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REVIEW OF CORRELATION BASED ALGORITHMS IN SIGNAL AND IMAGE PROCESSING FOR PATTERN IDENTIFICATION

Humera Rafique[1], Samina Rafique[2]

[1]Department of Mechatronic Engineering, SZAB Institute of Science and Technology, Karachi, Pakistan;
[2] (Department of Electrical Engineering), Bahria University, Islamabad, Pakistan

ABSTRACT: This paper presents a review of applications of Digital Correlation algorithms in pattern identification in signal and image processing. The paper not only describes the correlation function, and its different variants descriptively and mathematically and the algorithms in which these can perform discrimination among several signals, images, video frames and objects within them, it also explains the implementation of these by using synthetic signals, audio signals and images, including alphabetic characters. Apart from this, the paper also includes the pseudo code for better understanding of algorithms from the implementation point of view. The results of all described techniques are presented from unprocessed original signals to their processed from, graphically, and spatially.

As an extended version [11], this paper presents a few more applications of Digital Correlation related functions for more complex tasks and it has been found that, the Digital Correlation and its related functions are very useful and easy to implement for real life pattern identification problems with enough level of complexity with less computational efforts as an independent algorithm and as a part of a complex algorithm.

Keywords: Digital Correlation, Auto correlation function (ACF), Cross correlation function (CCF), Pattern identification, Feature extraction.

1. INTRODUCTION

For a period of four decades, pattern identification has been used for the classification of objects into categories, seen by a computing machine [1]. This sub-field of Image and Signal processing has several applications in medicine, weather forecasting, forensics, industrial inspection, communication, geology, agriculture and security. There are a number of techniques and algorithms to teach a computing machine by which these are able to identify objects within images and videos even in real time. There are several advanced and growing techniques for this purpose, while this paper represents a simple, easy and strong thus widely used technique that can serve the described purpose independently, as well as can be a part of many advanced and robust pattern identification algorithms and techniques, namely Correlation, its variants and related algorithms.

Correlation, or Discrete Time Correlation is a mathematical operation that is similar to Discrete Time Convolution apparently. This operation compares two signals, at a time, by measuring the degree of similarity of these signals. On the basis of this measure, further application dependent information can be obtained. Good, but advanced examples of applications are radar communication, under water sound navigation systems (SONAR), general communication systems, and medical imaging [2].

The mathematical definition of correlation has two versions, as the correlation itself has two forms: Autocorrelation and Crosscorelation. Autocorrelation, primarily a comparison of a signal with its own, while the Crosscorrelation compares a signal with some other signal. The connection between signals is often used to determine system characteristics, these signals are related to. [8]. Defining two signals $\boldsymbol{x_1(n)}$ and $\boldsymbol{x_2(n)}$, $\forall\ \boldsymbol{x_1(n) = x_2(n)}$, then the corresponding sequence is defined as

$$\mathbb{C}_{\mathbf{12}}(\boldsymbol{\ell}) = \boldsymbol{x_1(n)} \otimes \boldsymbol{x_2(n)} = \sum_{\boldsymbol{n=0}}^{\boldsymbol{N-1}} \boldsymbol{x_1(n)\, x_2(n-\ell)} \qquad (\mathbf{1})$$

where, index $\boldsymbol{\ell} = \cdots, -\mathbf{2}, -\mathbf{1}, \mathbf{0}, \mathbf{1}, \mathbf{2}, \ldots$ and defined as the time shift or lag, and subscript '12' represents the order of correlation, or the direction of shift of one signal. Reversing that order will not change the resultant, as correlation is a 'Commutative process'.

