

FLOOD ANALYSIS IN LANGAT RIVER BASIN USING STOCHASTIC MODEL

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ABSTRACT: This study analyzed the annual maximum stage readings of three rivers in Langat River Basin for flood forecasting using Autoregressive Integrated Moving-average (ARIMA) model. Model identification was done by visual inspection on the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF). The model parameters were computed using the Maximum Likelihood (ML) method. In model verification, the chosen criterion for model parsimony was the Akaike Information Criteria Corrected (AICC) and the diagnostic checks include residuals' independence, homoscedasticity and normal distribution. The best ARIMA models for the Dengkil, Kg. Lui and Kg. Rinching series were (1,1,0), (1,1,0) and (1,1,1) respectively, with their AICC values of 133.736, 55.348 and 42.292. Homoscedasticity was confirmed with the Breusch-Pagan test giving p-values of 0.145, 0.195 and 0.747 for the Dengkil, Kg. Lui and Kg. Rinching models respectively. Forecast series up to a lead time of eight years were generated using the accepted ARIMA models. Model accuracy was checked by comparing the synthetic series with the original series. Results show that the ARIMA models for the rivers and the forecast series were adequate. In conclusion, the Box-Jenkins approach to ARIMA modelling was found to be appropriate and adequate for the rivers. The flood forecast up to a lead time of eight years for the three models exhibit a straight line with near constant streamflow values showing that the forecast values were similar to the last recorded observation.

Keywords: Flood Analysis, ARIMA, Box-Jenkins Approach, Langat River Basin

1. INTRODUCTION

Floods have huge environmental and economic impact. According to a study conducted by KTA Tenaga in 2002, the total flood affected area in Malaysia in 2000 was about 9.04% of the total land area in Malaysia. The population in flood affected areas in 2000 was 22% of the total population at that time and the Annual Average Damage estimated was RM 915 million. On the other hand, the design of hydraulic structures such as dams and reservoirs also depends on the design flood of the particular river. An inaccurate design flood can lead to inefficiency of those hydraulic structures. Modifications to an existing structure are extremely costly and troublesome. Therefore, flood analysis is important to address these key issues.

There are two main approaches in performing a flood analysis; the rainfall-runoff analysis approach and the flood frequency analysis approach. The first approach uses rainfall statistics and a catchment model to estimate flood. This approach can be divided according to their spatial structure (lumped, semi-distributed or distributed) and time representation (event-based simulations and continuous simulations). For event-based simulations the rainfall-runoff model is fed by a design rainfall of a defined probability [1]. A very popular method is the revitalised flood hydrograph (ReFH) method [2], which is commonly used in

England and Wales. In the flood frequency analysis approach, only peak flow data is used to make the estimation. There are two prevalent methods for flood frequency analysis; the annual maximum series (AMS) method and the partial duration series (PDS) method. However, the AMS method is commonly used in frequency analyses compared to the PDS method [3]. The AMS method is easier to define and the assumption that annual maxima are independent is reasonable. In addition, [4] comprehended that hydrologic phenomenon behaves stochastically. Thus, there is another method called stochastic modelling which uses time series that has four main components which are the trend component, the periodic component, the catastrophic component, and the random component.

The study area is the Langat River Basin which spans two states in Malaysia, namely Selangor and Negeri Sembilan. The Langat River Basin is shown in Fig. 1. It has a catchment area of approximately 2,348 km². The Langat River is the main stream while other major tributaries include the Semenyih River, the Labu River and the Beranang River. Two dams are located at the upper region of the river basin; the Semenyih dam and the Langat dam. The Semenyih dam has a catchment area of 56.7 km² while the Langat dam has a catchment area of 41.1 km².

