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PROPAGATION OF HARMONICAL VIBRATIONS IN PEAT

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ABSTRACT: In order to check the reliability of man-made vibration prediction methods, vibration tests were performed on one of polders in the North-West of the Netherlands. The polder was chosen because it has a rather homogenous, thick and soft peat top layer. Here sufficient harmonical vibrations could be generated by a rather small shaker. The shaker was designed and manufactured in order to produce harmonical vibrations at the soil surface. It consists of two counter rotating electric vibrators (with rotating eccentric masses) in order to produce a vertically oscillating force. For the recordings of the vibrations, six 2D or 3D geophones were placed on the soil surface and one 2D geophone was placed on top of the shaker. The measured vibration amplitudes of the vertically oscillating shaker were compared with 1. Two different analytical methods used for the design of vibrating machine foundations, 2. The Confined Elasticity approach and 3. The Finite Element Method, for which Plaxis 2D software was used. Also the measured vibration amplitudes at the soil surface were compared with Barkan-Bornitz's solution and Finite Element Modeling.

Keywords: Soil Vibrations, Man-made Vibrations, Vibration Propagation, Soil Waves

1. INTRODUCTION

For most developing countries the urban environment is getting larger and denser. Therefore civil engineers have to pay more attention to the effect of man-made vibrations. According to research by Hölscher and Waarts [2] the reliability of prediction methods for man-made vibrations is unfortunately disappointingly low. In order to check this conclusion, a field test was performed to measure the vibration propagation with an experiment on site.

In order to produce harmonic vibrations at the soil surface, a shaker was designed and manufactured. For the recordings of the vibrations, six 2D or 3D geophones were placed on the soil surface and one 2D geophone was placed on top of the shaker.

The measured vibrations of the shaker and the soil surface will be compared with analytical and numerical (FEM) solutions.

2. SITE LOCATION AND STRATIGRAPHY

In order to produce sufficient harmonic vibrations on the soil surface with a rather small shaker, the potential site for the field test should fulfil two major conditions: it should be rather homogeneous and rather soft. Therefore a peaty site in the Netherlands was chosen. The test site is located about 10 km North-East from Amsterdam in the village of Uitdam. The test area is marked by letter "A" in Fig. 1.

Near the test area, other research has been made before, related to the strength of peat [3]. The area used for the peat strength research is marked by letter "B" in Fig 1. In there, geological investigations have been carried out in May 2012. Three boreholes were drilled and they are marked by "c1", "c2" and "c3" in Fig. 1 and Fig. 2.

Fig. 1 Location of the test site (Google, Map data)

The top layer is a thin clayey layer with a thickness varying between 0.2 m and 0.5 m. Below this layer there is a peat layer of 4.5 m thick. It was reported that the bulk density of the peat layer $\rho = 0.98 \pm 0.08$ t/m^3 [3].

3. EQUIPMENT AND SETUP

3.1 Shaker Design

In order to make the shaker transportable, two small counter rotating electric vibrators (with rotating eccentric masses) have been used to produce a vertically oscillating force. This type of vibrator is frequently used in geotechnics (for example sheet pile driving or soil densification).

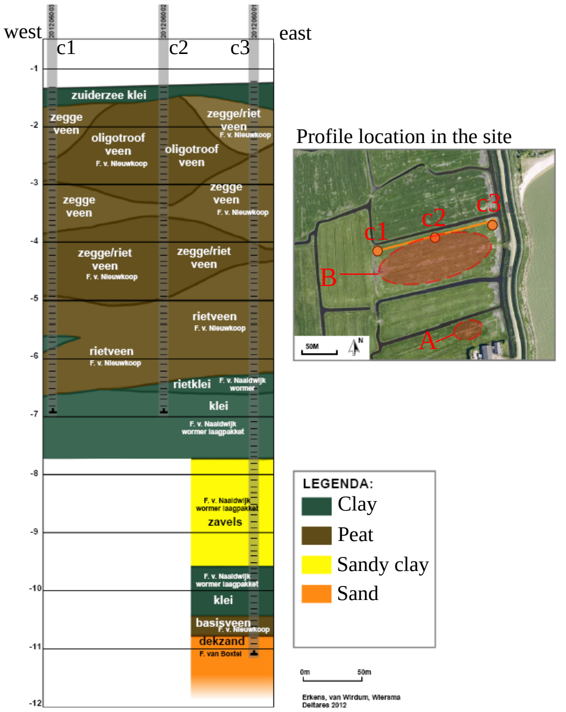

Fig. 2 Data from geological investigations [3]

