

THE APPLICATION OF CELLULAR AUTOMATA TO INVESTIGATE RUNOFF ON SURFACE OF COMPLEX TOPOGRAPHY UNDER DIFFERENT RAINFALL SCENARIO

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ABSTRACT: Overland flow development on a steep slope area is the major source of non-point pollution from rainfall event to receiving surface water bodies. The mechanism of runoff is a complex and non-linear system that can be influenced by infiltration excess, saturation excess, and subsurface flow. Because of these limitations of hydrological measurement techniques, the true complexity and heterogeneity of the hydrological process in the fields, the quantities of overland flow under rainfall are not fully to define despite their importance. In this research, we present an application of Cellular Automata (CA) in a computation model to simulate surface runoff on the complicated shape of the hillslope. The first contribution of research was applying the wall boundary and flooded condition into CA method to ensure the accuracy of the method physically. Besides, modified CA-rules were also developed to correct and speed up the implementation. In addition, these factors that effect to the behavior of runoff such as rainfall regime, an interception from covered vegetation (plant, tree, grass, bush), infiltration based on soil characteristic, and roughness of surface were also considered in the CA rules of computation model. The output of the mathematical model succeeded could be promising to treat rainfall-runoff on the real field. The accuracy of CA runoff method is also investigated and proved through convergence study.

Keywords: Cellular automata, Hillslope, Rainfall, Runoff, Overland flow

1. INTRODUCTION

Runoff is an important subject which causes environmental consequences such as flood, landslide, and soil erosion. The formation of overland flow on a steeply sloped area is denoted as a complex and non-linear system that might be influenced by infiltration excess, saturation excess, and the movement of subsurface water [1-3]. Overland flow is unstable and spatially varies as it comes from rainwater and decreases due to infiltration, where the two processes are not constant over time and location. The basis used to quantify the transformation from rainfall to overland flow is the main objective of the study to date.

Runoff generation processes are controlled by various environmental factors such as soil characteristics, climate, vegetation, and land cover. This induces these difficulties in comparing quantitatively hydrological responses in a variety of different environmental conditions [4]. Land cover influences the formation of overland flow on a steeply sloped area, where the soil with less dense vegetation tended to show a faster formation rate of overland flow compared to the soil with denser vegetation cover naturally [5]. As in [6] author indicated in his research that rainfall intensity, slope, and land cover types can also affect the soil moisture dynamics which led to overland flow formation. A

comprehension related to the mechanism of overland flow formation on steeply sloped area is required to establish a model of runoff and erosion that can assist researchers in determining appropriate management to reduce the amount of runoff and sediment to water bodies. It encourages research was conducted to ascertain the type of the mechanism of runoff generation in accordance with the characteristic of the steep-slope area.

Investigation of the rainfall-runoff process may bring us to the early prediction of these serious concerns. It is harder to define precisely overland flow than in-channel flow, and the use of hydraulic procedures in predicting overland flow and its characterization are faced with limitations [7]. The field researches to study about runoff are the practical and exact method by doing statistics. However, it is limited by a certain area and it will take time for investigation due to dependence on the weather and the huge size of the real field. The other manner as known as the small-scale of a real field that the weather and size conditions could be handled under control of laboratory. Nevertheless, this method is only suitable for some areas which scaled.

Since the limited monitoring and field survey, there are different types of simulation methods used in the analysis of the rainfall and run-off process was developed. In recent years, most interest in the

simulation of catchment behavior has been concentrated in the indirect type of simulation associated with mathematical catchment models. However, there are still simulation models could not be implemented at this time, such as the Stanford Watershed Model because of computation capacity and data limitations. Until 1970, with the digital revolution, the power of computers increased exponentially and, as a result, advances in watershed hydrology have occurred at an unprecedented pace during the past 35 years. Many of the advances after 1973 were due to improvements in computational facilities or new measurement techniques [8]. Last but not least, using a numerical method brings many benefits. The numerical approach can be implemented onto the various shape of the area, even any real field. By utilizing computation, it is easy and quick to predict the environmental phenomena. Cellular-Automata (CA), a numerical method which can predict the next state of a phenomenon by employing a set of rules applies on discretized lattice cells of a domain. Thus, this CA can be the very promising numerical way in order to investigate the rainfall-runoff.

