# ESTIMATING FLEXURAL RELIABILITY OF CARBONATED RC BRIDGE BEAMS USING PARTICLE FILTER

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**ABSTRACT:** Many of our reinforced concrete structures today are ageing at the same time subjected to carbonation. It occurs when atmospheric carbon dioxide reacts with the components of the hydrated cement. In this regard, the authors estimated the probability of flexural failure of a deteriorated reinforced concrete (RC) beams subjected to carbonation. In the reliability analysis, the resistance degrades over time due to a change in the concrete compressive strength caused by carbonation. The load was modeled as a uniformly distributed on a simply supported beam. The results of the Monte Carlo simulation of an example bridge showed a decrease in the compressive strength due to carbonation. To estimate the present state of the RC beams, particle filter was used in conjunction with observation data from rebound hammer tests on the bridge.

Keywords: Carbonation, Monte Carlo, Reinforce Concrete, Reliability Analysis

# 1. INTRODUCTION

Several studies have shown that carbonation can affect the durability of concrete (Chi, J. M. Huang, R. and Yang, C. C., 2002, Saetta, A.V., and Vitaliani, R. V., 2004) as well as its service life (Liang, M., Qu, W., and Liang, C., 2002, Liang, M., Huang, R., and Fang, S., 2013). Carbonation occurs when atmospheric carbon dioxide (CO<sub>2</sub>) reacts with the compounds of hydrated cement such as calcium hydroxide, dicalcium silicate, and tri-calcium silicate (Claisse, P. A., Elsayad, H. I., and Shaaban, I. B., 1999). As carbonation progresses to the concrete layer surrounding the reinforcing steel, the pH of this high alkaline layer drops initiating corrosion. Over time the surrounding concrete spalls or crack leading to a higher probability of serviceability failure or service life reduction.

The rate carbonation is affected by the temperature, relative humidity, water-cement ratio, aggregate-cement ratio and CO<sub>2</sub> concentration. Carbonation also shrinks the volume of concrete due to the change in the morphology of the C-S-H gel from fibrous to dense. This causes the coarse pores to change into much finer pores. Water also initiates carbonation process and occurs mostly on submerged foundation and underwater reinforced concrete (RC) columns. In this case the amount of carbonates in water is far less so the rate of carbonation is significantly lower (Gode, K. and Paeglitis, A., 2009). Reliability assessment on the effects of carbonation depth to the performance of RC structures have also been studied (Hagino, T., Akiyama, M., and Frangopol, D, 2014, Akiyama, M., Frangopol, D. M., and Yoshida, I, 2010; Na,

U.J., Kwon, S-J, Chaudhuri, S. M., and Shinozuka, M, 2011). However studies on how the probability of flexural failure ( $P_f$ ) degrades over time due to carbonation-induced change in the compressive strength of concrete is lacking.

Consider a time variant reliability problem shown in Fig. 1 where  $M_S$  and  $M_R(t)$  are the random load and resistance, respectively. Due to the deteriorating nature of  $M_R(t)$ , the probability of failure  $(p_f)$  increases over time. If the mathematical models (and the parameters) for  $M_R(t)$  and  $M_S$  are prescribed, simulation techniques such as Monte Carlo simulation will suffice to determine  $p_f$ .



Fig.1Time dependent reliability problem

In this paper, the authors estimate the  $p_f$  of a simply supported RC bridge using Monte Carlo simulation subjected to a random moment load and a carbonation-induced deteriorating moment resistance. In Figure 1 the resistance is

deteriorating while the load varies randomly. At a specific time  $t_1$  the  $p_f$  is low but at  $t_2$  it increases. The above results (at a specific time *T*) can be updated using particle filter when data related to carbonation such as compressive concrete strength and relative humidity are observed as shown in Figure 2.



Fig. 2 Updated  $P_f$  conditioned on the observation

Depending on the severity of the observed data, the previously estimated  $p_f$  can either be under or over estimated. Thus with this new information and the carbonation-induced deteriorating RC beam model an updated curve can be obtained. This paper is organized as follows: (*a*) the general filtering problem and its usage in the present problem of carbonation, (*b*) reliability analysis of a RC beam (*c*) analysis and results of the research and finally (*d*) some recommendations.

# 2. RELIABILITY ANALYSIS

### 2.1 Limit state function

Consider a limit state function of the form

$$g(t) = M_{R}(t) - M_{S}, (11)$$

with the probability of failure as

$$p_f = p(g(t) < 0).$$
 (12)

If the distribution and parameters of  $M_R(t)$  and  $M_S$  are known then Monte Carlo simulation will suffice to determine  $p_f$ (or the reliability index). If  $n_s$  is the total number of sample realizations of G while  $n_o$  is the sample realizations where G< 0, then the  $p_f$  is obtain using Eq. 13 below.

$$p_f \approx \frac{z_o}{z} \tag{13}$$

Furthermore if inspection data related to any of the random variables of  $M_R(t)$  can be obtained, then  $p_f$  can be updated using Bayesian updating technique such as particle filter (Garciano and Yoshida, 2011).