Now, defining two signals $\boldsymbol{x_1(n)}$ and $\boldsymbol{x_2(n)}$, $\forall$ $\boldsymbol{x_1(n) \neq\ x_2(n)}$, then the corresponding sequence is defined as

$$\mathbb{C}_{\mathbf{12}}(\boldsymbol{\ell}) = \boldsymbol{x_1(n)} \otimes \boldsymbol{x_2(n)} = \sum_{\boldsymbol{n=0}}^{\boldsymbol{N-1}} \boldsymbol{x_1(n)\, x_2(n-\ell)} \qquad (\mathbf{2})$$

where, index $\boldsymbol{\ell} = \cdots, -\mathbf{2}, -\mathbf{1}, \mathbf{0}, \mathbf{1}, \mathbf{2}, \ldots$ and defined as the time shift or lag, and subscript '12' represents the order of correlation, or the direction of shift of one signal. Again, 'commutative property' is valid here. Both equations look same, but results will be different because of similarities and dissimilarities of two signals being compared.

1.1 Process of Correlation

The *Correlation* is a process that seems like the process of *Convolution;* a process in which one of the signals to be convolved (in fact any one of them), will be *folded*, than *shifted*, and then *multiplied* with the other signal. All products (depends how many samples or elements of one signal in range of the other one, (i.e, one to many) will be added to deliver elements of the output signal respectively. This three step process is repeated multiple times, (as folding is perfumed once throughout the process), until the whole signal is covered. The *Correlation* applies the same process onto the signals, without *folding*. i.e. *Shifting, multiplication* and *Summation*. Due to this similarity of process, many computer programs use alternate commands for both operations by providing the folded version of the signal. i.e., $\boldsymbol{x_1(-n)}$.

2. APPLICATIONS OF CORRELATION FUNCTIONS IN SIGNAL AND AUDIO PROCESSING

This section will describe a few synthetic signal, and audio signal processing applications mainly focusing on signals' characteristic detection and identification:

2.1 Detection and Estimation of Periodic Signals Concealed in Random Noise

Many signals on communication lines suffer transmission impairments and noise interruption problems. A periodic signal e.g., has been received at a receiver as noise corrupted signal, can be identified in random noise by characterizing ACF of both signals. Fig.1-1 shows signals and their respective ACF plots. As a broad band signal (random noise) possesses narrower correlation peaks, thus easy to identify [9]. Fig. 1-2. Conclusively, the received signal is not 'just' noise, but it carries usable information too, to be extracted.

Pseudo Code flow:

- *Sampling frequency (fs), signal's frequency (f), time sweep (t),*
- *Sinusoid x(n), Random noise rn(n)*
- *acf(x), acf(rn), acf(x+rn)*
- *Plots*

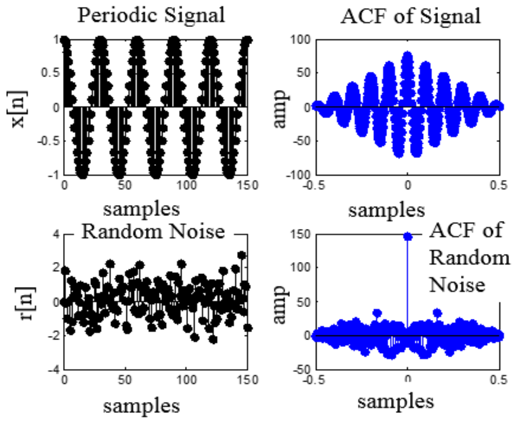


Fig. 1-1 Separate periodic cosine wave and random noise, and their respective ACF plots

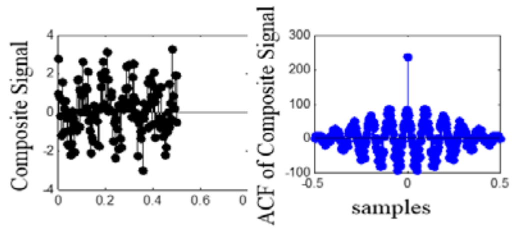


Fig. 1-2 Periodic signal embedded in random noise and their composite ACF

2.2 Estimation of Power Spectral Density of a Signal via Autocorrelation

The power spectral density (PSD) is an estimate of amplitude variations of a signal over a range of frequency. 'The Fourier transform of the autocorrelation function of an energy signal is equal to the energy spectral density of that signal' [5].

$$\boldsymbol{\mathcal{F}}[\mathbb{C}_{xx}(\boldsymbol{\ell})] = \boldsymbol{\mathcal{P}}_{xx}(\boldsymbol{\omega}) \quad (3)$$

The test signal is a sinusoid with three harmonic components. One power spectral density (PSD) has been computed by fast Fourier transform (FFT); the right hand side of Eq. (3). The ACF of the signal then brought to frequency domain via FFT.
Both PSDs of Eq. (3) are found comparable.