As in [9] found that almost CA models were primarily developed to study the landform evolution patterns but not to predict explicit quantitative estimations of runoff yields although in reality. CA model could simulate a complex geographic phenomenon through simple local interaction rules. The simulation accuracy and the computational complexity were determined by the size of the spatial cell, the selection of hydraulic parameters, the length of a time step, and the iteration times (runs) of the model. How to improve the performance of CA models and how to assess the effects of time step length and the number of iteration runs on the simulation results should be further studied for applying the CA model to solve the explicit runoff and soil erosion problems.

In this study, the mathematical model was developed to estimate the quantities of overland flow generation. Besides, model results help to increase the understanding of the effect of rainfall, soil characteristics, climate, vegetation, and land cover on runoff mechanism. In this model, the role of Cellular Automata (CA) explored and developed to emulate the alterations of water quality in runoff according to data of the hypothetical case of complex topography, rainfall, and runoff.

2. CA IMPLEMENTATION IN RUNOFF PROBLEMS – A DESCRIPTION

In this section, we present the application of the CA method on Rainfall-runoff problem. By following, we propose an algorithm that we can use to simulate the rainfall-runoff problem, as illustrated in Fig.1.

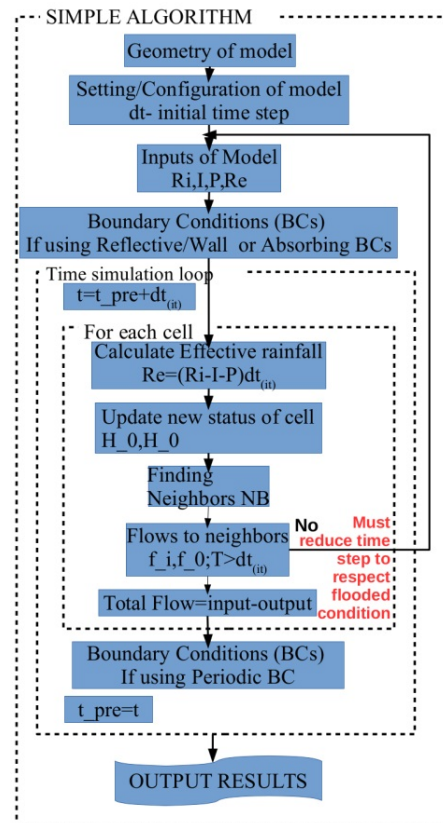


Fig.1 CA Rainfall-Runoff algorithm

2.1 Initial Configuration

Before going to detail about methodology, we consent that the whole investigated domain is discretized into a set of cells. Each cell contains the cell information including cell elevation $H(t=0)$ and water surface elevation h .

The first process of the algorithm (Fig.1) is the set of input geometry data of the model. For the numerical investigation, we need to choose a geometry of slope, which consist the sizes, height, a surface slope of a model, cell size of a discretized domain (L), and time step to observe the changing of the model (the second process of the algorithm).

Besides, factors that affect the behavior of problems are also considered such as rainfall regime, an interception from covered vegetation, infiltration based on soil characteristic, and roughness of a surface. For simplification, rainfall regime, interception parameter, infiltration parameter, and roughness coefficient were used as a constant in a short period of time simulation. Practically, our developed code is available for the real various rainfall databases, interception changed in time by the method LISEM [10], and infiltration prediction based on empirical infiltration models, such as the Philip equation (1957), Green and Ampt equation (1911), Horton equation (1940) and Holtan equation

(1961). For the more detail, we refer the reader to the literature [11].

As we known, boundary conditions (BCs) decide the accuracy of the numerical method. Due to the rainfall-runoff problems, we realize that there are three kinds of BCs could be applied to the model. For the huge field with the regular shape of the land, we can employ periodic BCs, and save computation by solving the small and representative area. At this point, the output water from this side of the boundary will be the input water for the opposite side and vice versa. In the field area, the absorbing BCs should be engaged for such a semi-isolate system that water can leave from any side of a boundary, and there is no entered water from outside boundaries as an input.