The vibrators are connected to a plate of 40 cm in diameter and 2 cm in thickness. Additional square plates can be added on site to vary the total mass of the system. The whole is tightened together with bolts. This makes a total weight of the shaker variable up to a maximum of 300 kg (which makes the vertical gravity force 2.94 kN) and the vertical oscillating force (due to the rotating eccentric masses) can vary up to 2.06 kN.

3.2 Vibration Tests Setup

The equipment used for the vibration tests, consists of a: 1) Shaker; 2) Frequency inverter; 3) Power generator; 4) Geophones; 5) Data acquisition box; 6) Laptop. All these components with their corresponding numbers are shown in Fig. 3.

4. ELASTIC PROPERTIES OF THE SITE

In order to evaluate small strain stiffness parameters of the peat, pressure (P) and shear (S) wave velocity measurements were carried out. This was done by hitting the shaker with a hammer and measuring the arrival times at the geophones. The geophone on top of the shaker records the input wave.

The P-wave velocity was measured from the arrival time differences between of the first radial vibration peaks and the S-wave velocity similarly but of the biggest vertical vibration peaks.

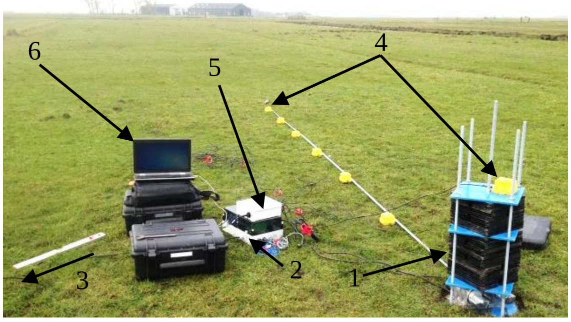

Fig. 3 Vibration test setup: 1) Shaker; 2) Frequency inverter; 3) Power generator; 4) Geophones; 5) Data acquisition box; 6) Laptop

The measured P-wave and S-wave velocities on site are respectively $v_p = 66.9$ m/s and $v_s = 17.4$ m/s. From the velocities of the body waves and the bulk density of the peat $\rho \approx 1$ t/m^3, according to Eq. 1, Eq. 2 and Eq. 3 the shear modulus $G = 303$ kN/m^2, the Poisson's ratio $\nu = 0.464$ and the Elastic modulus $E = 886$ kN/m^2 can be determined. The R-wave velocity can be calculated from the Poison's ratio and S-wave velocity: $v_r = 16.5$ m/s.

$$G = \rho v_s^2 \tag{1}$$

$$\nu = \frac{1}{2}\frac{\left(v_p / v_s\right)^2 - 2}{\left(v_p / v_s\right)^2 - 1} \tag{2}$$

$$E = 2G(1+\nu) \tag{3}$$

In order to compare the analytical solutions of the shaker vibration amplitude with the vibration amplitudes of the soil surface, the site will be modelled as an elastic isotropic homogenous half-space with the previously defined elastic properties.

5. SHAKER VIBRATION AMPLITUDES

In 1904 Lamb [4] solved the problem of the wave propagation in three dimensions (also known as the dynamic Boussinesq problem). Based on Lamb's work, Reissner in 1936 [5] first developed the vertical response of a uniformly loaded flexible circular area resting on an elastic half-space. The vertical displacement amplitude at the center of the flexible loaded area is, according to Reissner, defined by:

$$A_R = \frac{F_0}{G r_{pl}} \sqrt{\frac{f_1^2 + f_2^2}{\left(1 - b a_0^2 f_1\right)^2 + \left(b a_0^2 f_1\right)^2}}\ , \tag{4}$$

in which: F_0 – amplitude of the vertically exciting force; r_{pl} – radius of the loaded circular area; f_1, f_2 – displacement (compliance) functions; a_0 – dimensionless frequency; b – dimensionless mass ratio. The dimensionless frequency and mass ratio are calculated as follows:

$$a_0 = \omega r_{pl}\sqrt{\frac{\rho}{G}}\,, \tag{5}$$

$$b = \frac{m_{vib}}{\rho r_{pl}^3}\,, \tag{6}$$

where: ω – angular frequency; m_{vib} – total vibrating mass.