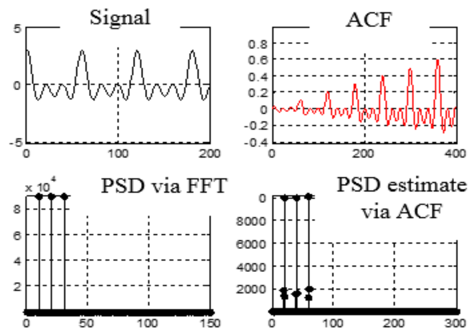


Fig. 2-1 Multi frequency sinusoid, its ACF and their respective PSDs

The Fourier transform relationship of the ACF and the power spectrum describes their mathematical equivalency, as they possess the same information. Though, the power spectrum is more accustomed and generally easily interpretable. The ACF is powerful for theoretical work, for determining the non-whiteness of data or residuals, for detecting periodic components in data, and for identifying the dominant power law noise type [6].

Pseudo Code flow:

- *fs, f1, f2=2f2, f3=3f1, t,*
- *Three Sinusoids x1, x2, x3, y = x1+x2+x3*
- *(magni(fft(acf(y)))^2, (magni(fft(y)))^2, plots*

2.3 The ACF of a Real Valued Signal is An Even Function, while CCF is An Odd Function. [2].

These properties of both functions have been implemented using a few periodic and aperiodic signals. *Plots in Fig 3-1 and Fig 3-2 can be plotted using built in commands of any computing software.*

i. The ACF is symmetric about the central lag i.e., an even function:

$$\mathbb{C}_{11}(-\boldsymbol{\ell}) = \mathbb{C}_{11}(\boldsymbol{\ell}) \qquad (4)$$

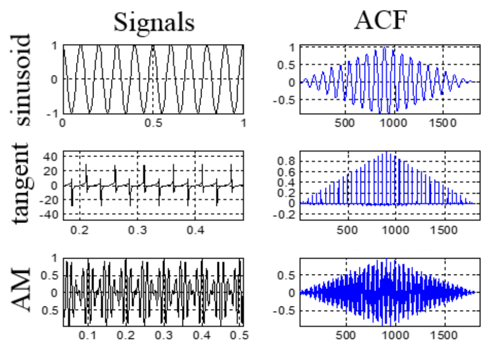


Fig. 3-1 Periodic signals and their respective ACF (even function symmetry)

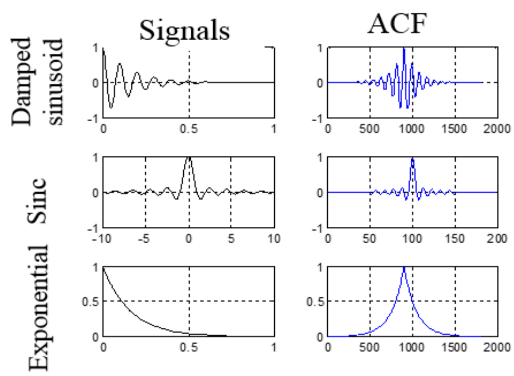


Fig 3-2 Aperiodic signals and their respective ACF (even function symmetry)

As signals in Fig. 3-1 and Fig.3-2 are of low frequencies, their ACF plots show a border range over entire lag sweep.

ii. The CCF is symmetric with amplitude reversal about the central lag i.e., an odd function:

$$\mathbb{C}_{12}(-\boldsymbol{\ell}) = -\mathbb{C}_{21}(\boldsymbol{\ell}) \qquad (5)$$

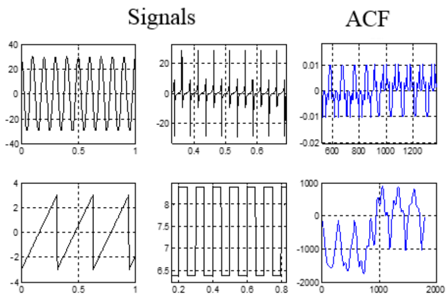


Fig. 3-3 Couple of CCF of two (different) signals (odd function symmetry)

Fig. 3-3 describes detection of odd function symmetry in different signals via CCF.

