-kt}\right) \quad (6)$$

where $H(t)$ is jet nozzle height as a function of time (m), H_i is nozzle height at an initial time (cm), H_f is final predicted scour distance on jet centerline (cm), k is a constant rate coefficient (min^{-1}), and t is time (min). The scour depth data were fit with Eq. (3) to obtain H_e that would correspond to infinite testing time (i.e. $H_e = H_i + H_f$). Eventually, the critical shear stress can be calculated using pre-determined H_e by substitution in Eq. (2).

Regarding k_d and constant a, these parameters can be simultaneously estimated by the "Linearization method" (hereafter called LM), in which the non-linear excess shear stress equation (Eq. 1) is rearranged to a linear form by taking the natural logarithm on both sides, as shown in Eq. (7). Plots of JET data transformed to $\ln(\varepsilon_r)$ and $\ln(\tau - \tau_c)$ on y and x-axes, respectively, were made. Then k_d and a are obtained from the y-intercept and slope of a linear fit (Eq. 7), respectively.

$$\ln(\varepsilon_r) = \ln k_d + a\ln(\tau - \tau_c) \quad (7)$$

4. VALIDATION OF THE PROPOSED METHOD

The proposed method was validated by conducting in-situ JET tests of U-Tapao river bank soil. A total of 30 JET tests at 12 study sites (Fig. 2a), selected based on accessibility to representative bank reaches, were performed for use with the non-linear erosion model and for determining the erosion parameters. In-situ JET testing followed the procedures introduced by Hanson and Cook, [6]. Disturbed samples from each JET test were collected to classify the bank soil based on the Unified Soil Classification System (USCS, ASTM D2487-98). The three existing methods and the proposed method were used to determine erosion parameters and will be discussed. Finally, the erosion rate equation (Eq. 1) was used with the JET data.

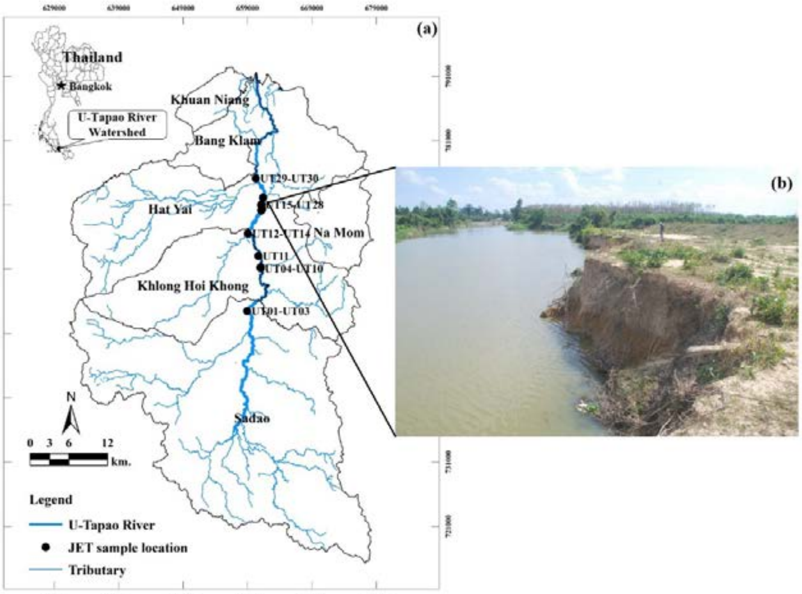

Fig. **2** U-Tapao river watershed, **(a)** selected bank locations along the middle part of the watershed, and **(b)** a typical composite bank of U-Tapao river

4.1 Description of U-Tapao River watershed

The U-Tapao River watershed, Southern Thailand (Fig. 2), is one of the biggest watersheds in Songkhla lake basin and the most important watershed of this region. Hat Yai city is the main city and a local center of economy, tourism, and different cultures along with over 700,000 residents. The U-Tapao river is the main river, approximately 112 km long, and flows northward through Hat Yai city into the Songkhla Lake. The tropical monsoon climate in Southern Thailand generates approximately 1,524 mm average annual rainfall, mainly from October to December [16].

4.2 Sample collection and testing

Figure 2a shows a group of sampling locations along the main channel of the U-Tapao river watershed. This study focused intentionally on the riverbank at the middle part of the watershed (approximately 36 kilometers) that has serious bank retreat problems. The average channel slope relevant in this study is about 1:10,000 (V: H).