The output flow will be deleted from memory. In our model testing, the wall-reflective BCs (higher cells) is utilized surrounding the considered area in order to ensure for an isolated system that water cannot go through the boundaries, but it will be collected and accumulated at somewhere. For example, the flooded area or the downslope of steep slope area near the water body. At this deal, we can easily collect the total output (discharge) flow of the field by using some lower cells at bottom of the area. In order to represent the roughness surface, we use the Manning's roughness coefficient ($n = 0.01$ for our tests), will be more detail in the next section. Since all, the effective rainfall can be obtained before each execution time step $Re = (R_i - I - P) dt$ and then the new status of each cell for the next time step is updated.

2.2 Set of Rules

As the previous discussion, using the CA method then the domain of the whole field will be discretized into lattice cells, and each cell brings its own information (elevation of cell and water surface). The transporting flows between cells will be considered and treated by following rules.

The first rule was applied in the model for searching satisfied neighbors of each cell (Fig.2) that to ensure the water flows from the higher level to the lower level until both of the levels are the same. In order to do that, the average considered cell and neighbors were used. First, the list of neighbors follows the rule of 8N, 4N or 4N+N. Then, the calculated average is compared to each neighbor in the list. If the neighbor has a higher level than the average value, that neighbor will be deleted from the list.

The process is continued until has no neighbor be removed. Note that the list following 8N rule allows water from the considered cell can move to any cell in eight around it. Meanwhile, the 4N rule only allows water from considered cells can flow to horizontal and vertical cells around it. The 4N+4

rule prioritizes for horizontal and vertical cells, if there is no satisfied cell, then it allows the rest.

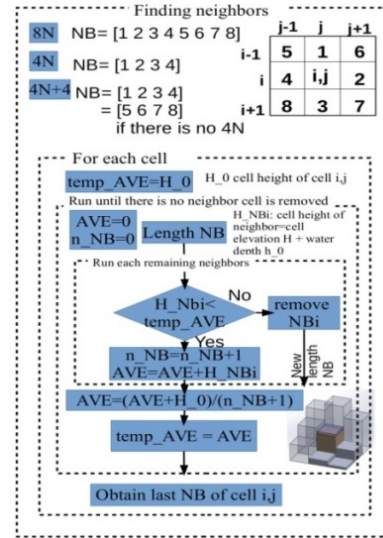


Fig.2 Finding neighbors

The second rule was considered for distribution of flow to the satisfied neighbors. In order to distribute the water to satisfied neighbors, we need to calculate the velocity of each flow from considered cell to its neighbors by using Manning's equation which based on water surface slope (s), water elevation (h) and the Manning's roughness coefficient (n)

$$V = \frac{h^{2/3} s^{1/2}}{n} \quad (1)$$

Since then we can find the travel time of flow from considered cell to its neighbor

$$T = \frac{D}{V} = \frac{nD}{h^{2/3} s^{1/2}} \quad (2)$$

Where D refers to the travel distance of flow, it can be L (cell size) if water flows to a horizontal and vertical neighbor, or $\sqrt{2}L$ if water moves to diagonal neighbors.

Note that, to ensure flooded condition (see next section), the computational time step must always smaller than travel time. Then, by taking the ratio between computational time step and travel time, we can get the actual flow transferred to the neighbors, as illustrated in Fig.3.

The last rule was developed for contribution total flow. It is easy to calculate the balance water elevation of each cell by the contribution inflows from the higher level neighbor and the drainage outflows to lower neighbors (Fig.4).

