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COMPARISON OF LEE-CARTER'S CLASSIC AND GENERAL MODEL FOR FORECASTING MORTALITY RATE IN INDONESIA

Lutfiani Safitri[1], Sri Mardiyati[2], and Hendrisman Rahim[3]

[1,2] Faculty of Mathematics and Natural Science, Universitas Indonesia, Indonesia; [3] PT.Asuransi Jiwasraya, Indonesia

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ABSTRACT: Assurance Company needs to know the mortality rate of a country to decide the value of the premium which has to be paid by the company. To do that, an assurance company requires a mathematical model which is able to represent problems in forecasting mortality rate. One of model which have been acknowledged by the actuary community in forecasting mortality rate is Lee-Carter model. This research forecasts the mortality rate in Indonesia by using Classical Lee-Carter Model as the basic model of Lee-Carter. Besides that, this research also used Generalized Lee-Carter model, which is an extension of the basic Lee-Carter model. The parameters in both models will be estimated by using Least Square method and the Newton Raphson method. The result of parameter estimation will be substituted into the model to obtain the estimated mortality rate and the result will be compared. Then, the parameter which depends on the year will be used in mortality forecasting by using the neural network. The final result of this research is a table of mortality rate in Indonesia for the next 3-years period.

Keywords: Mortality Rate, Forecasting, Least Square Method, Newton Raphson Method, Neural Network Method

1. INTRODUCTION

The mortality table is a table which showing the rate of death based on age group yearly. One of the value of the mortality table is to show the probability of a person living in the starting point of the year and dying before the year ends, this probability is called as the mortality rate. An assurance company needs to know the mortality rate in a country to decide the number of premium which should be paid. In order to get the mortality rate in the future, the thing which should do is forecasting it by estimating cumulatively what will happen in the future based on relevant data from the past.

Forecasting mortality has been researched by other researchers before. Some researchers have Forecasted the mortality rates of Malaysian population using Lee-Carter method [1], forecasting Hungarian mortality rates using the lee–carter method [2], and the mortality of the Italian population: smoothing techniques on the lee-carter model [3].

This research will forecast the mortality rate in Indonesia by using Classical Lee-Carter Model and General Lee-Carter Model. The parameters in both models will be estimated by using Least Square method and the Newton Raphson method. The result of parameter estimation will be substituted into the model to obtain the estimated mortality rate, and the result is compared. Then, the parameter which depends on year will be used in mortality forecasting by using the neural network.

2. CLASSIC LEE-CARTER MODEL

On 1992 Lee and Carter introduced a new model to forecast mortality rate, the model can be seen on equation below:

$$\ln\left(m_{x,t}\right) = a_x + b_x \cdot k_t + \varepsilon_{x,t} \tag{1}$$

with constrain:

$\sum_{t=1}^{T} k_t = 0$ dan $\sum_{x=1}^{N} {b_x}^2 = 1$

In this model $m_{x,t}$ is the central death rate or mortality rate on age x on a specific year t, with x= 1,2, …, N, shows the age, and t=1,2, …, T shows the year, a_x and b_x is the parameter which depends on age and k_t is the parameter which depends on the year and $\varepsilon_{x,t}$ is an independent error, with its mean 0 and variance σ^2 [4].

Eq. (1) states that the mortality rate on age x on year t. To forecast the rate of mortality on the next year needs estimation on all parameter in the model. Then will be the estimated parameter on Eq. (1) by using the Least Square method, so it will be gotten an equation which stating:

$$F(a_x\ ,\ b_x\ ,\ k_t\ ) = \sum_{x=1}^{N}\sum_{t=1}^{T}\left(\ln\left(m_{x,t}\right) - a_x - b_x\, k_t\right)^2 \tag{2}$$

From Eq. (2) will be the estimated value of parameter a_x, b_x, and $k_t$by using Newton Raphson Method. The idea from Newton Raphson is by guessing the initial value of the roots of Eq. (2) then it will be updated the root values of the equation until a minimum value of error is found, here are the steps:

a) Choose initial value: for each x and t, we choose $b_x = \frac{1}{N}$, $k_t = 0$ and $\hat{a}_x = \frac{\sum_{t=1}^{T}\ln(m_{x,t})}{T}$.