A general characteristic of the selected banks in this area is steep bank faces, as illustrated in Fig. 2b. Bank soils of the U-Tapao river are generally composed of both fine-grained soil (low and high plasticity clay, CH and CL, and low plasticity silt, ML) and coarse-grained soil (clayey sand, SC and silty sand, SM). Generally, the banks of U-Tapao river are naturally formed as a cohesive and composite bank. The sand layers are carried by underlying cohesive layer in the composite banks, and the cohesive banks are established by either clay layers or silt layers [16].

5. RESULTS AND DISCUSSION

5.1 Critical shear stress

Erosion parameters of the bank soils were initially determined by calculating critical shear stress (τ_c) from estimates of equilibrium scour depth (H_e). Figure 3 shows an example of estimating H_e using the exponential rise equation (Eq. 6). The H_e obtained from model fit across the experimental tests was used to determine τ_c values in Table 1. The τ_c on banks of U-Tapao spanned the range from 1.306 to 24.33 Pa. It is noted that the τ_c of the fine-grained soil had a wide range from the least to the largest among all samples, while the average (τ_{c-avg}) and standard deviation (τ_{c-SD}) of τ_c values were 8.82 and 6.29 Pa, respectively. In contrast, the non-cohesive soils showed a narrow range of τ_c values from 1.64 to 5.55 Pa with 3.48 and 1.633 Pa for τ_{c-avg} and τ_{c-SD}, respectively. This result confirms that variability of τ_c relates to spatial and temporal changes [20]. The obtained τ_c estimates were further compared with the three prior methods.

Figure. 4a shows in plots the τ_c estimates obtained from predicted H_e (Eq. 6) along with those from three prior methods (BS, IS, SS). The results indicate that τ_c of bank soils in this study was similar to estimates by SS in all cases, and also matched in some cases IS estimates, as shown in Fig. 4a. The results also indicate that the τ_c estimated in this study and from SS were more reliable than those from IS and BS methods. This is due to the fact that both these methods (proposed method and SS) considered scouring distance in making the estimates. In contrast, BS was based on hyperbolic logarithm function [19] that unavoidably overestimated the H_e resulting in underestimates of τ_c. Regarding IS, it seems that there were fluctuations of τ_c estimates caused by the optimization process.

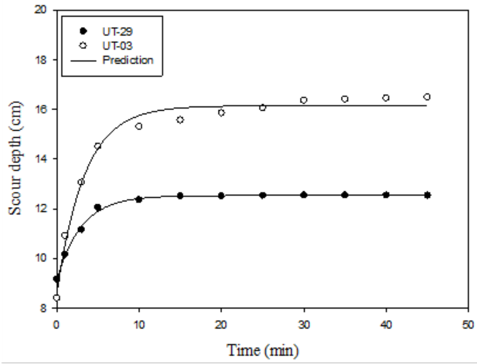

Fig. 3 Prediction of equilibrium scour depth using exponential rise equation

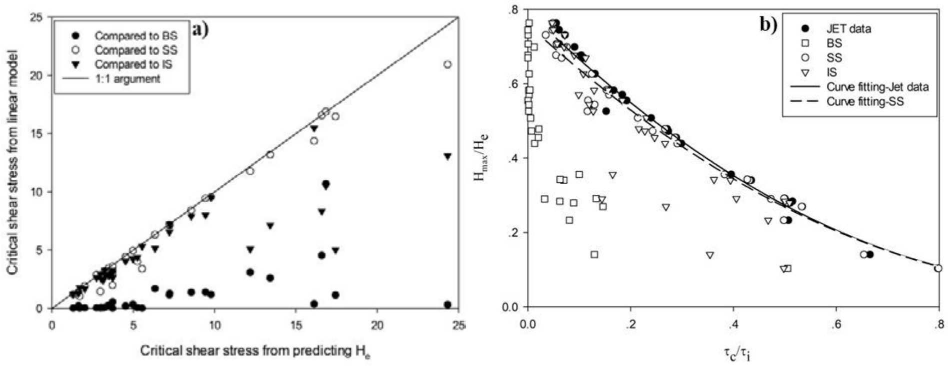

Fig. 4 Comparison of critical shear stress with three prior methods (a) and (b) normalized shear stress versus normalized scour depth (curve fit done with only SS method)