Note, that possessing even or odd symmetries in corresponding correlation functions, depend on the degree of similarity between the two signals.

2.4 Frequency Detection

A highly variant ACF time domain plot describes how fast the signal itself changes w.r.t time and vice versa. Fig 4.

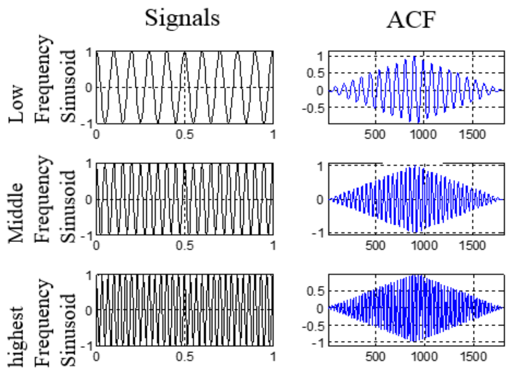


Fig. 4 Three signals (harmonics) and their respective ACFs, showing faster time changes as the signals go faster (increasing frequency)

2.5 Identifying Information Carrying Signal from Random Noise, with Occurrence in Time Domain

In addition to the identification of noise buried signal, Correlation can identify the exact localization of signal in time domain. Two wave audio signals ‘car horn’ and ‘train whistle’ are test signals. Fig. 5-1.

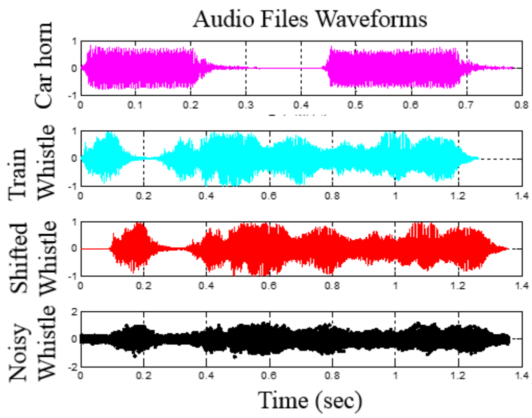


Fig. 5-1 Time series plots of wave files, and train sound buried in random noise

Getting their 'Welch Spectra' will not reveal the required information easily regarding (a) if noisy signal contains information (Train whistle) sound, and (b) when the information signal joins the random noise.

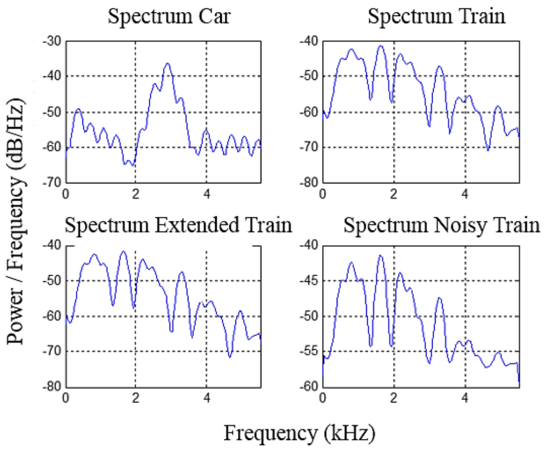


Fig. 5-2 Welch Spectra of separate and noisy signals

In Fig. 5-1 and 5-2 it is very difficult to get answers of (a) and (b), specially (b). While computing Cross correlation of two signals with the noisy signal will answer both. In Fig 5-3, CCF of car horn sound shows minor amplitudes (notice the difference of amplitude scales, at same scale this plot will show almost a straight line). The larger values show the presence (matching) of train whistle sound in noisy signal, while the position of largest peak from zeroth lag (i.e., 90.7 ms) describes the time when the train sound joins the random noise.