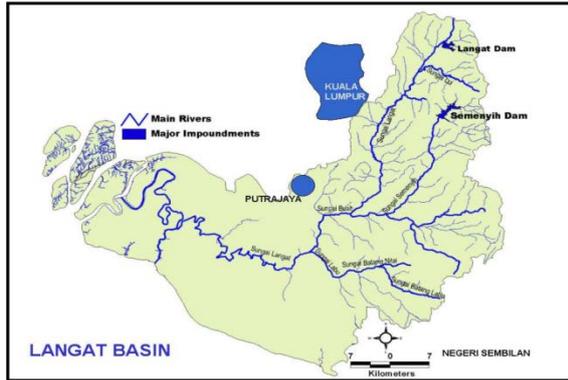


Fig. 1: The Langkat River Basin

The aim of this study is to mitigate the flood problems in Langkat River Basin through developing stochastic ARIMA models for the study rivers using Box-Jenkins approach and afterwards, forecast future annual maximum streamflow values in the study rivers using the developed ARIMA models.

2. ARIMA MODEL

The ARIMA modelling is actually an approach that has the flexibility to fit a model which is adapted from the data structure itself. With the help of the computed autocorrelation function and partial autocorrelation function, the time series' stochastic nature can be modelled and vital information such as trend, periodic components, random components and serial correlation can be obtained. The Box-Jenkins approach to ARIMA modelling is an iterative model building process where the best models have to be determined through trial and error. However, with the advent of computers and statistical software packages, this iterative process can be simplified. Commonly used software packages include Statgraphics, Minitab and Statistica.

The ARIMA model has three main components, namely Autoregressive (AR), Integrated (I) and Moving-Average (MA). The AR component represents the autocorrelation between current and past observations while the MA component describes the autocorrelation structure of error. The integrated component represents the level of differencing required to transform a non-stationary series into a stationary series [5]. A non-seasonal ARIMA model is usually denoted by (p,d,q) . The order of the AR component is denoted by p , the order of differencing is denoted by d and q is the order of the MA component.

Throughout the years researchers have used the ARIMA model for different scientific and technical applications. [6] described the random component of streamflow time series by examining the stochastic structure of the flow data for the Upper Delaware River. Forecasting monthly rainfall data using various ARIMA models was done by [7],

whereas [8] carried out streamflow prediction on a medium sized basin in Mississippi. The ARIMA model was applied to monthly data from Kelkit Stream watershed by [9]. [10] reviewed the performance of two stochastic models (Thomas-Fiering and ARIMA) on Yesilirmak River, Turkey.

There have been a lot of reviews on the performance of the ARIMA model. [11] argued that the ARIMA model is only suitable for short term forecasting. The ARIMA model needs a long input series to produce forecasts that are more accurate. Therefore, the ARIMA model may not work well for short input series. [12] showed that the performance of ARIMA is satisfactory in forecasting either a linear or non-linear interval series. It is also a good forecasting alternative to inter-valued time series.

2.1 Stationarity

The Box-Jenkins approach is a stationary time series approach. If a time series is non-stationary, differencing is required to make it stationary before the Box-Jenkins approach can be carried out. There are many ways to determine non-stationarity. The common tests used include unit root and trend tests.

2.1.1 ADF Test

The testing for unit root's presence in a time series is a normal starting point of applied work in macroeconomics. One of the popular tests for unit root is the Augmented Dickey-Fuller (ADF) test. This test is based on estimates from an augmented autoregression. One of the main issues in the ADF test is the choice of lag length k .

2.1.2 KPSS Test

Another well-known test for stationarity in econometrics is the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. It tests for the null hypothesis of stationarity as opposed to the ADF test which tests for the null hypothesis of non-stationarity. One of the important arguments against the use of tests with stationarity as the null hypothesis is that it is very difficult to control their size when the process is stationary and extremely autoregressive [13].

2.1.3 Mann-Kendall Trend Test

The Mann-Kendall trend test is commonly used to test the presence of trend in a time series. It is not a parametric test so the data do not have to be normally distributed and it has low sensitivity to sudden changes due to non-homogeneous time series. The Mann-Kendall S Statistic shows the behaviour of a trend. A positive S indicates an upward trend while a downward trend is indicated by a negative S. Another statistic obtained from the test is the

Kendall's tau, which measures the strength of the dependence between two variables. A positive value of Kendall's tau shows that the variables' ranks increase together while a negative value shows that as one variable's rank increases, the other variable's rank decrease.