b) Update parameter:

$$\hat{a}_x = \hat{a}_x + \frac{\sum_{x=1}^{N}\sum_{t=1}^{T}\left(\ln(m_{x,t}) - a_x - b_x\, k_t\right)^2}{\sum_{t=1}^{T} 2\,(\ln(m_{x,t}) - \hat{a}_x - \hat{b}_x\hat{k}_t)} \quad (3)$$

$$\hat{k}_t = \hat{k}_t + \frac{\sum_{x=1}^{N}\sum_{t=1}^{T}\left(\ln(m_{x,t}) - a_x - b_x\, k_t\right)^2}{\sum_{x=1}^{N} 2\hat{b}_x(\ln(m_{x,t}) - \hat{a}_x - \hat{b}_x\hat{k}_t)} \quad (4)$$

$$\hat{b}_x = \hat{b}_x + \frac{\sum_{x=1}^{N}\sum_{t=1}^{T}\left(\ln(m_{x,t}) - a_x - b_x\, k_t\right)^2}{\sum_{t=1}^{T} 2\hat{k}_t\,(\ln(m_{x,t}) - \hat{a}_x - \hat{b}_x\hat{k}_t)} \quad (5)$$

c) Calculate the error value, then repeat step b until minimum error is obtained.

The result of parameter estimation will be substituted into the model to obtain the estimated mortality rate. Then, the estimated mortality rate will be compared with actual data on the mortality rate. After comparing that data, the error value will be found. The parameter which depends on time will be used in mortality forecasting by using the neural network. After forecasting the mortality rate using a classical Lee-Carter model, then the mortality rate will be forecast using a general Lee-Carter model.

3. GENERAL LEE-CARTER MODEL

Renshaw and Haberman issued this model in 2006. In the modeling will be added cohort effect as the additional parameter. The model of general Lee-Carter model is:

$$\log(\mu_{x,t}) = a_x + b_x \cdot k_t + c_x \cdot v_{t-x} + \varepsilon_{x,t} \quad (6)$$

Where a_x, b_x, and c_x is the dependent parameter to the age, v_{t-x} shows the cohort effect and k_t is the dependent parameter to the year. Eq. (6) include in a generalized nonlinear model family with the link function to log [5] [6].

From the model then will be done some parameter assessment by using the same method from the previous, the least square so that the function becomes as follows:

$$F(a_x, b_x, k_t, c_x, v_{t-x}) = \sum_{x=1}^{N}\sum_{t=1}^{T}\left(\ln(m_{x,t}) - a_x - b_x\, k_t - c_x \cdot v_{t-x}\right)^2 \quad (7)$$

In the same way from the previous we will find the estimation of each parameter by using Newton Raphson method, here is the algorithm:

a. Choose initial value: for each x and t, we choose $b_x = c_x = \frac{1}{\omega+1}$, k_t and v_{t-x} we choose from value of equation: $\log(m_{x,t}) - \hat{a}_x = \hat{k}t + v_{t-x}$ and $\hat{a}_x = \frac{\sum_{t=1}^{T}\ln(m_{x,t})}{T}$.

b. Update parameters:

$$\hat{k}t = \hat{k}t + \frac{\sum_{x=1}^{N}\sum_{t=1}^{T}\left(\ln(m_{x,t}) - a_x - b_x\, k_t - c_x \cdot v_{t-x}\right)^2}{\sum_{x=1}^{N} 2\hat{b}_x(\ln(m_{x,t}) - \hat{a}_x - \hat{b}_x\hat{k}_t - c_x \cdot v_{t-x})} \quad (8)$$

$$\hat{b}_x = \hat{b}_x + \frac{\sum_{x=1}^{N}\sum_{t=1}^{T}\left(\ln(m_{x,t}) - a_x - b_x\, k_t - c_x \cdot v_{t-x}\right)^2}{\sum_{x=1}^{N} 2\hat{k}t(\ln(m_{x,t}) - \hat{a}_x - \hat{b}_x\hat{k}_t - c_x \cdot v_{t-x})} \quad (9)$$

