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REVERSIBLE JUMP MCMC METHOD FOR HIERARCHICAL BAYESIAN MODEL SELECTION IN MOVING AVERAGE MODEL

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ABSTRACT: Moving average (MA) is one of the mathematical models that is often used to model data in various fields. Noise in the MA model is often assumed to be normally distributed. In application, it is often found that noise is exponentially distributed. The parameter of the MA model includes order, coefficient, and noise variance. This paper proposes a procedure to estimate the MA model parameter which contains noise with a normal and exponential distribution where the order is unknown. The estimation of parameters of the MA model parameter is carried out in a hierarchical Bayesian framework. Prior distribution for the parameter is selected. The likelihood function for data is combined with prior distribution for the parameter to get posterior distribution for the parameter. The parameter dimension is a combination of several different dimensional spaces so that the posterior distribution for a parameter has a complex form and the Bayes estimator cannot be determined explicitly. The reversible jump Markov Chain Monte Carlo (MCMC) method is proposed to determine the Bayes estimator of the MA model parameter. The performance of the method is tested using a simulation study. The simulation result shows that the reversible jump MCMC method estimates the MA model parameter well. The reversible jump MCMC method can calculate the MA model parameter simultaneously and produce an invertible MA model.

Keywords: Bayesian, Moving Average, Exponential Noise, Normal Noise, Reversible Jump MCMC

1. INTRODUCTION

The autoregressive (AR) model is a time series model used to model data in different areas of life. The AR model with normally distributed noise has been extensively studied by various researchers. Also, the AR model with exponential noise has been investigated by various researchers. A genetic algorithm is used to estimate the exponential AR model [1]. An AR(1) model whose noise is exponentially distributed is studied in [2]. A robust Bayesian method is used to obtain the optimal Bayes estimator for AR models whose noise is exponentially distributed [3]. In the above studies, the order of the AR model is assumed to be known. An AR model whose error is exponential distributed but the model order is unknown is studied in [4].

Moving average (MA) model is a time series model that is similar to the AR model. The MA model is also used to model data in different areas of life. An MA model is used as a continuous quality control analysis for routine chemical tests [5]. The MA procedure is optimized using the MA bias detection simulation procedure. An MA filter is used to accelerate the acceleration signal and determine the location of the damaged steel beam [6].

In various studies, the noise in the MA model is often assumed to be normally distributed, for example [7]-[11]. While the noise of the MA model with exponential distribution has not been widely investigated by researchers. In the studies above, the order of the MA model is assumed to be known. But in the application of the MA model, the MA model order is unknown.

Reversible jump Markov Chain Monte Carlo (MCMC) [12] has been applied in many areas including in signal processing and in time series data analysis. The reversible jump MCMC algorithm is used for model selection. The reversible jump MCMC algorithm is used to select a piecewise AR model that has a normally distributed noise [13]. The reversible jump MCMC is used for species selection [14]. The reversible jump MCMC is used for the selection of the number and locations of the pseudo points [15]. The reversible jump MCMC is used to select the instrument calibration model [16]. The reversible jump MCMC is used for the selection of variables in regression [17], [18]. The reversible jump MCMC is used to select non-linear models in the Volterra system [19]. The reversible jump MCMC is used to estimate AR model order [4].

This study proposes the reversible jump MCMC method to estimate MA model parameters where the order is unknown. This study discusses parameter estimation of MA models that have normal or exponential noise. The parameters of the MA model include the order of the MA model, MA model coefficient, and noise variance.

2. METHOD

The parameter estimation is done in a Bayesian framework. Bayesian estimation requires a prior distribution and likelihood function. The prior distribution and likelihood function are combined to obtain a posterior distribution. Under the quadratic loss function, the Bayes estimator is obtained by calculating the mean of the posterior distribution. Because the posterior distribution has a complex form, the Bayes estimator cannot be determined analytically. An MCMC is used to determine the Bayes estimator by creating a Markov chain whose limit distribution is close to the posterior distribution. This Markov chain is used to determine the Bayes estimator. In this study, the order of the MA model is also a parameter that is estimated based on the data. This makes the dimensions of the Markov chain a combination of several different dimensioned spaces. So MCMC cannot be used directly. Therefore the reversible jump MCMC is used to solve the problem. The estimation procedure is shown in Figure 1

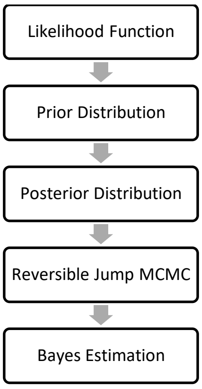

Fig. 1 Estimation Procedure

First, determining the likelihood function. Second, the selection of the prior distribution. Third, determining posterior distribution. The fourth determination of the Bayes estimator by using the reversible jump MCMC.