2.2 Independence

The basic assumption is that the residuals of an ARIMA model are white noise. A white noise series have uncorrelated random shock with zero mean and constant variance. If the residuals are independent, it means that there is no more information that could be extracted from the series. One of the ways to determine the independence is to visually inspect the correlogram of the residuals. If the correlogram shows values that are close to zero, the residuals are uncorrelated and independent.

2.3 Homoscedasticity

Homoscedasticity is the term used to define that the variance of the disturbance term in each observation is constant. If the residuals are homoscedastic, their variances are stable. The probability of the disturbance terms reaching a given positive or negative value will be the same in all observations, which means that they have the same dispersion.

2.4 Transformation

Many statistical analyses are done based on the assumption that the population being investigated is normally distributed with a common variance. In situations where the relevant assumptions are violated, a few options are available:

- i. Ignore the violation of the assumptions and continue with the analysis;
- ii. Decide on a correct assumption in place of the violated one and proceed with the new assumption taken into account;
- iii. Design a new model that retains the important aspects of the original model and satisfies the assumptions;
- iv. Select a distribution-free method that can be used even if the assumptions are violated.

Most researchers have opted for the third option which includes applying a transformation to the original data. One of the popular transformation methods is the Box-Cox transformation. In ARIMA modelling, if the normality assumption for the residuals is not true, it is usually well satisfied when a Box-Cox transformation is done onto the original observations [14].

2.5 Forecasting

Forecasting can be categorized into short-term forecasting and long-term forecasting. Short-term forecasting can predict values that are a few time periods (a few years) into the future. Long-term forecasting on the other hand, can predict values for time periods that extend far beyond that. In terms of applications, long-term forecasts are used for strategic planning while short-term forecasts are used for project developments as well as operation management. Statistical methods are good for short-term forecasting because the historical data normally exhibit inertia and do not show drastic changes [15]. Short-term forecasting is based on identifying, modelling and extrapolating the patterns found in the data.

3. METHODOLOGY

The general ARIMA (*p,d,q*) model is:

$$U_t = \phi_1 U_{t-1} + \phi_2 U_{t-2} + \dots + \phi_p U_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \dots - \theta_q \epsilon_{t-q} \tag{1}$$

$$U_t = X_t - X_{t-d} \tag{2}$$

ϕ_p = autoregressive parameter

ϵ_t = residual

θ_q = moving-average parameter

X = dependent variable

U = d-th difference of the dependent variable.

3.1 Plotting the Series and Its ACF and PACF

The main tools used for identification of model were the visual displays of the series, which included the autocorrelation function (ACF) and the partial correlation function (PACF). By using the annual maximum stage readings as the input time series, the autocovariance function (c_k), the autocorrelation coefficients (r_k) and the partial correlation coefficients ($\hat{\phi}_k(k)$) were calculated and the series with its ACF and PACF were plotted using XLSTAT. The number of lags k should fall between $N/4$ and N , therefore the chosen number of lags in this study was sufficient.

$$c_k = \frac{1}{N} \sum_{t=1}^{N-k} (x_t - \bar{x})(x_{t+k} - \bar{x}), \quad 0 \leq k \leq N \tag{3}$$

$$r_k = \frac{c_k}{c_0} = \frac{\sum_{t=1}^{N-k} (x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^N (x_t - \bar{x})^2} \tag{4}$$

$$\hat{\phi}_{k+1}(k+1) = \left[r_{k+1} - \sum_{j=1}^k \hat{\phi}_k(j) r_{k+1-j} \right] / \left[1 - \sum_{j=1}^k \hat{\phi}_k(j) r_j \right] \tag{5a}$$

$$\hat{\phi}_{k+1}(j) = \hat{\phi}_k(j) - \hat{\phi}_{k+1}(k+1)\hat{\phi}_k(k-j+1) \quad (5b)$$

The ACF and PACF were then analysed to determine behaviour and stationarity of the series. If all the ACF and PACF values are insignificant and fall within the confidence band, it indicates that the observations are independent. In such a case the time series is a white noise process and no modelling could be performed. A stationary time series has a rapidly decaying ACF. If the ACF is slow decaying, it indicates that the series may be non-stationary and requires differencing. Further tests should be carried out to confirm the non-stationarity.