The τ_c estimates from all methods were further subjected to normalization. The final predicted scour depth (H_f) from JET tests and τ_c were normalized by the corresponding equilibrium scour depth and the initial applied shear stress (τ_i), respectively. Figure 4b shows in a scatter plot the normalized shear stress (τ_c/τ_i) in relation to the scour depth (H_f/H_e). The result reveals that there was a good agreement between τ_c/τ_i and H_f/H_e fit well by a second-order polynomial (line plotted in Fig. 4b, R^2 = 0.977) in the JET results. The findings reveal that the τ_c can immediately be estimated using $H_{\max}$ from JET data with Fig. 4b. However, it must be kept in mind that this graph can be used with the scour depth range 1.00-20.00 cm. The corresponding ranges of H_f/H_e andwere τ_c/τ_i 0.10-0.80 and 0.05-0.80, respectively.

5.2 Erodibility coefficient and an indicator of non-linearity

Erodibility coefficient (k_d) and exponent a are defined as the rate of erosion and an indicator of non-linearity. Mathematically, both parameters relate to how fast the bank soil is eroded. These parameters can be calculated by LM (Eq. 7) applied to experimental JET data. Figure 5 depicts some example plots of the excess shear stress equation based on LM, from which both k_d and a are then obtained. The average k_d and a ranged within 0.02-2.53 cm^3/N·s. and 0.168-0.697, respectively.

Figure 6 illustrates the relationship between the computed τ_c and k_d for U-Tapao river bank soils, using the proposed method of this study along with the prior methods. Most of the k_d values were less below those obtained from BS (0.00-42.01 cm^3/N·s), IS (0.00-192.94 cm^3/N·s), or SS (1.74-89.07 cm^3/N·s). The SS and IS tend to give larger estimates of k_d than LM and BS methods. The results confirm again that SS and are methods gave high estimates of erodibility coefficients, resulting in overestimates of erosion. The LM improved the k_d estimates from those of SS and IS methods. According to Hanson and Simon [22], based on the LM method, erosion susceptibility of U-Tapao riverbank soil ranges from very erodible to resistant soil. More than half of all samples were classified as moderately erodible soil, as seen in Fig. 6.

Table 1 Prediction of equilibrium scour depth and computed critical shear stress

Sample	USCS	Jet-setting		Curve fit results			H_e (cm)	τ_c (Pa)
		H_i (cm)	τ_i (Pa)	H_f (cm)	k	R^2		
UT01	CL	11.21	21.37	18.32	0.084	0.931	29.53	1.31
UT02	SM	11.56	15.47	15.02	0.1652	0.956	26.58	2.02
UT03	CL	8.64	12.93	7.50	0.2956	0.986	16.14	3.50
UT04	CL	7.66	30.81	4.50	0.1783	0.981	12.16	12.19
UT05	ML	6.25	61.64	14.47	0.1555	0.974	20.73	3.73
UT06	CL	9.16	18.89	11.15	0.1536	0.945	20.31	3.16
UT07	CL	8.30	15.17	5.88	0.0822	0.977	14.18	4.53
UT08	ML	9.36	22.06	11.76	0.0433	0.938	21.12	2.75
UT09	ML	10.84	20.80	11.65	0.0991	0.958	22.49	3.26
UT10	SM	9.00	15.73	18.03	0.0923	0.994	27.02	1.71
UT11	CL	8.94	19.00	3.60	0.1119	0.991	12.54	9.78
UT12	CL	6.19	67.22	6.07	0.137	0.972	12.25	16.12
UT13	SC	7.06	34.49	10.10	0.1351	0.963	17.16	5.26
UT14	CL	8.07	20.16	1.86	0.0927	0.996	9.93	13.42
UT15	CL	4.15	132.43	5.30	0.1631	0.982	9.45	24.33
UT16	SM	7.46	41.22	20.06	0.1638	0.999	27.51	2.98
UT17	SM	7.89	18.04	18.16	0.349	0.986	26.05	1.64
UT18	ML	8.15	11.11	7.15	0.068	0.990	15.31	3.03
UT19	ML	5.42	60.76	14.20	0.3635	0.971	19.62	3.34
UT20	ML	7.83	14.17	2.94	0.069	0.994	10.78	7.23
UT21	SM	4.96	35.93	10.10	0.1092	0.995	15.06	3.73
UT22	SM	8.74	28.87	10.65	0.0718	0.992	19.39	5.55
UT23	CL	7.74	34.84	3.14	0.0668	0.982	10.88	17.44
UT24	ML	7.30	16.61	3.70	0.2893	0.974	11.00	7.22
UT25	CL	10.33	22.11	4.75	0.1745	0.977	15.07	9.44
UT26	ML	8.59	12.68	3.15	0.2382	0.972	11.74	6.34
UT27	CL	8.60	21.13	0.87	0.13	0.944	9.46	16.85
UT28	SM	8.78	17.25	6.20	0.1639	0.968	14.98	4.98
UT29	CH	9.15	31.09	3.37	0.3347	0.997	12.53	16.59
UT30	CL	8.26	16.90	3.47	0.2374	0.933	11.73	8.57