3. RESULTS AND DISCUSSION

The Bayesian method is used to estimate the parameters. Bayesian estimation requires a likelihood function and prior distribution.

3.1 Likelihood Function

Let $x_1, \cdots, x_n$ be n data following the MA model:

$$x_t = \sum_{j=1}^{q} \theta_j z_{t-j} + z_t \tag{1}$$

Here, q is model order, $t = 1,2, \ldots, n$ and $\theta^{(q)} = (\theta_1, \ldots, \theta_q)$ is the coefficient vector. Table 1 shows the relationship between orders and the number of coefficients of the MA model. A relationship between order and coefficient of the MA model is illustrated in Table 1.

Table 1: Relationship between order and coefficient of the MA model.

Order q	Coefficient $\theta^{(q)}$
1	(θ_1)
2	(θ_1, θ_2)
3	$(\theta_1, \theta_2, \theta_3)$
4	$(\theta_1, \theta_2, \theta_3, \theta_4)$
…	…
q	$(\theta_1, \theta_2, \theta_3, \ldots, \theta_q)$

3.1.1 First case: exponential noise

Random variable z_t is the independent variable and the exponential distribution with parameter λ. For example, for $n = 250$, $q = 2$, $\theta_1 = -1.34$, $\theta_2 = 0.36$, and $\lambda = 2$ then the value x_t is presented in Figure 2.

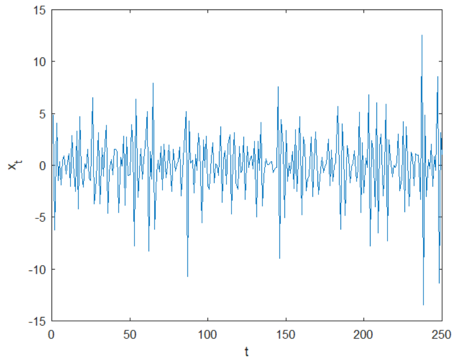

Fig. 2 MA model data with exponential noise.

The probability function z_t is

$$f(z_t|\lambda) = \lambda exp - \lambda z_t \tag{2}$$

The variable transformation is used to transform from variable z do variable x. So $z_t = x_t - \sum_{j=1}^{q} \theta_j z_{t-j}$ and $\frac{dz_t}{dx_t} = 1$.

Therefore, the probability function of x_t is

$$f(x_t|\lambda) = \lambda exp - \lambda \left(x_t - \sum_{j=1}^{q} \theta_j z_{t-j} \right) \tag{3}$$

Suppose that $x = (x_{q+1}, \ldots, x_n)$. By taking $z_1 = \cdots = z_q = 0$, the likelihood function of $x_1, \cdots, x_n$ can be approximated by :

$$\begin{aligned} &L(x|q, \theta^{(q)}, \lambda) \\ &= \prod_{t=q+1}^{n} f(x_t|\lambda) \\ &= \lambda^{n-q} exp - \lambda \sum_{t=q+1}^{n} \Bigg(x_t \\ &\qquad - \sum_{j=1}^{q} \theta_j z_{t-j} \Bigg) \end{aligned} \tag{4}$$

Let I_q is the invertibility region and $\rho^{(q)} =$

$(\rho_1, \dots, \rho_q)$ is the sample inverse partial autocorrelation vector. By using the transformation

$$G: \theta^{(q)} \in I_q \to \rho^{(q)} \in (-1,1)^q$$

An MA model with order q is invertible if and only if $\rho^{(q)} \in (-1,1)^q$. Finally, the approximate likelihood function of x can be written by :

$$L(x|q, \rho^{(q)}, \lambda) = \lambda^{n-q} exp - \lambda \sum_{t=q+1}^{n} \left(x_t - \sum_{j=1}^{q} G^{-1}(\rho_j) z_{t-j} \right) \tag{5}$$

where G^{-1} is the inverse transformation of the transformation G.