3.2 Stationarity Tests

Unit root tests such as the Augmented Dickey-Fuller (ADF) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test were carried out to test the presence of a unit root while the Mann-Kendall trend test was performed to check for the presence of a trend. The presence of a unit root or a trend should indicate non-stationarity of the series. The significance level used was 5%. If the series is non-stationary, differencing is required to transform it into a stationary series. On the other hand, if the series is stationary, the series is modelled as an ARMA process instead, which requires no differencing.

3.3 Differencing

The series was initially differenced once ($d = 1$) and the ACF and PACF of the differenced series were plotted and analysed. If the ACF and PACF decay rapidly then it indicates stationarity is achieved. Another indicator is the standard deviation of the differenced series. The optimum differenced series should have the lowest standard deviation. The differenced series was then differenced again ($d = 2$) to check for under-differencing or over-differencing. Similarly, the ACF and PACF were plotted and analysed. The lag 1 ACF and PACF of an over-differenced series will be lower than negative 0.5. If the standard deviation of the current series is lower than that of the previous series, then the current series has the optimum order of differencing. It is noteworthy that some researchers argue that the effect of over-differencing is much less serious than the effect of under-differencing.

3.4 Identifying p and q

Having identified the optimum order of differencing (d), the next step was to identify the order of the autoregressive and moving-average parameters. The ACF (symbolized as ρ_k) and the

PACF for the optimum differenced series were analysed to determine the p and q .

3.5 Choosing the Best ARIMA Model

The previous step gave an indication of the order of p and q that should be fitted in the model. However, it was recommended to try a few different values of p and q to get the best model while preserving the parsimony of the parameters. To test for the parsimony of parameters, the corrected Akaike Information Criteria (AICC) was used. The model with the minimum AICC was selected as the best model. The XLSTAT software can find the best model based on the AICC values calculated for a range of p and q . In this study the maximum p selected was 3 and the maximum q selected was also 3. The model with the minimum AICC was then subjected to diagnostic checks.

3.6 Diagnostic Checks

After the best initial model was determined, the next step was running the diagnostic checks. Its purpose was to verify the proposed model's validity. Before any checking was done onto the residuals, the values of the estimated ARIMA parameters first have to be in an interval computed using the Hessian standard errors. If the values are out of that interval, then they are not significant and the ARIMA model should not be used. The first checking on the residuals was to test for independence so that the residuals at any lag will not affect the value of residual at the next lag. The next criterion that required checking was residuals homoscedasticity, which means having a stable set of variances and then third checking was done to determine whether the residuals' distributions are approximately normal. The residuals have to be approximately normal in order to produce a good forecast confidence interval.

3.7 Series Comparison and Forecasting

The best model that passed the diagnostic checking will then have its synthetic series compared to the original data series. This determined the degree of resemblance between the synthetic series and the original data series. If the pattern of the synthetic series appears similar to the pattern of the original series, then the fitted model is a good model. The final step was to generate a forecast of future values. The ARIMA model can predict future values as well as its confidence interval using the calculated model parameters. In this study the chosen number of forecasted values was eight, which means that the values were forecasted for the next eight years after the last observation.

4. RESULTS AND DISCUSSION

The ACF and PACF for Dengkil , Kg. Lui and Kg. Rinching station are presented in Fig. 2, Fig. 3 and Fig. 4, respectively.