Third plot of Fig. 5-1 shows this time delay clearly.

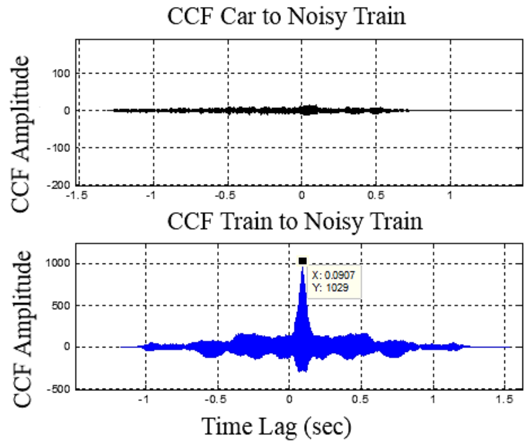


Fig. 5-3 CCF of both sounds with Noisy sound

Pseudo Code flow:

- *x = audioread(car.wav), get fs*
- *y = audioread(train.wav),*
- *y1 = zeropad(y,10ms)*
- *rn = random, z = addmix(rn,y1), plots*
- *welchspectrum(x,y,y1,z), plots*
- *c1 = ccf(x,z), c2 = ccf(y,z), plots*
- *timedelay = maxi(c2)/fs*

(Note: fs for the whole system should be the same)

3. APPLICATIONS OF CORRELATION FUNCTIONS IN IMAGE AND VIDEO PROCESSING

This section will describe a few image processing and alphabetic character identification applications:

3.1 Alphabetic Character Identification using Cross-Correlation Maxima

To demonstrate the above the system will follow the algorithm shown in Fig. 6-1.

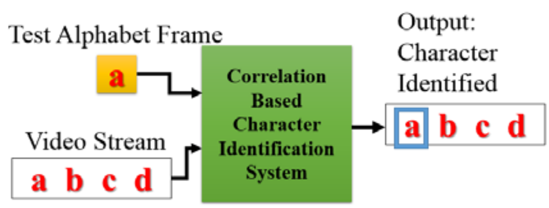


Fig. 6-1 Basic Block Diagram of Alphabetic Character Identification System

To implement the above block diagram, a series of .png images has been created. Fig. 6-2. These images have been converted into an avi file, each image as frame [10]. The created video file run these images one by one with an adjustable time delay.

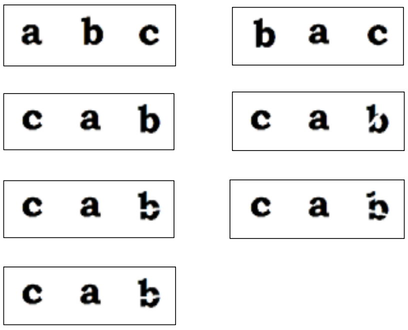


Fig. 6-2 Individual images of three alphabets in different order, standard and broken forms

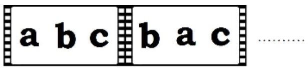


Fig. 6-3 Individual images run as avi file

Correlation Based Alphabet Identification System Block Algorithm [3]

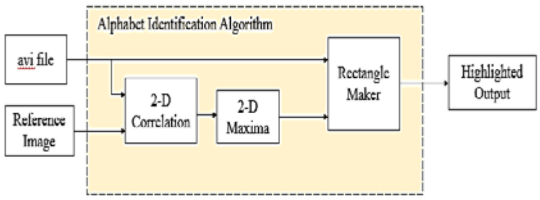


Fig. 6-4 Simplified Block Diagram of Algorithm

Each frame of video contains three alphabets and reference of target image contains a specific alphabet. The target image is compared frame by frame, alphabet by alphabet by computing the 2D correlation and then its maxima.

The alphabet with highest maxima is then encircled with a rectangle box as output per frame. The system has been tested with different fonts and is even able to identify the broken characters as well.