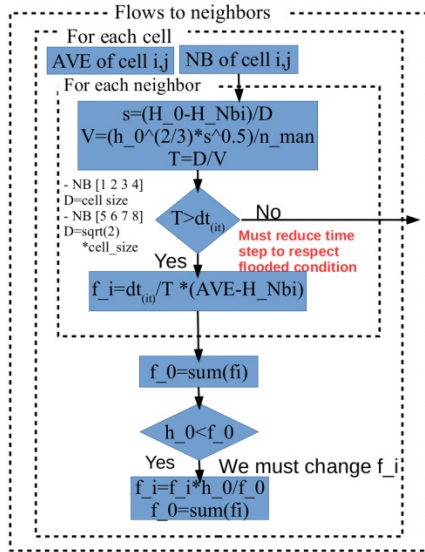


Fig.3 Distribution of flow to the neighbor

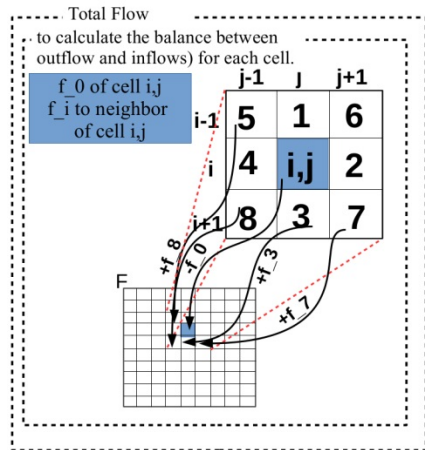


Fig.4 Contribution total flow

2.3. Flooded Condition – dt

If the traveling of flow is sufficient time for a certain time step ($T \leq dt$), it means the flow finish its traveling after T and then wait until finishing of dt. That is unphysical! Thus, we use travel time more than iteration time dt to make sure the flooded condition [12]. Then, the dt is chosen as follows $dt < 99\%T$.

Practically, we chose initial time step, and then we check the travel time T for each cell at each time step, if any T smaller than current dt, then dt will be recalculated to a smaller value. Vice versa, if T much bigger than dt (about 10 times) it will costly computation. At that point, we will increase the time step for saving computational cost.

3. NUMERICAL EXPERIMENTS BY SIMPLIFIED CONFIGURATIONS

In this section, an analytical solution using the kinematic wave modeling for runoff problems [13] will be referred for validation of the method. Considering plane with 500 m long, 100 m wide, and 5% slope where the surface roughness and rainfall regime were assumed constant in time and space. There are two kinds of effective rainfall will be applied to impervious and infiltration plane: 100 mm/h and 120 mm/h, respectively. And two regime rainfalls will be investigated: 25 minutes, and 5 minutes of rain durations for both cases (See Table 1).

Table 1 Cases configuration

Parameters	Case 1	Case 2	Case 3	Case 4
Field size (m x m)	100x500			
Slope	5 %			
Rainfall (m/h)	0.1	0.1	0.12	0.12
Rain duration (min)	25	5	25	5
Infiltration (m/h)	No	No	0.02	0.02
Resolution	100x500			
Simulation Time (min)	50	25	50	25
Boundary	Wall			
Flow-direction	4+4N			
Manning's coefficient n	0.01			

For numerical configuration, the cell size 1mx1m (means resolution 100x500] and the Manning's $n=0.01$ were set for our all simulations. The domain uses wall boundaries surrounding the main area which have higher cell elevation to prevent the water come out from the top, and cells with lowest cell elevation to collect discharged water. The 4+4N rule is used for flow-direction.

3.1. Results

The analytical solutions in [13] were used to compare to numerical results in CA method.

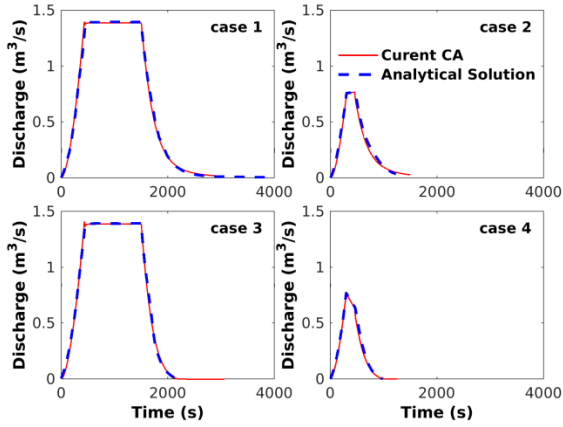


Fig.5 Numerical results of discharge compared to the analytical solution

Fig. 5 shows the good agreement of the current method with reference. The discharge of cases one and three reach the stable state at the same value $1.4 \text{ m}^3/\text{s}$ during the period of 7th -25th minute of rainfall duration, after that they reduced and exits from saturation stage of rainfall. For case two, we also see the stable state of discharge at value $0.78 \text{ m}^3/\text{s}$ during the short period right after rainfall stopped from 5th minute to around 7th minute. However, case four result shows the discharge decreased right after rainfall stop due to soil infiltrating that makes the water on surface released. In order to get more understanding about the saturation stage of rainfall, we investigate the result in term of runoff-rate as illustrated in Fig.6. It was easy to see that the runoff rate results are quite similar to discharge results. However, in this view, we can easily realize that at the saturation stage of case one and three, the runoff rate value was equal to rainfall rate of 100 mm/h . Obviously, when the saturation stage has achieved the speed of water input is the same with water output. Look at case two, the stable state has also happened, but the value of runoff rate is about 55 mm/h , smaller than rainfall-rate (100 mm/h). Hence, we can call that status is counterfeit saturation.