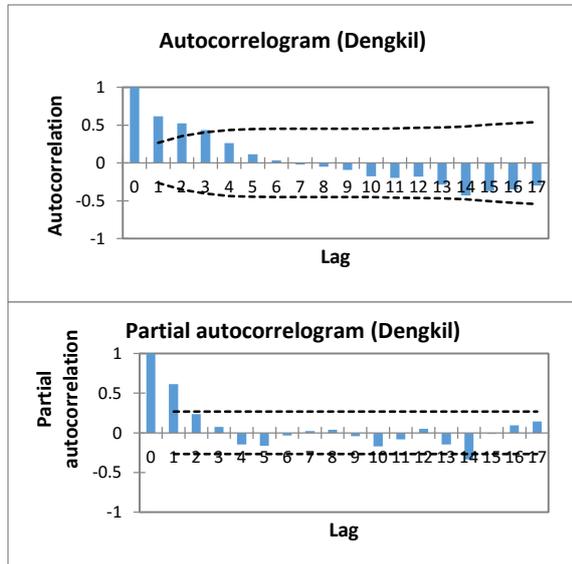


Fig. 2: ACF and PACF of Dengkil Series

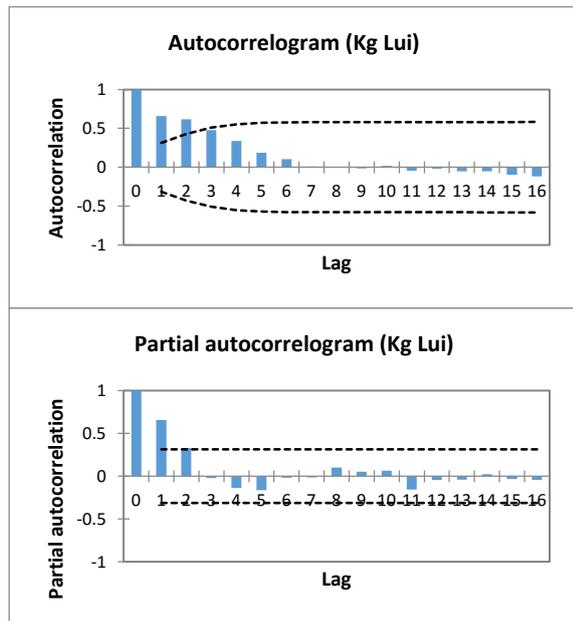


Fig. 3: ACF and PACF of Kg.Lui Series

The ACF plots for the Dengkil series, the Kg. Lui series and the Kg. Rinching series exhibited slow decay, indicating the possibility of non-stationarity. Since all the ACF and PACF values are significant and do not fall within the confidence band, it indicates that the time series are not white noise processes and modelling could be performed.

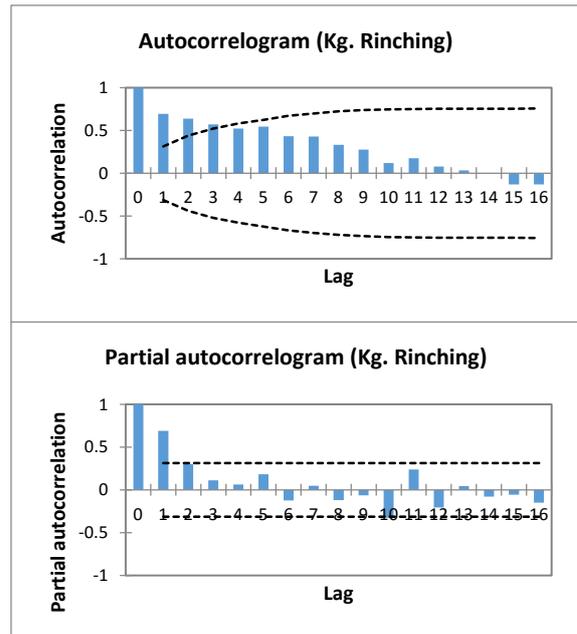


Fig. 4: ACF and PACF of Kg.Rinching Series

4.1 Stationarity Tests

Stationarity tests were carried out for the remaining three series to confirm the initial presumption that they were non-stationary. The results for the ADF test, KPSS test and Mann-Kendall trend test are presented in Table 2.