Table 1 Results of 2D Correlation Maxima Algorithm for letter ‘b’

Frame	Font 1	Output	Comments
1	a b c	a $\boxed{b}$ c	Recognized
2	a c b	a c $\boxed{b}$	Recognized
3	c a b*	c a $\boxed{b*}$	Recognized

* Broken alphabet

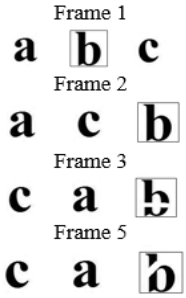


Fig. 6-5 Results of 2D Correlation Maxima Algorithm for letter ‘b’ in ‘*Times New Roman’*. Frame 3 and 5 show broken alphabets recognition*

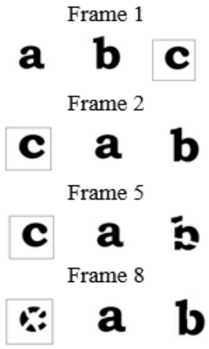


Fig. 6-6 Results of 2D Correlation Maxima Algorithm for letter ‘c’ in ‘Bookman Old Style’. Frame 8 shows badly broken alphabet recognition.

Pseudo Code flow:

- *x1 … x7 = images with 3 alphabets each*
- *abc.avi = videomake(x1,x2,…x7)*
- *mov1 = readvideio(abc.avi)*
- *y =2dcorr(frame1,tartgetimage1)*
- *ym = 2dmaxi(y)*
- *rect-shape = compare(frame1,ym),*
- *output*

Table 2 Correlation Maxima Factors of Alphabets

Alphabet	Matching Alphabet	Correlation maxima Factor (Ratios)
a	a	247
b	b	201
c	c	201

3.2. Sharpening of Colored Image using Luminance Processing with 2D Correlation Filter

A high pass image filter can make transitions of an image more critical making the image sharper interns. A colored RGB image if converted into Y’CbCr image so that its luma intensity should be filtered in the described manner. Fig. 7-1.

Algorithm [3]

A 'Y'CbCr' version of an 'RGB' image (bird.jpg) has been made intentionally blurred using averaging correlation filter of mask size 15×15. The intensity layer of image has been processing by a high pass correlation filter and then converted back to its RGB form. The output images show two levels of sharpening.

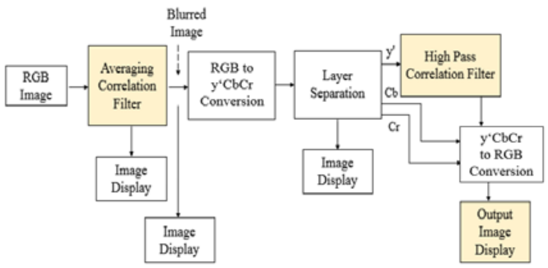


Fig. 7-1 Block Diagram of Luminance Processing Algorithm

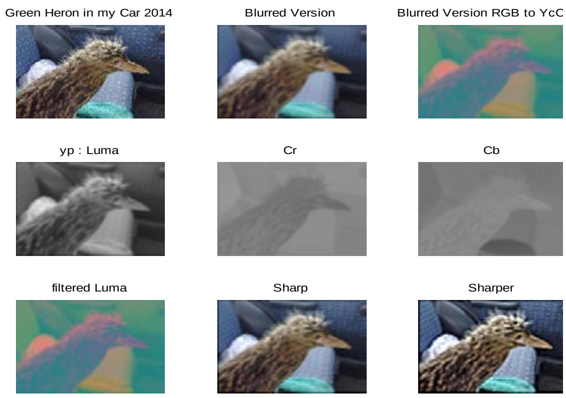


Fig. 7-2 Results of high pass filtering of luminance intensity of a colored image

3.3. Digital Image Edge Detection with Correlation Integral Filter

The human and machine vision require measurements of the amount of the reflected light from the object or scene of interest, to acquire the physical features e.g., location of the object boundaries, the structure, color, texture of the surface, from a 2D scenario for a 3D vision sense. The fundamental information regarding the physical properties described, can be extracted from 'change of intensities' and 'the localization of edges' [12].