3.1.2 Second case: normal noise

The random variable z_t is a mutually independent variable and is normally distributed with mean 0 and variance σ^2. For example, if $n = 250$, $q = 2$, $\theta_1 = -1.34$, $\theta_2 = 0.36$, and $\sigma^2 = 2$ then the value x_t is presented in Figure 3.

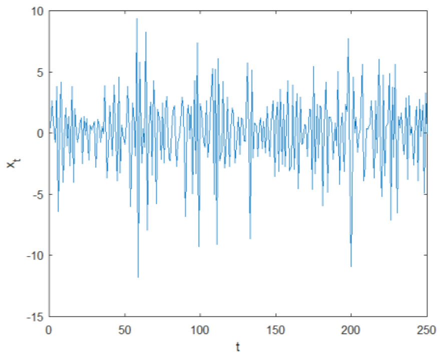

Fig. 3 MA model data with normal noise.

The probability function of z_t is

$$f(z_t|\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} exp - \frac{1}{2\sigma^2} z_t^2 \tag{6}$$

The variable transformation is used to transform from variable z do variable x. So $z_t = x_t - \sum_{j=1}^{q} \theta_j z_{t-j}$ and $\frac{dz_t}{dx_t} = 1$. Therefore, The probability function of x_t is

$$f(x_t|\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} exp - \frac{1}{2\sigma^2} \left(x_t - \sum_{j=1}^{q} \theta_j z_{t-j} \right)^2 \tag{7}$$

Suppose that $x = (x_{q+1}, \dots, x_n)$. By taking the values $z_1 = \cdots = z_q = 0$, The likelihood function of $x_1, \cdots, x_n$ can be approximated by:

$$L(x|q, \theta^{(q)}, \sigma^2) = (2\pi\sigma^2)^{\frac{n-q}{2}} \exp - \frac{1}{2\sigma^2} \sum_{t=q+1}^{n} \left(x_t - \sum_{j=1}^{q} \theta_j z_{t-j} \right)^2 \tag{8}$$

3.2 Bayesian

Before obtaining a posterior distribution, the prior distribution is selected.

3.2.1 First case: exponential noise

The prior distribution is taken as follows. The binomial distribution is chosen as the distribution for the order q $(q = 1, \dots, q_{max})$

$$\pi(q|\mu) = C_q^{q_{max}} \mu^q (1-\mu)^{1-q} \tag{9}$$

where q_{max} is the maximum for q and μ $(0 < \mu < 1)$ is a hyperparameter. The uniform distribution is chosen as the distribution for the vector coefficient $\rho^{(q)}$

$$\pi(\rho^{(q)}|q) = U(-1,1)^q \tag{10}$$

Also, the Jeffrey distribution is selected as the distribution for parameter λ

$$\pi(\lambda) \propto \frac{1}{\lambda}$$

Then, the hyperprior distribution for μ is a uniform distribution at an interval $(0,1)$.

Let $\pi(q, \rho^{(q)}, \lambda, \mu)$ be the prior distribution for $(q, \rho^{(q)}, \lambda, \mu)$. Because of the conditional distribution of the parameter $(q, \rho^{(q)}, \lambda)$ is given the hyperparameter μ is

$$\pi(q, \rho^{(q)}, \lambda|\mu) = \frac{\pi(q, \rho^{(q)}, \lambda, \mu)}{\pi(\mu)}, \tag{11}$$

The prior distribution for $(q, \rho^{(q)}, \lambda, \mu)$ can be written as:

$$\pi(q, \rho^{(q)}, \lambda, \mu) = \pi(q, \rho^{(q)}, \lambda|\mu)\pi(\mu) \tag{12}$$

Let $\pi(q, \rho^{(q)}, \lambda, \mu|x)$ be the posterior distribution for $(q, \rho^{(q)}, \lambda, \mu)$. According to Bayes theorem, the posterior distribution for $(q, \rho^{(q)}, \lambda, \mu)$ is given as follows

$$\pi(q, \rho^{(q)}, \lambda, \mu|x)$$
$$\propto L(x|q, \rho^{(q)}, \lambda)\pi(q, \rho^{(q)}, \lambda, \mu)$$
$$\propto L(x|q, \rho^{(q)}, \lambda)\pi(q, \rho^{(q)}, \lambda|\mu)\pi(\mu)$$

However, the Bayes estimator cannot be determined analytically because of the posterior distribution of the parameter $(q, \rho^{(q)}, \lambda, \mu)$ has a complex form. To solve this problem, the reversible jump MCMC algorithm is used.