Table 2: Results of Stationarity Tests

| Station | ADF test | KPSS test | Mann-Kendall trend test | Remarks |
|--------------|----------|-----------|-------------------------|----------------|
| | p-value | p-value | p-value | |
| Dengkil | 0.350 | 0.001 | 0.438 | Non-stationary |
| Kg. Lui | 0.138 | 0.005 | 0.072 | Non-stationary |
| Kg. Rinching | 0.411 | 0.030 | <0.0001 | Non-stationary |

The tests confirmed that all the data series were non-stationary. The Augmented Dickey-Fuller test and the KPSS test showed that all three series had unit roots. The Mann-Kendall test also detected a trend in the Kg. Rinching series. A series that has either a unit root or a trend was considered as non-stationary and therefore required differencing.

4.2 Differencing the Series

The series were differenced once and twice to obtain the optimum d . The standard deviations of the original and differenced series are shown in Table 3.

Table 3: Standard Deviations of Original Series and Differenced Series

| Order, d | Dengkil | Kg. Lui | Kg. Rinching |
|------------|---------|---------|--------------|
| 0 | 1.035 | 0.597 | 0.658 |
| 1 | 0.903 | 0.490 | 0.481 |
| 2 | 1.521 | 0.858 | 0.829 |

The ACF and PACF of the once differenced ($d = 1$) series decayed rapidly compared to the ACF and PACF of the original series. Comparing the standard deviations of the series, the minimum standard deviations were obtained from the series with $d = 1$. The results also showed that the first lags of the twice differenced ($d = 2$) series were lower than -0.5 , indicating over-differencing. Therefore, the optimum level of differencing for the three series was one and the d value used in the ARIMA model would be one.

4.3 ARIMA Modelling and Diagnostic Checking

The AICC for ARIMA models were computed with p starting from one to three and q starting from zero to three. The models tested were (1,1,0), (1,1,1), (1,1,2), (1,1,3), (2,1,0), (2,1,1), (2,1,2), (2,1,3), (3,1,0), (3,1,1), (3,1,2) and (3,1,3). For each station, the model having the minimum AICC was chosen as the best model. The best models along with their estimated parameter values are tabulated in Table 4. The results showed that the preliminary models determined from the ACF and PACF of the differenced series were indeed the best models

Table 4: Best ARIMA Models

| | Dengkil | Kg. Lui | Kg. Rinching |
|------------|---------|---------|--------------|
| Best model | (1,1,0) | (1,1,0) | (1,1,1) |
| AICC | 133.736 | 55.348 | 42.292 |
| MSE | 0.672 | 0.169 | 0.137 |
| AR(1) | -0.395 | -0.532 | 0.241 |
| MA(1) | - | - | -1.000 |
| Constant | -0.023 | 0.044 | -0.047 |

The Hessian standard errors were calculated and all the estimated parameters successfully fell within the significance interval. The RACF and RPACF for the best ARIMA models were plotted. The RACF and RPACF for all the three series fell within the confidence interval. They were not significant and this showed that the residuals were independent,

therefore satisfying the first residual criterion. The next requirement was residuals' homoscedasticity and Table 5 shows the results of Breusch-Pagan test. Fig.5 shows the distribution of the standardized residuals for the Dengkil series.

Table 5: Results of Breusch-Pagan Test

| Station | p-value | Remarks |
|--------------|---------|---------------|
| Dengkil | 0.145 | Homoscedastic |
| Kg. Lui | 0.195 | Homoscedastic |
| Kg. Rinching | 0.747 | Homoscedastic |