Among many techniques, an 'integral correlation filter' technique is used for speedy analysis and edge detection of images [13].

An integral filter is a correlation filter, as it does not rotate prior to computation of the filtered image like an ordinary filter does. It performs correlation filtering of an integral image with an appropriate box type filter. [3]. Fig. 8-1 and Fig. 8-2 show the simplified algorithm of an integral filter processing of an image or a video frame, while Fig. 8-3 shows the results of integral correlation filtering.

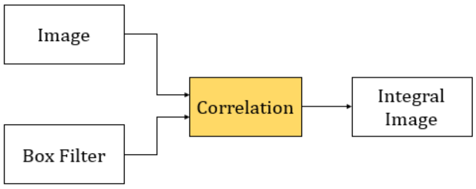


Fig. 8-1 Integral image processing

An integral image *ii(x,y)* can be represented as:

$$\boldsymbol{ii}(\boldsymbol{x},\boldsymbol{y}) = \sum_{x' \le x, y' \le y} \boldsymbol{i}(\boldsymbol{x}',\boldsymbol{y}') \qquad \textbf{(6)}$$

Where, *i(x',y')* is a subset of original image.

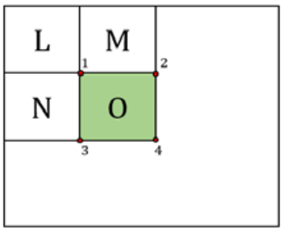


Fig. 8-2 Integral image algorithm: The sum of the pixels within rectangle 'O' can be computed with four array references. The value of the integral image at location 1 is the sum of the pixels in rectangle L. The value at location 2 is L + M. at location 3 is L +N, and at location 4 is L + M + N + O. The sum within O is 4 + 1 – (2+3). [13].

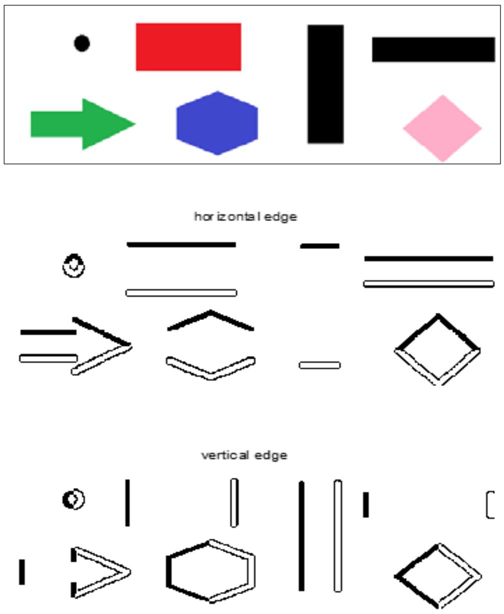


Fig. 8-3 Top-Bottom: Original image, horizontal edges highlighted, vertical edges highlighted.

3.4. Digital Image Registration with Normalized Cross Correlation

Image registration is the process of lining up two or more images of the similar view acquired differently, w.r.t to time of acquisition, view and by different sensing mechanisms. It geometrically lines up two images or video frames, the reference and the acquired image. It has many applications in image processing, specifically in medical imaging, remote sensing, computer vision and pattern matching. [14].

There are various image registration algorithms, with variety of classification, here the area-based methods, also called, correlation-like method or template matching method has been presented. This classical method, uses image intensities for comparison and does not perform any structural analysis. This method, covers mainly the translation changes, but slight rotation and scaling problems. [14]. Fig 8-4 shows the algorithm and Fig 8-5 shows the results following the same sequence.