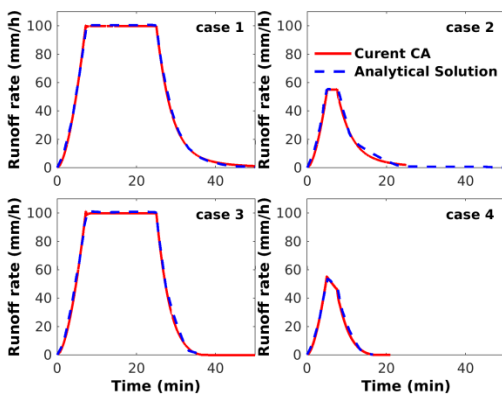


Fig.6 Numerical results of runoff rate compared to the analytical solution

3.2. Convergence

As we know, the iterative method is convergence if the number of computation iteration increase then we get a small error. However, the discretization method is convergence if the particle-size and time step reducing may lead the result getting close to the exact solution.

Based on that, in order to study the convergence of method, we investigate the various resolutions for the case of impervious plane 500 m long, 100 m wide, 5% slope, and $n = 0.01$ under 5 minute and 25 minute of rainfall duration (Case 1 and Case 2 in Table 1). The resolutions that we used $[2 \times 10]$, $[5 \times 25]$, $[10 \times 50]$, $[25 \times 125]$, $[50 \times 250]$, $[100 \times 500]$ corresponding to cell-size $50 \text{ m} \times 50 \text{ m}$, $20 \text{ m} \times 20 \text{ m}$, $10 \text{ m} \times 10 \text{ m}$, $4 \text{ m} \times 4 \text{ m}$, $2 \text{ m} \times 2 \text{ m}$, $1 \text{ m} \times 1 \text{ m}$ (getting finer).

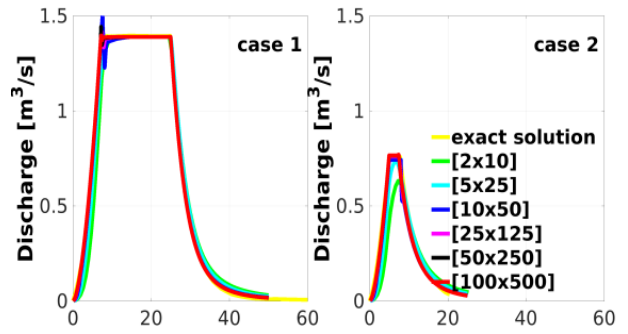


Fig.7 Numerical results of discharge at various resolutions compared to the analytical solution

For the case of 5 minutes rainfall duration, when the cell-size finer we get the better result that close to the analysis solution. But for the case of 25 minutes, it is difficult to follow and conclude (Fig.7). Due to that problem, we consider the error of each simulation by using absolute error, as in

$$e = |\tilde{\alpha} - \alpha| \quad (3)$$

Here, $\tilde{\alpha}$ refers to a numerical (approximated) solution, α is the exact solution.

If α is nonzero, we can use relative error, as in

$$e = \frac{|\tilde{\alpha} - \alpha|}{|\alpha|} \quad (4)$$

The Fig.9 shows that relative errors were quite big at the beginning and after rainfall, but the absolute errors (Fig.8) was so small after rainfall if they were compared to 100 mm/h of rainfall rate. For all cases, the relative error is smaller than 5% at the middle and finer resolution gives a better result.