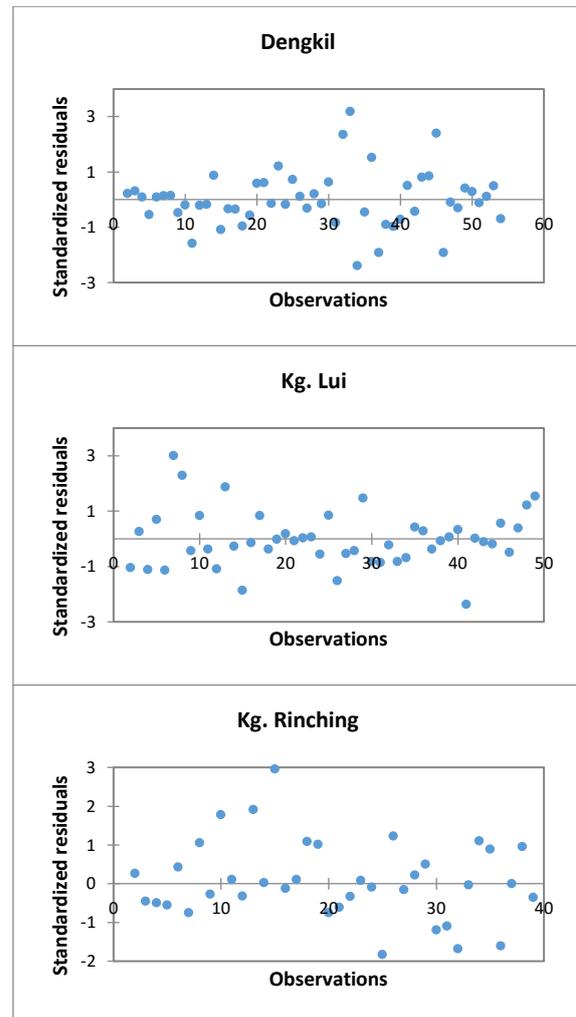


Fig. 5: Distribution of Standardized Residuals

The residuals were homoscedastic which mean that they had constant variances. It was important for the residuals to be homoscedastic because it determined whether the model's ability to predict variable values was consistent. A model with heteroscedastic residuals cannot give results that are trustworthy and transformation of the data is required. The third criterion for diagnostic checking was the distribution of the residuals. The residuals

were subjected to normality tests and histograms were also plotted to give a visual representation of their distributions. The results of normality tests are presented in Table 6, while the histograms are shown in Fig. 6.

Table 6: Results of Normality Tests

| Station | Shapiro-Wilk test | Anderson-Darling test | Jarque-Bera test |
|--------------|-------------------|-----------------------|------------------|
| | p-value | p-value | p-value |
| Dengkil | 0.017 | 0.012 | 0.007 |
| Kg. Lui | 0.140 | 0.066 | 0.064 |
| Kg. Rinching | 0.315 | 0.223 | 0.331 |