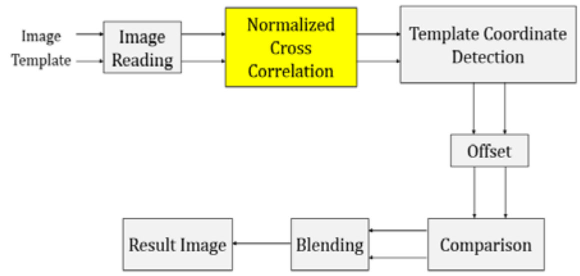
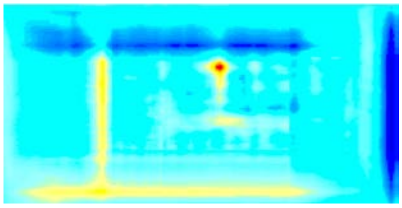


Fig. 8-4 The algorithm of Image Registration using Normalized Cross correlation. [3].

(a) Template

(b) Original Image [15]



(c) Template coordinate detection within the main image, the red color shoes the highest matching coordinates.

(d) Fitting the template at extracted size of a zero image of size of the original image using results of (c).

(e) Final blending of template with original image for comparison

Fig. 8-4 The Results following the algorithm of Image Registration using Normalized Cross correlation (a-e).

Fig. 8-6 shows the result of same algorithm applied to a natural scene image. From the final processed image, the imperfection of the process can be perceived from the blurry output. While the synthetic image of Fig. 8-4 shows a perfect result. The reason of imperfection in result can be understood from its coordinate detection plot in Fig. 8-7. The smooth intensities of a natural image makes it difficult to uniquely identify the highest value of normalized cross correlation. The 3D view of the plot in Fig. 8-8 further explains the problem.

With careful localization of sub images this discrepancy may be fixed. Fig. 8-9 and 8-10 show the results with manipulation of sub image coordinates and the 3D coordinate detection plot respectively.

Fig. 8-5 The Test image of a natural scene. [16].

Fig. 8-6 The Results of image blend following the same algorithm using a natural image.

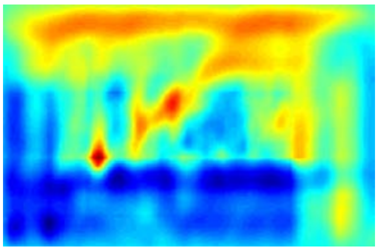


Fig. 8-7 Template coordinate detection plot.

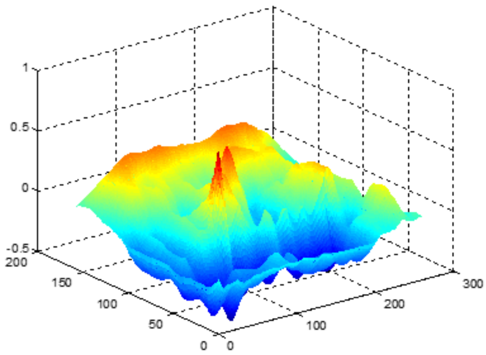


Fig. 8-8 3D view of Fig. 8-7: See the two peaks at nearly similar height.

Fig. 8-9 Result of sub image coordinate manipulation.

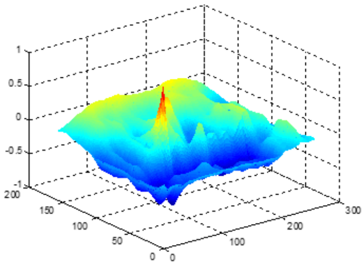


Fig. 8-10 3D view of peak detection plot for Fig. 8-9: A unique peak is clearly seen.

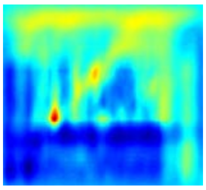


Fig. 8-11 Top view of Fig. 8-10.

4. CONCLUSION

This study has successfully described the Correlation and its related functions along with various applications in signal, audio, image and video processing applications. Selected algorithms and their implementation methodologies in the form of pseudo code have also been described. The digital correlation and its related algorithms have been found very useful, easy and simple to understand and implement for many pattern recognition algorithms. It can detect simple features of the signal ranging from periodicity detection and time lag to complex tasks of identifying objects within a video frame.

Over a wide range of applications, it works as an individual algorithm, or may be a part of more complex algorithms.

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Corresponding Author: Humera Rafique