$$\hat{v}_{t-x} = \hat{v}_{t-x} + \frac{\sum_{x=1}^{N}\sum_{t=1}^{T}\left(\ln(m_{x,t}) - a_x - b_x\, k_t - c_x \cdot v_{t-x}\right)^2}{\sum_{x=1}^{N} 2\hat{c}_x(\ln(m_{x,t}) - \hat{a}_x - \hat{b}_x\hat{k}_t - c_x \cdot v_{t-x})} \quad (10)$$

$$\hat{c}_x = \hat{c}_x + \frac{\sum_{x=1}^{N}\sum_{t=1}^{T}\left(\ln(m_{x,t}) - a_x - b_x\, k_t - c_x \cdot v_{t-x}\right)^2}{\sum_{x=1}^{N} 2\hat{v}_{t-x}(\ln(m_{x,t}) - \hat{a}_x - \hat{b}_x\hat{k}_t - c_x \cdot v_{t-x})} \quad (11)$$

c. Calculate the error value, then repeat step b until minimum error is obtained.

4. NEURAL NETWORK METHOD

After we get the estimation value from the model, then we forecast the mortality rate in Indonesia by using a Neural Network. Based on the encyclopedia of machine learning and data mining, neural network is a learning algorithm by replicating how the human brain works. The learning system works based on the value of the nodes which is simulated as synapsis and neuron [7].

This paper will use feedforward neural network by using multi-layer net which contains a layer in the hidden layer. The algorithm which will be used is the backpropagation algorithm, and activation function which will be used is a sigmoid function. The Neural Network structure which will be used in this paper will be shown in figure 1.

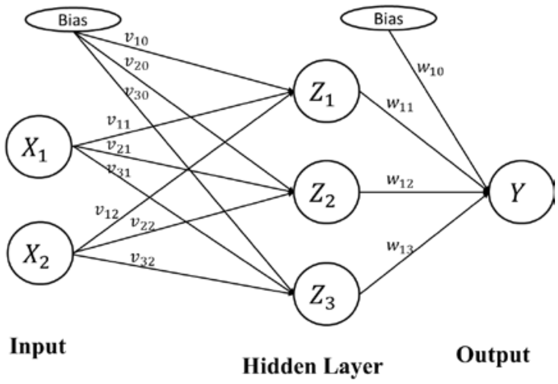

Fig. 1 Structure Multilayer Feedforward Neural Network

5. MAIN RESULT

The data we used in this research is the mortality rate of Indonesia from 1950-2015 with five years data range, and the group age is 0, 1-5, 6-10, 11-15 and so on until 86-90.

Table 1 Mortality Rate in Indonesia

		1	2	…	13
	Age Group	1950-1955	1955-1960	…	2011-2015
1	0	0.22061	0.18733	…	0.02553
2	1-5	0.03605	0.02804	…	0.00136
3	6-10	0.00553	0.00472	…	0.00066
	…	…	…	…	…
18	81-85	0.20386	0.19746	…	0.14275
19	86-90	0.30937	0.30388	…	0.25093

This data is taken from the united nation website (https://esa.un.org/-unpd/wpp/Download/Standard/Mortality/).

From the Indonesia mortality rate data in table 1 will be found parameter estimation a_x , b_x and k_t on Eq. (1) by using Least Square method and Newton Raphson method. By using the same method will also be found parameter estimation a_x, b_x, c_x, k_t and V_{t-x} on Eq. (5).

The result of parameter estimation a_x and b_x from the classical model will be shown in Table 2, the result of parameter estimation a_x , b_x and c_x from the general model will be shown in Table 3 and the result of parameter estimation k_t and V_{t-x} from both model will be shown in Table 4.