3.2.2 Second case: normal noise

The distribution of priors for order q given μ is a binomial distribution. The prior distribution for the coefficient vector $\rho^{(q)}$ is a uniform distribution. Whereas the prior distribution for parameter σ^2 is an inverse gamma distribution with parameters $a/2$ and $b/2$:

$$\pi(\sigma^2|a,b) = \frac{\left(\frac{b}{2}\right)^{\frac{a}{2}}}{\Gamma\left(\frac{a}{2}\right)}(\sigma^2)^{-\left(1+\frac{a}{2}\right)} exp - \frac{b/2}{\sigma^2} \quad (13)$$

Here, a = 2 and the prior distribution for b is Jeffrey's distribution. Furthermore, the prior distribution for μ is a uniform distribution at the interval (0,1).

Let $\pi\big(q,\rho^{(q)},\sigma^2,\mu,b\big)$ be a prior distribution for $\big(q,\rho^{(q)},\sigma^2,\mu,b\big)$. A prior distribution for $\big(q,\rho^{(q)},\lambda,\mu\big)$ can be written as follows:

$$\pi\big(q,\rho^{(q)},\sigma^2,\mu,b\big) = \pi(q|\mu)\pi\big(\rho^{(q)}\big|q\big)\pi(\sigma^2|b)\pi(\mu)\pi(b) \quad (14)$$

Let $\pi\big(q,\rho^{(q)},\sigma^2,\mu,b\big|x\big)$ be a posterior distribution for $\big(q,\rho^{(q)},\sigma^2,\mu,b\big)$. According to the Bayes theorem, the posterior distribution for $\big(q,\rho^{(q)},\sigma^2,\mu,b\big)$ is given as follows:

$$\pi\big(q,\rho^{(q)},\sigma^2,\mu,b\big|x\big) \propto L\big(x\big|q,\rho^{(q)},\sigma^2\big)\pi\big(q,\rho^{(q)},\sigma^2,\mu,b\big) \propto L\big(x\big|q,\rho^{(q)},\sigma^2\big) \pi(q|\mu)\pi\big(\rho^{(q)}\big|q\big)\pi(\sigma^2|b)\pi(\mu)\pi(b)$$

Also, the Bayesian estimator cannot be determined analytically because of the posterior distribution of the parameters $\big(q,\rho^{(q)},\sigma^2,\mu,b\big)$ has a complex form. Like in exponential noise case, a reversible jump MCMC algorithm is used to solve this problem.

3.3 Reversible Jump MCMC

3.3.1 First case: exponential noise

Suppose that $M = \big(q,\rho^{(q)},\lambda,\mu\big)$. The MCMC method for simulating the distribution $\pi\big(q,\rho^{(q)},\lambda,\mu\big|x\big)$ is a method that produces ergodic Markov chain $M_1,\dots,M_m$ which has a stationary distribution $\pi\big(q,\rho^{(q)},\lambda,\mu\big|x\big)$. The Markov chain $M_1,\dots,M_m$ which has a stationary distribution $\pi\big(q,\rho^{(q)},\lambda,\mu\big|x\big)$. Furthermore, Markov chain $M_1,\dots,M_m$ is used to estimate the parameter M. To realize this, the Gibbs algorithm is adopted. The simulation of distribution $\pi\big(q,\rho^{(q)},\lambda,\mu\big|x\big)$ consists of three steps: First, simulate $\mu \sim B(q+1, q_{max}-q+1)$. Second, simulate $\lambda \sim G(\alpha,\beta)$ where $\alpha = n-q+1$ and $\beta = \big(\sum_{t=q+1}^{n}\big(x_t - \sum_{j=1}^{q} G^{-1}(\rho_j)z_{t-j}\big)\big)^{-1}$. Third, simulate $\pi\big(q,\rho^{(q)}\big|x\big)$. The distribution $\pi\big(q,\rho^{(q)}\big|x\big)$ has a complex for so that simulation of the distribution of $\pi\big(q,\rho^{(q)}\big|x\big)$ cannot be done exactly. The value of q is unknown, the MCMC algorithm cannot be used to simulate the distribution $\pi\big(q,\rho^{(q)}\big|x\big)$. Here, the reversible jump MCMC [12] is adopted.