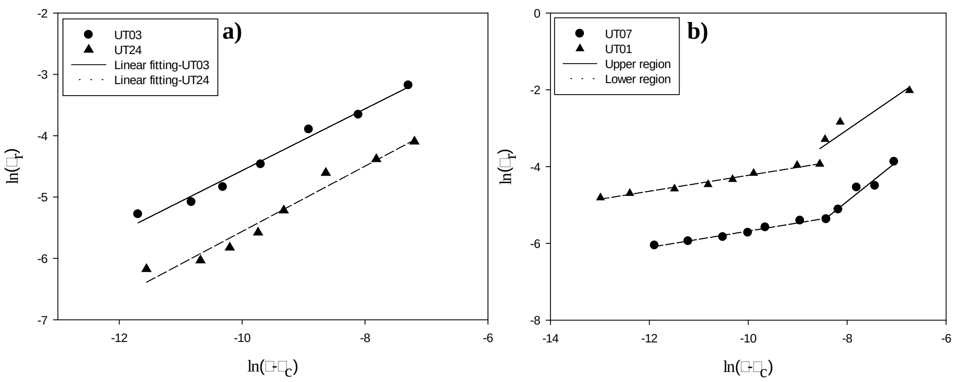

Fig. 5 An example of a) linear fits and b) broken stick shape of the excess shear stress

The parameter a adjusts the non-linearity of the erosion model to fit the JET data over the range of applied shear stresses used in experimental testing. Many studies in the literature show that the rate of erosion is well described by a power law [10]. Typically the exponent a has been larger than unity. However, in this current study $a < 1$, as a result of different testing methodology.

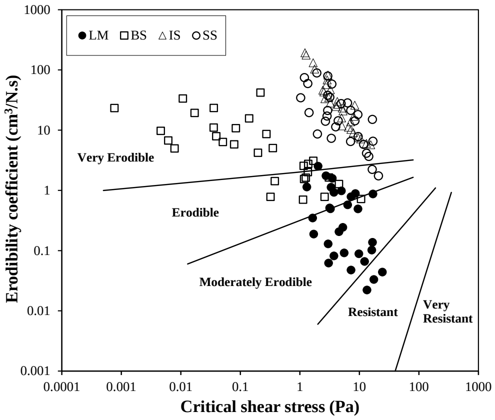

Fig. 6 Erodibility classification

5.3 Interpretation of the proposed nonlinear model

The proposed method was successfully fit to the JET data and the erosion parameters are shown in Fig. 6. The coefficient of determination (R^2) was in the range 0.800-0.985 for a wide range of applied shear stresses. This implies that the excess shear stress equation in LM can be used to determine the erosion parameters based on the JET test.

The results are remarkable that 30% of all samples showed two linear segments (often known as "broken stick"), as seen in Fig. 5b. It seems that heterogeneous bank soil and high applied shear stress were possibly affected by the bank soil samples during the test. When JET testing with a similar type of soil samples was performed with different initial applied shear stresses, data with the higher applied shear stresses was well fit by LM (Fig.5). This suggests that the bank soil had homogenous behavior and the non-linear excess shear stress equation could reliably predict erosion rates from JET data.