Bycroft [6] provided a solution for forced vibrations of a rigid circular plate attached to the surface of an elastic half-space for large values of the frequency (a_0 > 1.5, which is also the case in this article). Using Eq. (4), according to Kruijtzer [7] the compliance functions can be simplified to:

$$f_1 \approx \frac{3}{4a_0^2}\,, \tag{7}$$

$$f_2 \approx \frac{1.93}{a_0^3}\,. \tag{8}$$

Hsieh [8] modified in 1962 Reissner's solution and proved, that it is possible to have a mechanical analogue in a form of single-degree of freedom system. Later Lysmer and Richart [9] proposed a frequency independent mechanic analogue.

Lysmer modified the mass ratio:

$$B = \left(\frac{1-\nu}{4}\right)\frac{m_{vib}}{\rho r_{pl}^2} = \left(\frac{1-\nu}{4}\right)b\,. \tag{9}$$

And suggested an analogous spring stiffness K and damping ratio D.

$$K = \frac{4Gr_{pl}}{1-\nu}\,, \tag{10}$$

$$D = \frac{3.4r_{pl}^2}{1-\nu}\sqrt{G\rho}\,. \tag{11}$$

Using this, the vibration amplitude can be written as:

$$A_L = \frac{F_0/K}{\sqrt{\left[1-\left(\omega^2/\omega_n^2\right)\right]^2 + 4D\left(\omega^2/\omega_n^2\right)}}\,, \tag{12}$$

in which: ω_n – natural angular frequency of the mechanical analogue system:

$$\omega_n = \sqrt{\frac{K}{m_{vib}}}\,. \tag{13}$$

Another analytical solution was suggested by Verruijt [10]. He suggests to neglect here the horizontal displacements (while they are very small compared to the vertical ones), which is called the confined elasticity approach. This approach was first proposed by Westergraad [11] and generalised for elastodynamics by Barends [12]. In this way the vibration amplitude of a rigid circular plate on a confined elastic half space becomes:

$$A_C = \left|\mathrm{Re}\begin{pmatrix} \dfrac{\omega}{\omega_c}\dfrac{\exp(i\omega/\omega_c)}{\sin(\omega/\omega_c)} - \\ \dfrac{16B}{\pi m_c(1-\nu)}\left(\dfrac{\omega}{\omega_c}\right)^2 \end{pmatrix}^{-1}\right| A_s\,, \tag{14}$$

in which: ω_c – characteristic frequency; m_c – material constant, A_s – static displacement.

$$\omega_c = \sqrt{\frac{4G}{\rho r_{pl}^2}}\,, \tag{15}$$

$$m_c = \sqrt{\frac{2(1-\nu)}{1-2\nu}}\,, \tag{16}$$

The static displacement is defined as follows:

$$A_s = \frac{F_0}{\pi r_{pl}}\frac{m_c}{\lambda + 2G}\,, \tag{17}$$

in which λ – is the Lame constant:

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}\,. \tag{18}$$

These three methods described above will be compared to the measured vibration amplitude of the shaker on site.

6. VIBRATION AMPLITUDES OF THE SOIL SURFACE

In this article only the vertical vibration amplitudes at the surface will be investigated. Barkan [1] suggested to distinguish between the near-field and the far-field.

For the near-field, vertical vibration amplitudes on the surface may be determined by:

$$A_{nf,s} = \frac{F_0 \omega}{v_s G} \cdot \sqrt{f_{1,s}^2 + f_{2,s}^2}\,, \tag{19}$$

where $f_{1,s}$ and $f_{2,s}$ – displacement (compliance) functions for the surface. Barkan [1] advised to use displacement functions solved by Shekhter [13].