Let $\xi = (q,\rho^{(q)})$ be the actual point of the Markov chain. There are 3 types of transformation used: order birth, order death and, coefficient change. Next, let N_q be the probability of transformation from q to q + 1, let D_q be the probability of transformation from q + 1 to q, and let C_q be the probability of transformation from q to q.

The transformation of the birth of order will change the MA model coefficient, from q to q + 1. Let $\xi = (q,\rho^{(q)})$ be the actual point and $\xi^* = (q+1,\rho^{(q+1)})$ is the new point. The birth of order from $\xi = (q,\rho^{(q)})$ to $\xi^* = (q+1,\rho^{*(q+1)})$ is defined in the following way. Select random point $v \sim U(-1,1)$. Then, create a new point $\xi^* = (q+1,\rho^{*(q+1)})$ with $\rho^{(q+1)} = \{\rho_1^* = \rho_1.,\dots,\rho_q^* = \rho_q, \rho_{q+1}^* = v\}$

Conversely, the transformation of the death of order will change the MA model coefficient, from q+1 to q. Let $\xi = (q+1,\rho^{(q+1)})$ be the actual point and $\xi^* = (q,\rho^{*(q)})$ is the new point. The death of order from $\xi = (q+1,\rho^{(q+1)})$ to $\xi^* = (q,\rho^{*(q)})$ is defined in the following way. Create a new point $\xi^* = (q,\rho^{(q)})$ with $\rho^{(q)} = \{\rho_1^* = \rho_1,\dots,\rho_q^* = \rho_q\}$.

Suppose that $P_n(\xi,\xi^*)$ and $P_d(\xi,\xi^*)$ are respective the acceptance probability for the birth of order and the acceptance probability for the death of order. The acceptance probability for the birth of order is as follows:

$$P_n(\xi,\xi^*) = min\left\{1, \frac{(\beta^*)^{n-q}}{\beta^{n-q+1}} \frac{1}{n-q} \frac{q+1}{q_{max}-q}\right\} \quad (15)$$

The acceptance probability for the death of order is as follows:

$$P_d(\xi,\xi^*) = min\left\{1, \frac{(\beta^*)^{n-q+1}}{\beta^{n-q}} (n-q) \frac{q_{max}-q}{q+1}\right\} \quad (16)$$

Transformation of the change of the coefficient will not change the order. This transformation only will change the MA coefficient. Let $\xi = (q,\rho^{(q)})$ be the actual point and $\xi^* = (q, \rho^{*(q)})$ is the new point. The change of coefficient from $\xi = (q, \rho^{(q)})$ to $\xi^* = (q, \rho^{*(q)})$ is defined in the following way. Select an index randomly $j \in \{1, \dots, q\}$, and select a point randomly $u \sim U(-1,1)$. Then a new point $\xi^* = (q, \rho^{*(q)})$ is created with $\rho^{*(q)} = \{\rho_1^* = \rho_1, \dots, \rho_{j-1}^* = \rho_{j-1}, \rho_j^* = u, \rho_{j+1}^* = \rho_{J+1} \dots, \rho_q^* = \rho_q\}$. Let $P_c(\xi, \xi^*)$ be the acceptance probability for the change of coefficient. The acceptance probability for the change of coefficient is as follows:

$$P_c(\xi, \xi^*) = min\left\{1, \left(\frac{\beta^*}{\beta}\right)^{\alpha}\right\} \tag{17}$$