Table 2 The estimated values a_x and b_x From Classical Lee-Carter Model

X	Age Group	$\boldsymbol{a_x}$	$\boldsymbol{b_x}$
1	0	-0.38072	0.041105
2	1-5	-0.82811	0.072152
3	6-10	-0.9185	0.080028
4	11-15	-0.93582	0.081537
5	16-20	-0.8462	0.073728
6	21-25	-0.80907	0.070493
7	26-30	-0.79887	0.069605
8	31-35	-0.77768	0.067758
9	36-40	-0.74286	0.064725
10	41-45	-0.70116	0.061091
11	46-50	-0.65084	0.056707
12	51-55	-0.59849	0.052145
13	56-60	-0.54428	0.047422
14	61-65	-0.47749	0.041603
15	66-70	-0.42148	0.036723
16	71-75	-0.3634	0.031662
17	76-80	-0.30299	0.026399
18	81-85	-0.24432	0.021287
19	86-90	-0.17352	0.015119

After obtaining the value of each parameter estimation of the mortality rate from the actual data and estimated value will be compared. Figure 2 will show the result of comparison on classical Lee-Carter model, and Figure 3 will show the result of comparison on General Lee-Carter model.

Table 3 The estimated values a_x, b_x and c_x From General Lee-Carter Model

X	Age Group	$\boldsymbol{a_x}$	$\boldsymbol{b_x}$	$\boldsymbol{c_x}$
1	0	-0.28215	0.139356	0.052632
2	1-5	-0.50755	0.250549	0.052632
3	6-10	-0.56295	0.27788	0.052632
4	11-15	-0.57357	0.283117	0.052632
5	16-20	-0.51864	0.256019	0.052632
6	21-25	-0.49588	0.244792	0.052632
7	26-30	-0.48963	0.24171	0.052632
8	31-35	-0.47664	0.235301	0.052632
9	36-40	-0.4553	0.224775	0.052632
10	41-45	-0.42974	0.212164	0.052632
11	46-50	-0.3989	0.196951	0.052632
12	51-55	-0.36682	0.181122	0.052632
13	56-60	-0.33359	0.164732	0.052632
14	61-65	-0.29266	0.144538	0.052632
15	66-70	-0.25833	0.127603	0.052632
16	71-75	-0.22273	0.110041	0.052632
17	76-80	-0.1857	0.091776	0.052632
18	81-85	-0.14974	0.074036	0.052632
19	86-90	-0.10635	0.052632	0.052632

Table 4 The estimated values of k_t and V_t

t	Year Group	Classical Lee- Carter	Generalized Lee-carter	
		$\boldsymbol{k_t}$	$\boldsymbol{k_t}$	$\boldsymbol{V_{t-x}}$
1	1950-1955	-60.898	0.076923	-20.3472
2	1955-1960	-62.4732	0.076923	-20.6876
3	1960-1965	-64.0425	0.076923	-21.029
4	1965-1970	-65.6078	0.076923	-21.363
5	1970-1975	-67.3293	0.076923	-21.7226
6	1975-1980	-69.6288	0.076923	-21.9155
7	1980-1985	-71.8412	0.076923	-22.1242
8	1985-1990	-73.325	0.076923	-22.3706
9	1990-1995	-74.9707	0.076923	-22.705
10	1995-2000	-76.6227	0.076923	-23.1117
11	2000-2005	-77.2879	0.076923	-23.5735
12	2005-2010	-78.4987	0.076923	-24.0522
13	2010-2015	-79.9709	0.076923	-24.3252