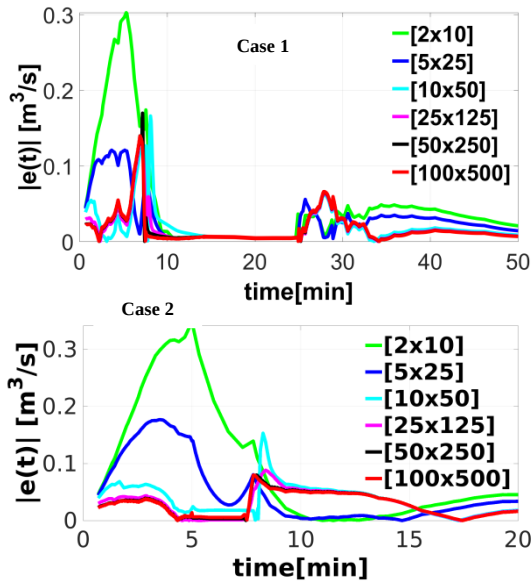


Fig.8 Comparison of absolute error of discharge between numerical results and analytical solution at a different resolution for case 1 and case 2

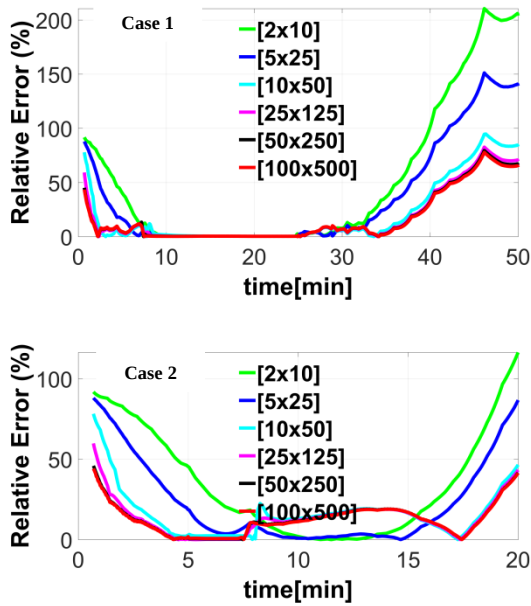


Fig.9 Comparison of relative error of discharge between numerical results and analytical solution at a different resolution for case 1 and case 2

Then we can conclude that the accuracy of the method is acceptable. However, to know more detail about the effect of the resolution to the accuracy of the method, the convergence orders by using the norm of the error were investigated. Convergence orders were determined by computing the L1, L2, and L ∞ of the norm for the relative error, as in

$$L^P(e) = \|e_p\| = \frac{\|\tilde{\alpha} - \alpha\|}{\|\alpha\|_p} \quad (5)$$

Where for any vector x:

$$L^P = \|\underline{x}_p\| = \left(\sum_{i=1}^n |x_i|^P \right)^{\frac{1}{P}} \quad (6)$$

It means one-norm:

$$L^1 = \|\underline{x}_1\| = \sum_{i=1}^n |x_i| \quad (7)$$

two-norm:

$$L^2 = \|\underline{x}_2\| = \left(\sum_{i=1}^n |x_i|^2 \right)^{\frac{1}{2}} = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2} \quad (8)$$

infinity norm:

$$L^\infty = \|\underline{x}_\infty\| = \max(|x_1|, |x_2|, \dots, |x_n|) \quad (9)$$

Fig.10 shows that bigger resolution (finer cell-size) leads to a smaller value of norm of relative error (better accuracy). For the case of 25min rainfall regime, we can see the early convergence at a resolution of [25x125] with the order 5.10-2 (L1 and L2). L ∞ is quite big due to taking maximum of error.

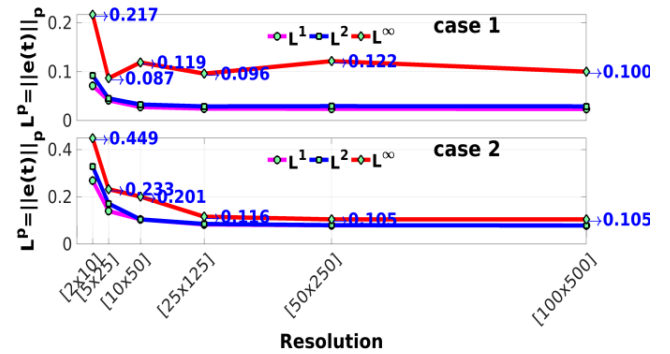


Fig.10 Norm of relative error

We can conclude that the method is convergence. The method can apply onto the complex shape of areas and get a better result by using finer resolution. Nevertheless, better resolution leads to more expensive computation. In order to get more detail about computational cost, the computational speed was studied in the next section.