3.3.2 Second case: normal noise

Let $N = \left(q, \rho^{(q)}, \sigma^2, \mu, b\right)$ be the actual point of the Markov chain. The MCMC method for simulating the distribution $\pi\left(q, \rho^{(q)}, \sigma^2, \mu, b \middle| x\right)$ is a method that produces ergodic Markov chain $N_1, \dots, N_m$ which has a stationary distribution $\pi\left(q, \rho^{(q)}, \sigma^2, \mu, b \middle| x\right)$. The Markov chain $N_1, \dots, N_m$ can be considered as a random variable having a distribution $\pi\left(q, \rho^{(q)}, \sigma^2, \mu, b \middle| x\right)$. Furthermore, Markov chain $N_1, \dots, N_m$ is used to estimate the parameter N. To realize this, the Gibbs algorithm is adopted. The simulation of distribution $\pi\left(q, \rho^{(q)}, \sigma^2, \mu, b \middle| x\right)$ consists of 4 steps: First, simulate $\mu \sim B(q + 1, q_{max} - q + 1)$. Second, simulate $b \sim G(\frac{\alpha}{2}, \frac{1}{2\sigma^2})$. Third, simulate $\sigma^2 \sim G(\gamma, \delta)$ where $\gamma = \frac{n - p_{max}}{2}$ and $\delta = \frac{\beta}{2} + \frac{1}{2}\sum_{t=p_{max}+1}^{n}\left(x_t - \sum_{j=1}^{q} G^{-1}(\rho_{j)} z_{t-j}\right)^2$. Fourth, simulate $\pi\left(q, \rho^{(q)} \middle| x\right)$. The distribution $\pi\left(q, \rho^{(q)} \middle| x\right)$ has a complex form so that simulation of the distribution of $\pi\left(q, \rho^{(q)} \middle| x\right)$ cannot be done exactly. Since the value of q is unknown, the MCMC algorithm cannot be used to simulate the distribution $\pi\left(q, \rho^{(q)} \middle| x\right)$. Here, the reversible jump MCMC algorithm (Green, 1995) is adopted.

Let $\xi = (q, \rho^{(q)})$ be the actual point of the Markov chain. There are 3 types of transformation used: order birth, order death and order change. Next, let N_q be the probability of transformation from q to q + 1, let D_q be the probability of transformation from q + 1 to q, and let C_q be the probability of the transformation from q to q.

The transformation of the birth of order will change the MA model coefficient, from q to q + 1. Let $\xi = (q, \rho^{(q)})$ be the actual point and $\xi^* = (q + 1, \rho^{(q+1)})$ is the new point. The birth of order from $\xi = (q, \rho^{(q)})$ to $\xi^* = (q + 1, \rho^{*(q+1)})$ is defined in the following way. Select random point $v \sim g(v)$ where

$$g(v) = \begin{cases} v + 1 & -1 < v < 0 \\ 1 - v & 0 < v < 1 \end{cases} \tag{18}$$

Then, create a new point $\xi^* = (q + 1, \rho^{*(q+1)})$ with $\rho^{(q+1)} = \{\rho_1^* = \rho_1., \dots, \rho_q^* = \rho_q, \rho_{q+1}^* = v\}$.

Conversely, the transformation of the death of order will change the MA coefficient, from q+1 to q. Let $\xi = (q + 1, \rho^{(q+1)})$ be the actual point and $\xi^* = (q, \rho^{*(q)})$ is the new point. The death of order from $\xi = (q + 1, \rho^{(q+1)})$ to $\xi^* = (q, \rho^{*(q)})$ is defined in the following way. Create a new $\xi^* = (q, \rho^{(q)})$ with $\rho^{(q)} = \{\rho_1^* = \rho_1, \dots, \rho_q^* = \rho_q\}$. Suppose that $P_n(\xi, \xi^*)$ and $P_d(\xi, \xi^*)$ are respective the acceptance probability for the birth of order and the acceptance probability for the death of order. The acceptance probability for the birth of order is as follows:

$$\begin{aligned} &P_n(\xi, \xi^*) \\ &= min\left\{1, \left(\frac{\delta^*}{\delta}\right)^{-\gamma} \frac{q_{max-q}}{q + 1} \frac{\mu}{1 - \mu} \frac{1}{2}\right\} \end{aligned} \tag{19}$$

The acceptance probability for the death of order is as follows:

$$\begin{aligned} &P_d(\xi, \xi^*) \\ &= min\left\{1, \left(\frac{\delta^*}{\delta}\right)^{-\gamma} \frac{q + 1}{q_{max} - q} \frac{1 - \mu}{\mu} 2\right\} \end{aligned} \tag{20}$$