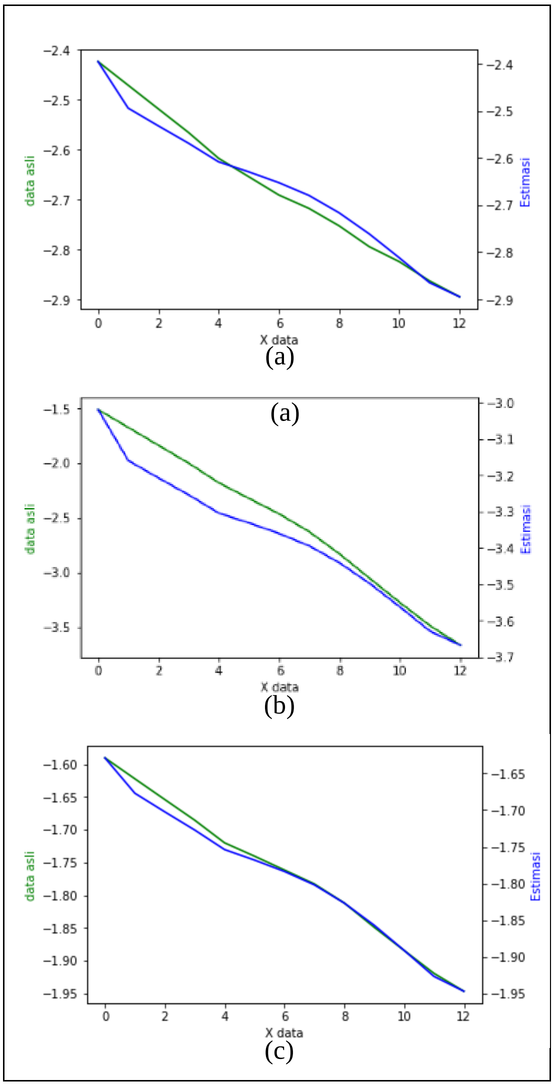

Fig. 2. The fitted and estimated value of mortality rate by using Newton Raphson of the model (1) (a) group age 0 (b) group age 1-5 dan (c) group age 86-90

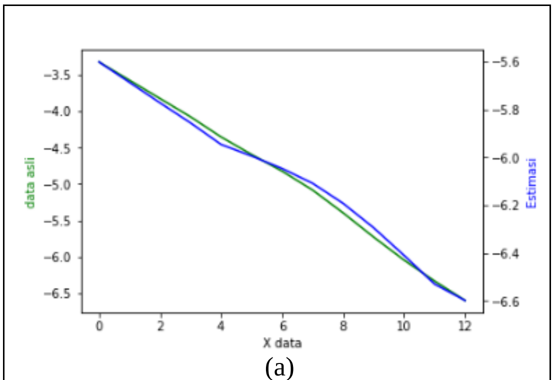

Fig. 3a The fitted and estimated value of mortality rate by using Newton Raphson of the model (5) (a) group age 0

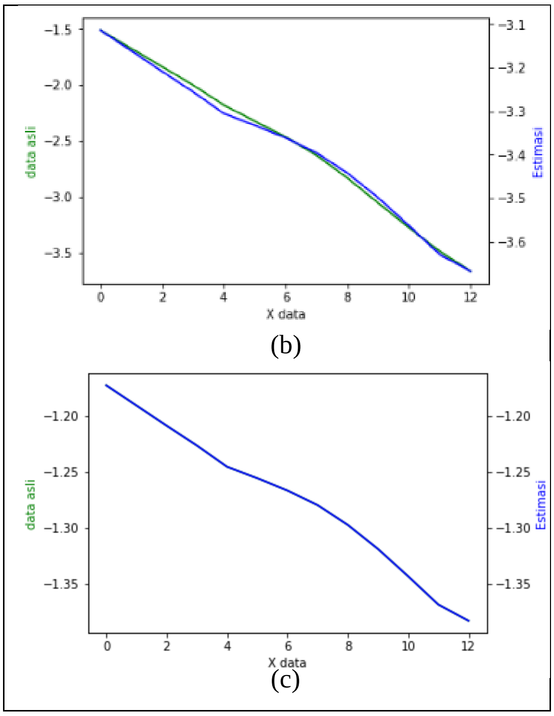

Fig. 3b. 3c.. The fitted and estimated value of mortality rate by using Newton Raphson of the model (5) (b) group age 1-5 dan (c) group age 86-90

Figure 2 and Figure 3 is shows that the estimated parameters obtained can adequately describe the actual data, with the error value will be shown in Table 5.

Table 5 Mean Absolute Percentage Error

X	MAPE Classical Lee-carter	MAPE General Lee-Carter
1	0.059434784424185899	0.063085618921268002
2	0.0089187715596411154	0.0094096005527437871
3	0.0008499532789508194	0.001146237259458239
4	0.0001344988555393873	0.00040489218245692561
5	0.0004247861942399127	0.00020330944802639291
6	0.0006083349158750781	0.00025449171005959795
7	0.0007108205429791112	0.00025967405641031435
8	0.0008595219048727523	0.0002711726241199909
9	0.0011845264526483424	0.00040317726523466171
10	0.0017208019693909934	0.00074368329548227035
11	0.0027838062032514927	0.0015108220359291282
12	0.0039208408670199368	0.0022550918669219421
13	0.005546147754330163	0.0033632025836276736