3.3. Computational Speed

All simulation was run with computer core i7, 4Gb RAM. The computational time that spent on various resolutions were shown in Table 2. Computational time for 50min of investigation may take double times to 25min of the investigation.

Table 2 Computational speed of various resolutions

Resolution	Case 1 ^a	Case 2 ^b
[2x10]	3s	4s
[5x25]	15s	30s
[10x50]	30s	1 min
[25x125]	7 min	>15 min
[50x250]	35 min	>1.5 h hour
[100x500]	>10 hour	>24 h hour

a: Rainfall: 5 min, Simulation duration: 25 min

b: Rainfall: 25 min, Simulation duration: 50 min

The most expensive process in the algorithm (Fig.2) is the second rule due to the searching neighbors. The searching neighbors process took 40% of total time computation. The first and third rules took the same 26-28% of total time computation. Other processes took only 4-5% of the total computational time (see Fig. 11).

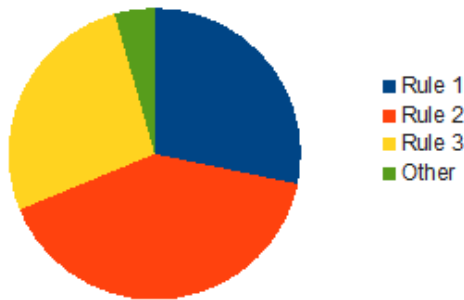


Fig.11 Computational speed for each rule

4. A HYPOTHETICAL CASE OF COMPLEX TOPOGRAPHY - A PERSPECTIVE

Using the FFT function in Matlab 2013, we create randomly a topography map (Fig.12) with the size of 50mx50m, resolution [50x50], slope 30%, infiltration rate of 20mm/h, and rainfall regime 2m/h during 5min, investigation in 10 minutes.

The simulation result is illustrated in Fig.13, from left to right and up to down, the rainfall has just started (Fig.13 a), water cover the land and concentrate at the bottom (Fig.13 b), fill up until the top (Fig 13 c), rainfall is stopped, the water on the surface is reduced (Fig 13 d).

For more comprehensible can find the video at:

<https://www.youtube.com/watch?v=lGkOPDFoAIE>

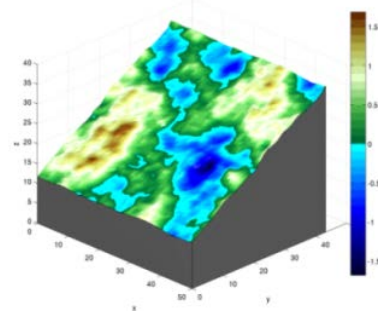


Fig.12 The random geometry of hillslope for testing

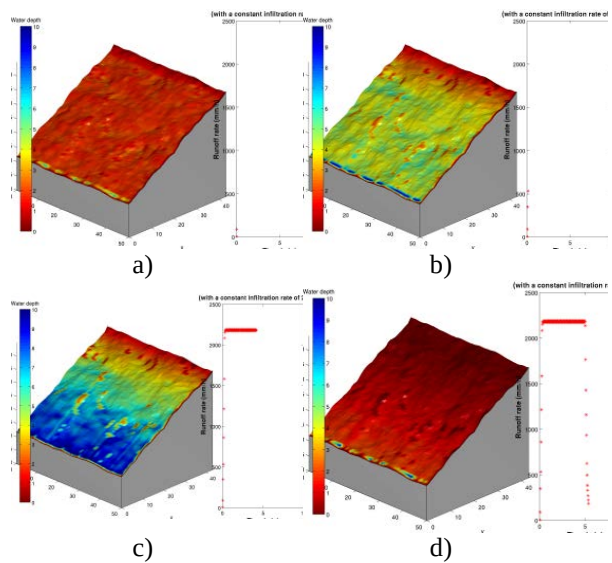


Fig.13. Historical recording of runoff flows

5. CONCLUSIONS

The numerical results of current CA are in good agreement with the analysis. The proposed method is convergence in term of resolutions. The finer resolution makes the result more accurate, however more expensive as well. The CA method is very promising for solving rainfall-runoff on the complex surface of geometry such as a real field.

6. ACKNOWLEDGMENTS

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