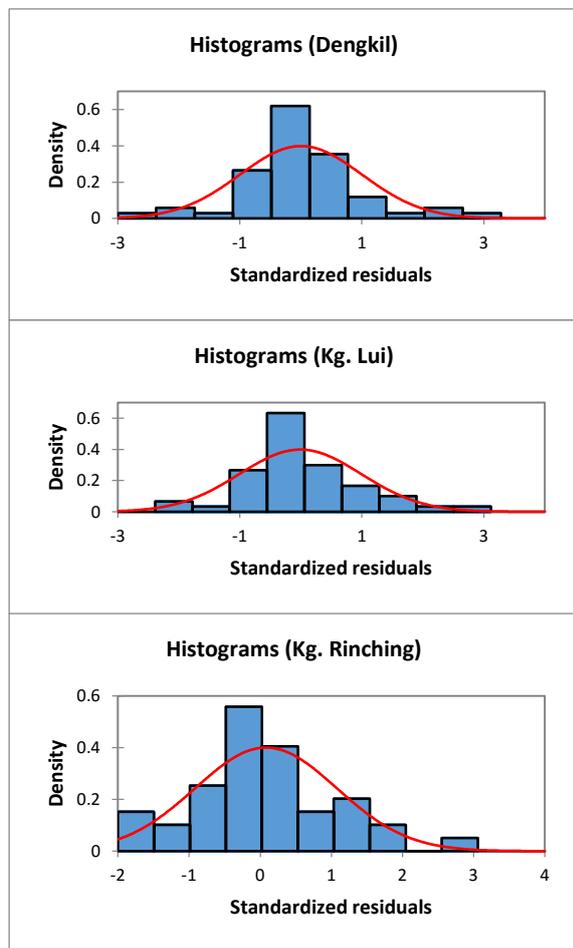


Fig. 6: Histograms of Residuals

Both the normality tests and histograms showed that the Kg. Lui series and the Kg. Rinching series had normally distributed residuals. The Dengkil series however, failed the normality tests but its histograms showed that it was very close to a normal distribution, which was good enough. The normality of residuals' distribution was important to produce a satisfactory confidence interval for the forecast.

4.4 Comparison of Series and Forecasting

The synthetic series generated by the ARIMA models were compared to the original series to check for model accuracy. Forecast series were also generated for a lead time of eight years with 95% confidence intervals. Fig. 7 shows the original series, the synthetic series and the forecast series for the three stations.

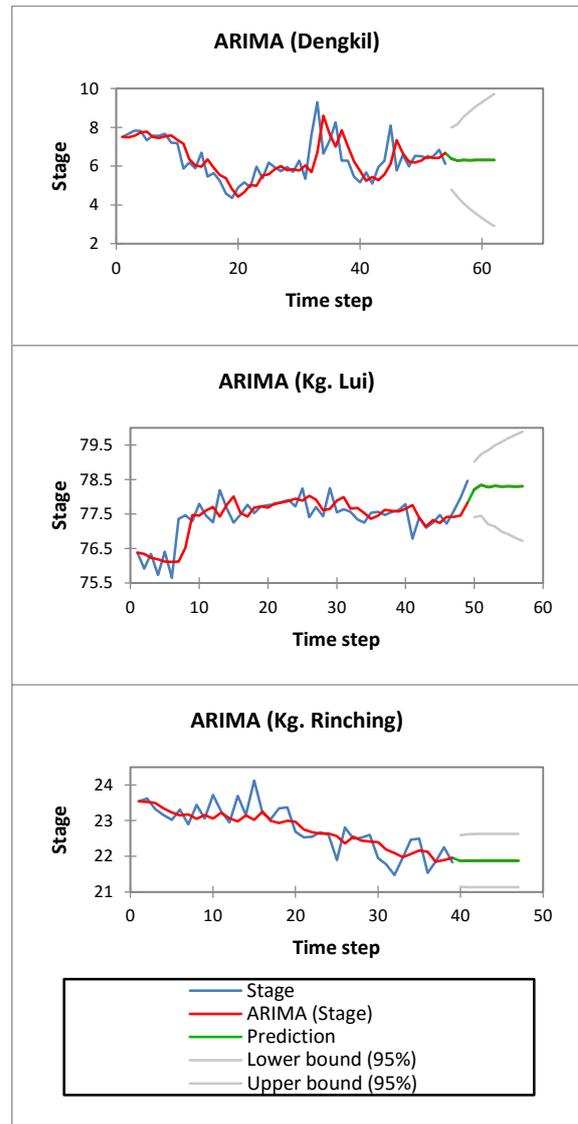


Fig. 7: Original, Synthetic and Forecast Series

5. CONCLUSION

Statistical modelling was successfully performed onto the study rivers using the autoregressive integrated moving-average (ARIMA) method. Forecast series were also generated by the models to give sequences of future stage and streamflow values. The best ARIMA models for the other three series, Dengkil, Kg. Lui and Kg. Rinching series

were (1,1,0), (1,1,0) and (1,1,1) respectively. The ARIMA model is suitable for short term forecasting because the ARMA family models can model short term persistence very well.

In conclusion, the Box-Jenkins approach for ARIMA modelling was found to be appropriate and adequate for the rivers under study in Langat River Basin. The flood forecast up to a lead time of eight years for the three models exhibited a straight line with near constant streamflow values showing that the forecast values were similar to the last recorded observation.

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