For the far-field, an assumption was made that vibrations are caused only by R-waves, giving the following vertical vibration amplitudes:

$$A_{ff,s} = \frac{k_r F_0}{2G} \cdot \frac{k_s^2 \sqrt{k_r^2 - k_p^2}}{\partial g_k / \partial k_r} \sqrt{\frac{2}{\pi k_r r}}\,, \tag{20}$$

in which k_p, k_s and k_r are P-, S- and R-wave numbers; g_k – function of wave numbers:

$$g_k = \left(2k_r^2 - k_s\right)^2 - 4k_r \sqrt{k_r^2 - k_p^2}\sqrt{k_r^2 - k_s^2}\,. \tag{21}$$

The distance, where $A_{nf,s} = A_{ff,s}$ is the end of the near-field and beginning of the far field.

Another method to predict vibration amplitudes on the soil surface was suggested by Bornitz [14]:

$$\hat{u}_1 = \hat{u}_0 \left(\frac{r_0}{r_1}\right)^n e^{-\alpha(r_1 - r_0)}, \tag{22}$$

where $\hat{u}_0$ is the amplitude of vibration at distance r_0 from the source, $\hat{u}_1$ is the amplitude of vibration at distance r_1 from the source, n is the geometrical damping factor, α is the material absorption coefficient. The benefit of the Bornitz's equation is limited, because the vibration amplitude $\hat{u}_0$ at the distance r_0 should be known a priori. However, the second part (i.e. exponential part) of the equation represents the material damping law which will be used together with Eq. 19 and Eq. 20. In this way the material damping will be considered. Based on Coelho [15] a material damping will be used for peat of D = 1 %.

Furthermore by assuming that, 1) most of the vibrations on the soil surface are caused by R-waves [16], [17], and 2) damping is frequency independent, the material absorption coefficient can be determined by:

$$\alpha = \omega D / v_r\,. \tag{23}$$

The field test was performed with frequency f = 24 Hz, therefore, the absorption coefficient here is α = 0.09 m^{-1}.

The Barkan-Bornitz equation will be used to predict the vibration amplitudes on the soil surface in the vicinity of the shaker.

7. NUMERICAL SIMULATION

The field test was modeled with the Finite Elements Method (FEM). Plaxis 2D software was used. A 2-dimensional, axial symmetric model was built. The used geometry and mesh of the model can be seen in Fig. 4. The modelled area is 30 m in both length and depth. Measurement points for displacement recording were placed from radius r = 1.2 m to r = 6.2 m (reflecting positions of the geophones on the site). The soil is modelled with 15-node triangle elements.

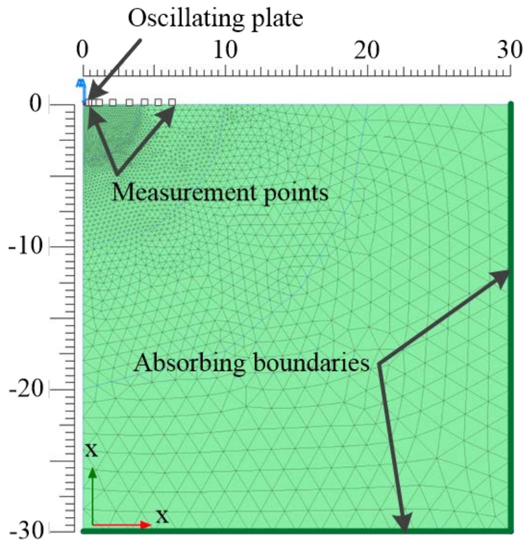

Fig. 4 FEM model: geometry and mesh

Elasticity properties defined from the P- and S-wave velocity measurements were used for the calculations. The shaker was defined as a plate element with axial stiffness EA = 21 GN/m, bending stiffness EI = 17.5 MN/m and weigth w = 21.94 kN/m/m. The selected weight corresponds to the total vibrating mass m_{tot}.

The general force-displacement matrix in Plaxis 2D is based on the following equation:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\}, \tag{24}$$

where $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices respectively, $\{u\}$ is the displacement vector (with the first and second derivatives) and $\{F\}$ is the force vector.

In Plaxis 2D, the Rayleigh damping is used, where $[C]$ is a function of the mass and stiffness, defined by:

$$[C] = \alpha_R [M] + \beta_R [K], \tag{25}$$

in which α_R and β_R determines the influence of mass and stiffness respectively.