Transformation of the change of the coefficient will not change the order. This transformation only will change the MA coefficient. Let $\xi = (q, \rho^{(q)})$ be the actual point and $\xi^* = (q, \rho^{*(q)})$ is a new point. The change of coefficient from $\xi = (q, \rho^{(q)})$ to $\xi^* = (q, \rho^{*(q)})$ is defined in the following way. Select an index randomly $j \in \{1, \dots, q\}$, and select a point $u_i \sim f(u_i)$ where

$$f(u_i | \rho_i) = \frac{5}{\pi\sqrt{1 - u_i}} \tag{21}$$

for $u_i \in \left(\sin\left(\rho_i - \frac{\pi}{10}\right), \sin\left(\rho_i + \frac{\pi}{10}\right)\right)$. The new point $\xi^* = (q, \rho^{*(q)})$ is created with $\rho^{*(q)} = \{\rho_1^* = \rho_1, \dots, \rho_{j-1}^* = \rho_{j-1}, \rho_j^* = u, \rho_{j+1}^* = \rho_{J+1} \dots, \rho_q^* = \rho_q\}$. Let $P_c(\xi, \xi^*)$ be the acceptance probability for the change of coefficient. The acceptance probability for the change of coefficient is as follows:

$$\begin{aligned} &P_c(\xi, \xi^*) \\ &= min\left\{1, \left(\frac{\delta^*}{\delta}\right)^{-\gamma} \left(\frac{1 - \rho_i}{1 + u_i} \frac{1 + \rho_i}{1 - u_i}\right)^{1/2}\right\} \end{aligned} \tag{22}$$

3.4 Simulation

Simulation studies were carried out to determine the performance of the reversible jump MCMC algorithm in estimating the parameters of the MA model. Table 2 presents the parameter values of the MA model. The MA model order is 3. The error is assumed to have a normal distribution with mean 0 and variance 2.

Table 2: Parameter values of the MA model

q	$\theta^{(q)}$	σ^2
3	(0.5348, -0.0391, -0.7460)	2

Two hundred and fifty synthesis data is made using Eq. (1). The synthesis data is presented in Figure 4.

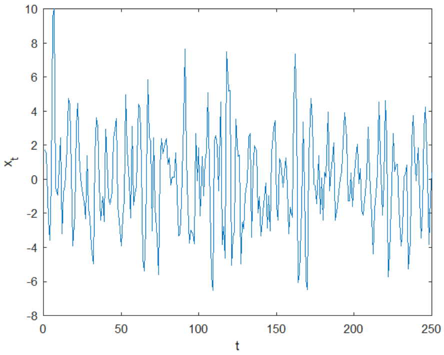

Fig. 4 Data synthesis

Then, synthesis data is used as input for the reversible jump MCMC algorithm to estimate the parameters of the MA model. The algorithm is run in 50000 iterations with 10000 iterations of a burn-in period. The output algorithm is as follows. The order histogram is presented in Figure 5.

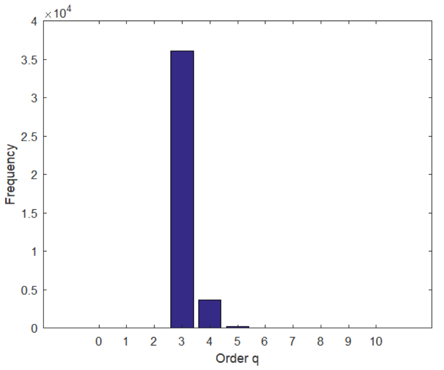

Fig. 5 Histogram of order q

From Figure 5, it can be seen that the third order reaches the highest frequency. This shows that the estimated MA model order is 3. Based on the MA model (3), then the MA model coefficients and error variance are estimated. The results of the MA model parameter estimation are presented in Table 3.

Table 3: Value of parameter estimation of MA model

$\hat{q}$	$\widehat{\theta^q}$	$\hat{\sigma}^2$
3	(0.5306, -0.0177, -0.7251)	2.2014

The estimator of the parameters in Table 3 approaches the true value of the parameter in Table 2. This shows that the reversible jump MCMC algorithm can properly estimate the parameters of the MA model. This algorithm can be used to estimate the MA model even though the order is unknown. The algorithm produces an invertible MA model.

4. CONCLUSION

This study discusses a new method for estimating the parameters of the MA model that is normal and exponential if the order is unknown. The reversible jump MCMC algorithm is an alternative method that can be used to estimate the parameters of the MA model even though the order is unknown. The advantage of this method is that both the order and coefficients of the MA model can be estimated simultaneously. Also, the MA model produced is the MA model that verifies invertibility region.

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6. REFERENCES

[1] Shi Z. and Aoyama H., Estimation of the Exponential Autoregressive time series model by using the genetic algorithm, Journal of Sound and Vibration, Vol. 205, No. 3, 1997, pp. 309-321.