The plot of excess shear stress shows a broken stick (Fig. 5b). The erosion rate was high at the soil surface due to high applied shear stress where low bulk caused low τ_c and high k_d [24]. Moreover, some samples also had mass erosion at the initial time of the JET test, as a result of loose soil on the surface. This is similar to the suggestion in a previous study on τ_c based on BS, in which τ_c less than 0.1 Pa was recognized as mass erosion [21] due to too high applied initial shear stress in the testing. However, this may result from a shortcoming in the JET test procedures, so the testing process should be revised to make it independent of the test conditions.

5.4 Model verification

The excess shear stress equation (Eq. 1) was employed to verify the reliability of LM in comparison to the three prior methods. Figure 7a shows the relationship between erosion rate (y-axis) and applied shear stress (x-axis) in the JET tests. The results show that the LM model can be used to determine the erosion parameters better than the prior methods, although some prior methods (SS and IS) may be appropriate for some experimental cases.

In contrast, as described above, bank soil can be either heterogeneous and/or have mass erosion from high initial applied shear stress in the testing. Predictions of erosion rate based on the proposed nonlinear model cannot be suitable with a single range of applied shear stresses, as shown in Fig. 7b. However, note that the critical shear stress in the calculation of erosion rate obtained from the lower region is associated with low applied shear stress. Therefore, a further study should consider separately the higher range of applied shear stresses during JET tests.

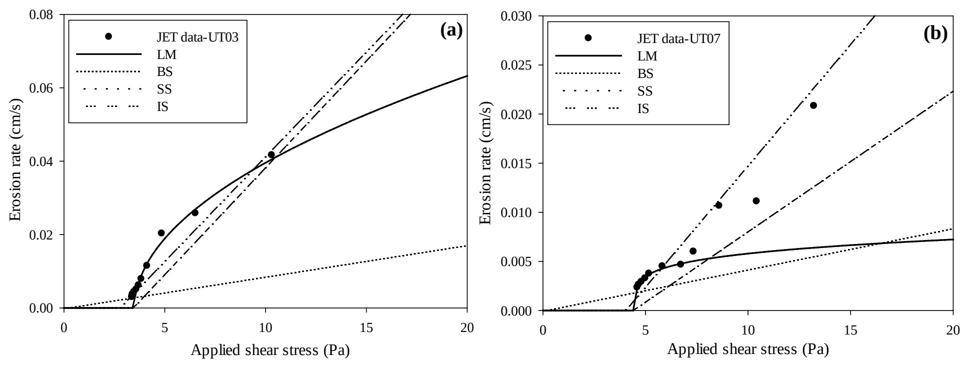

Fig. 7 Prediction of erosion rate based on JET tests, with non-linear erosion model compared to the linear model, (a) linear response and (b) broken stick response

6. CONCLUSIONS

An alternative method based on non-linear excess shear stress equation for determining erosion parameters was successfully developed in this study. A series of submerged jet tests were conducted on the bank soils along the U-Tapao river. Analysis of these data gave the following conclusions.

1. Exponential rise equation can be properly used to predict equilibrium scour depth from submerged jet data, with a high coefficient of determination (R^2 = 0.931-0.999). The τ_c of bank soil along the U-Tapao river ranged within 1.306-24.33 Pa, similar to the estimates by SS method. For fine-grained soil (CL, ML, and CH), the τ_c showed a wide range of values. However, the variability of τ_c due to spatial and temporal effects were confirmed.

2. The LM can be successfully applied to JET data, with the coefficient of determination (R^2) in the range 0.800-0.985 across a wide range of applied shear stresses. This indicates that the LM can be used to determine the erosion parameters based on JET tests.

3. The k_d and a parameters were in ranges 0.02-2.53 $cm^3/N \cdot s$ and 0.168-0.697, respectively. Most of the k_d estimates were below those from SS and IS methods, while lower estimates were provided by LM and BS methods. Erosion susceptibility of bank soils ranges from very erodible to resistant soil, and over half of the samples were moderately erodible soil.

4. The LM had better reliability than linear models in predicting erosion rates and can indicate mass erosion. Based on this study it is recommended that the JET testing processes should be revised.

This study provides an alternative approach for determining erosion resistance of the bank soil. At high shear stress range, an occurrence of mass erosion has unavoidably occurred. Further research should be considered insight for the involvement of high shear stress during the JET experiment which LM and another non-linear erosion models should be simultaneously verified.

7. Acknowledgments

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