The relationship between the Rayleigh damping coefficients and the damping ratio is:

$$\alpha_R + \beta_R \omega^2 = 2\omega D. \tag{26}$$

Solving Eq. 26 for two target frequencies and two target damping ratios yields:

$$\alpha_R = 2\omega_1\omega_2 \frac{\omega_1 D_2 - \omega_2 D_1}{\omega_1^2 - \omega_2^2}, \tag{27}$$

$$\beta_R = 2\frac{\omega_1 D_1 - \omega_2 D_2}{\omega_1^2 - \omega_2^2}. \tag{28}$$

Nevertheless, in this simulation there is only one frequency $\omega = \omega_1 = \omega_2$ and one damping ratio $D = D_1 = D_2$, therefore Eq. 27 and Eq. 28 can be simplified and the coefficients can be defined by:

$$\alpha_R = D\omega, \tag{29}$$

$$\beta_R = D / \omega. \tag{30}$$

In this case, for the vibration frequency $f = 24$ Hz, $\alpha_R = 1.508$ and $\beta_R = 6.63 \cdot 10^{-5}$.

8. PREDICTIONS AND MEASUREMENTS

First of all the measured amplitude of the shaker vibration is compared to the predicted by the analytical methods. Predicted and Measured amplitude ratios (P/M) are calculated in order to evaluate how good the predictions are. Table 1 shows that independently from the method used for a prediction, the amplitude of the shaker vibration has been predicted with at least 92 % accuracy.

Table 1 Comparison between the predicted and the measured shaker vibration amplitudes

Method	Amplitude [μm]	P/M ratio [-]
Measured	*292*	*1*
Reissner	293	1.00
Lysmer	309	1.06
Confined Elasticity	316	1.08
FEM	302	1.04

The highest P/M ratio is for the Confined Elasticity approach, which can be explained by the fact that horizontal deformations of the soil were neglected. It is also worth to mention that all methods over-predicted the vibration amplitude. Not only the measured amplitudes of the shaker, but also of the soil surface in the vicinity of the shaker, have been compared with the predictions of the analytical approach and also the FEM calculations.

The measured and the predicted vibration amplitudes can be found in Fig. 5.

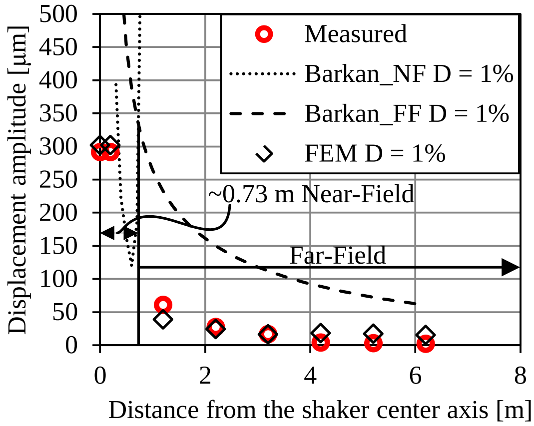

Fig. 5 Vibration amplitudes in the vicinity of the shaker

The P/M ratios between the predicted and the measured surface vibration amplitudes for the far-field can be seen in Table 2.

Table 2 Comparison between the predicted and the measured surface vibration amplitudes

Distance [m]	P/M ratio: Barkan-Bornitz Far-Field	P/M ratio: FEM
1.2	4.2	0.6
2.2	6.6	0.9
3.2	8.7	1.0
4.2	31.4	4.5
5.2	36.5	5.5
6.2	48.9	7.2

The near-field of Barkan-Bornitz's method ends before the first measurement point (Fig. 5), therefore only P/M values for the far-field are calculated. The analytical approach strongly over-predicts the vertical vibrations, this can be due to the fact that the method assumes only R-waves in the far-field, which could have led to misjudge of the destructive interference caused by body waves.

The FEM results are much better, but still under-predict the vibration amplitudes for the first 3 meters, and over-predicts further away. The weaker damping just next to the shaker and higher damping than expected further away from the shaker may be explained by the fact, that there is less vibration caused by the Rayleigh waves closer to the shaker and more vibrations caused by the Rayleigh waves further away from the shaker. This implies a different damping per different waves.