[2] Sad N., On Exponential Autoregressive Time Series Models, J. Math, Vol. 29, Vol. 1, 1999, pp. 97-101.

[3] Larbi L. and Fellag H., Robust Bayesian Analysis of an Autoregressive Model with Exponential Innovations, Afr. Stat., Vol. 11, No. 1, 2016, pp. 955-964.

[4] Suparman, A New Estimation Procedure Using Reversible Jump MCMC Algorithm for AR Models of Exponential White Noise, International Journal of GEOMATE, Vol 15, No. 49, 2018, pp. 2186-2982.

[5] Rossum H.H.V. and Kemperman H.,

Implementation and Application of Moving Average as Continuous Analytical Quality Control Instrument Demonstrated for 24 Routine Chemistry Assays, Clinical Chemistry & Laboratory Medicine, Vol. 55, Issue. 8, 2017, pp. 1142-1151.
[6] Kordestani H., Xiang Y-Q, and Ye X-W., Output-Only Damage Detection of Steel Beam Using Moving Average Filter. Shock & Vibration, Vol. 28, No. 1, 2018, pp. 1-13.
[7] Middleton R.H., Rojas A.J., Freudenberg J.S., and Braslavsky J.H., Feedback Stabilization Over a First Order Moving Average Gaussian Noise Channel, IEEE Transactions on Automatic Control, Vol. 54, No. 1, 2009, pp. 163-167.
[8] Smadi M.M. and Alodat M.T., Bayesian Threshold Moving Average Models, Journal of Modern Applied Statistical Methods, Vol. 10, Issue 1, 2011, pp. 262-267.
[9] Yang W., Wang X., Ling N., and Hu S., Discrete Dynamics in Nature and Society, 2012, pp. 1-24.
[10] Safi S.K., Generalized Heteroskedasticity ACF for Moving Average Models in Explicit Forms, J. Stat. Oper. Res, Vol.9, No. 4, 2013, pp. 379-391.
[11] Dejamkhooy A., Dastfan A., and Ahmadyfard A., Modeling and Forecasting of Non Stationary Voltage Flicker and Its Source Detection in Power System, J. Electrical System, Vol. 10, No. 2, 2014, pp. 223-237.
[12] Green P.J., Reversible Jump MCMC Computation and Bayesian Model Determination. Biometrika, Vol. 82, 1995, pp. 711-732.
[13] Punskaya E., Andrieu C., Doucet A., and Fitzgerald W.J., Bayesian Curve Fitting Using MCMC with Applications to Signal Segmentation, IEEE Transactions on Signal Processing, Vol. 50, No. 3, 2002, pp. 747-758.
[14] Grummer J.A., Bryson R.W., and Reeder T.W., Species Delimitation Using Bayes Factors: Simulations and Application to the *Sceloporus Scalaris* Species Group (Squamata: Phrynosomatidae). Syst. Biol, Vol. 63, No. 2, 2014, pp. 119-133.
[15] Pourhabib A., Liang F., and Ding Y., Bayesian Site Selection for Fast Gaussian Process Regression. IIE Transactions, Vol. 46, 2014, pp. 543-555.
[16] Fox M. and Ickert R.B., Model Selection During Sample-Standard-Bracketing Using Reversible Jump Markov Chain Monte Carlo, Journal of Analytical Atomic Spectrometry, Vol. 30, Issue 10, 2015, pp. 2208-2213.
[17] Newcombe P.J., Ali R.H., Blows F.M. Provenzano E., Pharoah P.D., Caldas C., and Richardson S., Weibull Regression with Bayesian Variable Selection to Identify Prognostic Tumour Markers of Breast Cancer Survival, Statistical Methods in Medical Research, Vol. 26, Issue. 1, 2017, pp. 414-436.
[18] Steward R.M. and Rigdon S.E., Risk-Adjusted Monitoring of Healthcare Quality: Model Selection and Change-Point Estimation. Quality & Reliability Engineering International, Vol. 33, Issue 5, 2017, pp. 979-992.
[19] Karakus O., Altinkaya M.A., and Kuruoglu E.E., Bayesian Volterra System Identification Using Reversible Jump MCMC Algorithm, Signal Processing, Vol. 141, 2017, pp. 125-136.