Table 5 continued

14	0.00646790157923332	0.0034638587169624024
15	0.0071899980234531096	0.0033219807212230322
16	0.0053995622244533397	0.0014734023378508015
17	0.004523024725092941	0.002471282476604612
18	0.0049007822355427253	0.0048440450705464723
19	0.0094942980726671622	0.0

After obtaining the estimated value of each parameter then will be forecasting parameters that depend on the year by using a neural network method. Table 6 shows the forecasting value for the next 3-year period for Classical Lee-carter and General Lee-Carter models:

Table 6 Forecasting Value of k_t and V_t

t	Year Group	Classical Lee-Carter	General Lee-Carter
14	2015-2020	-78.50400971	-23.9917
15	2020-2025	-77.96545991	-24.0231
16	2025-2030	-77.22038052	-24.0361

After obtaining the parameters forecasting which depending on the year for the next few years, we will calculate the mortality rate in Indonesia by using the Classical Lee charter model which results will be shown in table 7 and by using the general lee charter model which results will be shown in Table 8.

Table 7 The result of forecasting Mortality rate in Indonesia with Neural Network and Classical Lee-Carter Model

Age Group	Year Group	2015-2020	2020-2025	2025-2030
		1	2	3
0	0	0.027115	0.027722	0.028584
1-5	1	0.001515	0.001575	0.001662
6-10	2	0.000746	0.000779	0.000827
11-15	3	0.000651	0.00068	0.000723
16-20	4	0.001315	0.001368	0.001445
21-25	5	0.001759	0.001827	0.001926
26-30	6	0.001905	0.001978	0.002083
31-35	7	0.00225	0.002333	0.002454
36-40	8	0.002956	0.003061	0.003212
41-45	9	0.004099	0.004236	0.004433
46-50	10	0.006081	0.00627	0.00654
51-55	11	0.009167	0.009428	0.009802
56-60	12	0.014022	0.014385	0.014902

Table 7 continued

61-65	13	0.023671	0.024208	0.02497
66-70	14	0.036722	0.037456	0.038495
71-75	15	0.057902	0.058898	0.060304
76-80	16	0.092978	0.094309	0.096183
81-85	17	0.14728	0.148979	0.15136
86-90	18	0.256558	0.258656	0.261586

Table 8 The result of forecasting Mortality rate in Indonesia with Neural Network and General Lee-Carter Model

	Year Group	2015-2020	2020-2025	2025-2030
Age Group		1	2	3
0	0	0.026743	0.026626	0.026578
1-5	1	0.001482	0.00147	0.001465
6-10	2	0.000728	0.000721	0.000719
11-15	3	0.000635	0.000629	0.000627
16-20	4	0.001285	0.001275	0.001271
21-25	5	0.001721	0.001708	0.001702
26-30	6	0.001865	0.001851	0.001845
31-35	7	0.002203	0.002187	0.00218
36-40	8	0.002897	0.002877	0.002869
41-45	9	0.004022	0.003996	0.003985
46-50	10	0.005976	0.005939	0.005924
51-55	11	0.009021	0.00897	0.008949
56-60	12	0.013818	0.013747	0.013718
61-65	13	0.02337	0.023264	0.02322
66-70	14	0.036309	0.036164	0.036104
71-75	15	0.05734	0.057143	0.057061
76-80	16	0.092226	0.091961	0.091851
81-85	17	0.146321	0.145982	0.145841
86-90	18	0.255375	0.254953	0.254779

6. CONCLUSIONS

The fitting model can be seen in Fig. 2 (Classical Lee Carter) and Fig. 3 (General Lee Carter). From that picture, we can see that Fig. 3's estimation value can give the value which is approaching the actual value compared to Fig.2. In table 4 we can see the MAPE from each model, from that table shows that the error value of Lee-Carter Model smaller than the error value of General Lee-Carter model for starting point age. However, when the age increased, the General Lee Carter model shows the smaller error value. It shows that on the starting point age, Classical Lee Carter model is better to forecast the mortality rate in Indonesia, While the General Lee Carter model can give better forecasting than Classical Lee Carter model to forecast the mortality rate in the middle and older age.

7. ACKNOWLEDGMENTS

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