9. CONCLUSIONS

The shaker vibration amplitudes can be predicted by three analytical approaches: Reissner, Lysmer and the Confined Elasticity approach and by numerical (FEM) calculations. The accuracy depends on the method used for the prediction, and ranges from 92% to 100%. This shows that the amplitude of the shaker can be predicted accurately enough for geotechnical purposes.

The soil surface vibration amplitudes can be predicted by Barkan-Bornitz's analytical approach and by numerical (FEM) calculations. The Barkan-Bornitz's approach over-predicted the amplitudes between 4.2 and 48.9 times. The FEM under-predicted the amplitudes for the first three meters and over-predicted up to 7.2 times for the last three meters. This shows that the amplitudes of the surface cannot be predicted accurately. This confirms the conclusion made by Hölscher and Waarts [2], that the reliability of man-made vibration prediction methods is disappointingly low. This shows a demand of more research in the man-made vibration field.

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11. REFERENCES

[1] Barkan DD, Dynamics of bases and foundations. New York: McGraw-Hill Book Company Inc., 1962, Chapter VIII.

[2] Hölscher P and Waarts PH, "Reliability of vibration prediction and reducing measures", Final Report. Delft Cluster, Delft, 2003.

[3] Zwanenburg C, "De bepaling van sterkte-eigenschappen van veen", Geotechniek, Vol. Juli, 2013, pp. 26-32.

[4] Lamb H, "On the propagation of tremors over the surface of an elastic solid, Phil. Trans. Royal Soc.", series A., Vol. 203, 1904, pp. 1-42.

[5] Reissner E, "Staionare axilasymmetriche durch eine schuterende Masse erregte Schwingungen eines homohenen elastischen Halbraumes", Ingenieur Archiv, Vol 22, Dec. 1936, pp. 381-396.

[6] Bycroft GN, "Soil-Structure Interaction at High Frequency Factors", Int. J. of Earthquake Engineering and Structural Dynamics, Vol. 5, 1977 pp. 235-248.

[7] Kruijtzer G, The vertical motion of foundations and pontoons, Delft: VSSD, Reprinted 2006, Appendix 1.1.

[8] Hsieh TK, "Foundation Vibrations", in Proc. Of inst. Of Civil Engineers, Vol. 22, 1962, pp. 211-226.

[9] Lysmer J and Richard FE, "Dynamic response of footings to vertical loading", J. of Soil Mechanics and Foundations Division, ASCE, Vol. 92 No. SM1, January 1966, pp. 65-91.

[10] Verruijt A, Soil Dynamics. Delft, 2006.

[11] Westergraad HM, "A problem of elasticity suggested by a problem in soil mechanics: soft material reinforced by numerous strong horizontal sheets", Contributions to the Mechanics of Solids, Stephen Timoshenko 60th Anniversary Volume. New York: Macmillan, 1938.

[12] Barends FBJ, "Dynamics of elastic plates on a flexible subsoil", LGM-Mededelingen, Vol 21, 1980, pp. 127-134.

[13] Shekhter OY, "Consideration of Intertial Properties of Soil in the Computations of Vertical Forced Vibrations of Massive Foundations" NII, Symposium 12, Vibratsii Osnovaniy y Fundamentov, Moscow, 1948.

[14] Bornitz G, Über die Ausbreitung der von Großkolbenmaschinen erzeugten Bodenschwingungen in die Tiefe. Berlin: Springer, 1931.

[15] Coelho BEZ, Dynamics of railway transition zones in soft soils, PhD Thesis, TU Delft, 2010.

[16] Miller GF and Pursey H, "On the Partition of Energy between Elastic Waves in a Semi-Infinite Solid", Proc. Of the Royal Society of London, Series A, Mathematical and Physical Sciences, Vol 233, No. 1192, 1955, pp. 55-69.

[17] Macijauskas D and Van Baars S, "Decomposition of measured ground vibrations into basic soil waves", in Proc. of the 3rd Int. Symposium on Computational Geomechanics (ComGeo III), Krakow, Poland, 21-23 August, 2013.

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