# 2.2 Resistance of RC beam

### 2.2.1 Moment Resistance of beam

The moment resistance  $M_R(t)$  of a RC rectangular section can be described as

$$M_{R}(t) = 0.85 f_{c}(t)(b)(a)(d - a/2)$$
(14)

where *a* is the depth of the stress block, effective *d* of the reinforcement, *b* is the width of the RC section and fc(t) is the compressive strength of concrete at time *t*. In the research, three bridges were considered, but in this paper only one will be shown. The design parameters of the chosen bridge are as follows:

### 2.2.2 Dungon Bridge

This bridge runs through a small river and the columns are under water. The beams were also classified under both exposure class 1 and 3 with a mean RH of 53%. It has been in service for 33 years and is still being used for vehicular traffic.

Table 1	Bridge	Charac	teristics
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Variables	Value	
service life (years)	33	
length in (m)	27	
<i>w</i> (kN/m)	11	
span (m)	13.5	
base (mm)	400	
height (mm)	550	
concrete cover (mm)	75	
depth (mm)	475	
no. of steel reinforcements	4	
reinforcement diameter	28	
(mm)		
$f_y$ (MPa)	276	
relative humidity (mean)	53%	
exposure class classification	RH < 70, $e_{c1}$ /	
	e <sub>c3</sub>	

For the exposure class  $(e_c)$ , four classes for carbonated-induced corrosion were used (Silva 1984):

 $e_{c1}$  – permanently dry / wet concrete,

 $e_{c2}$  – concrete with long periods in contact with water,

 $e_{c3}$  – concrete in open air structures sheltered from rain,

 $e_{c4}$  – concrete in open air structures not sheltered from rain.

The value of  $e_c$  depends on the distance of the structure from the shoreline or any other large body of water.

### 2.2.3 Carbonation rate

The relationship between carbonation depth and the compressive strength of concrete can be modeled as follows (Breccolotti M., Bonfigli, M. F. and Materazzi, A. L., 2013 and AIJ, 1983):

$$f_c(t) = \frac{f_c}{1 + a_{11}y^{a_{12}}}$$
(15)

where:  $a_{11} < 0$  and  $a_{12} > 0$  are constants (shown in Tables 2 and 3), *y* is the carbonation depth and  $f_c$  is the 28<sup>th</sup> day compressive strength of concrete.

Table 2  $a_{11}$  values

Age (days)	а	
less than 28	1.0	
28-182	0.9 - 1.0 (varies linearly)	
greater than 182	0.9	
(Tanigawa, Baba and Mori, 1984)		

Table 3 Proposed  $a_{12}$  by AIJ

Age (days)	<i>a</i> <sub>12</sub>
28	1.0
50	0.87
70	0.84
100	0.78
200	0.72
500	0.67
1000	0.65
3000	0.63

(AIJ, 1983)

The depth of carbonation *y* can be determined as follows:

$$y = k\sqrt{t} \tag{16}$$

where k is the carbonation coefficient (mm/year <sup>0.5</sup>). The carbonation rate (when  $RH \le 70\%$ ) is

estimated using the following equation (Silva, 2014):

$$k = 0.556c - 3.602e_c - 0.148f_c + 18.734 \quad (17)$$

Where *c* is the carbon dioxide content in perecent. However if RH > 70% Eq. 18 is used.

$$k = 3.355c - 0.019C - 0.042f_c + 10.83$$
 (18)

where C is the clinker content (kg/m<sup>3</sup>).

#### 2.3 Load

The maximum moment of a simply supported beam subjected to a uniform load is given as:

$$S = M_s = wl^2/8 \tag{19}$$

where w is the deterministic uniformly distributed load and l is the length of the beam

# 2.4 Observation Equation

The relationship between the rebound number and the compressive strength can be represented by the equation (Shang 2012) below

$$f_c(t) = 0.032509 R^{1.94172} \times 10^{-0.00789y(t)}$$
(20)

where *R* is the rebound number and y(t) is the carbonation depth at time *t*.

# 3. ANALYSIS AND RESULTS

### 3.1 Time updating process

To evaluate the resistance, a total of 1000 sample realizations for every random variable were generated for each time step until t = 100 years. The parameters used in the Monte Carlo simulation are provided in the table below.

Table 4.Statistics of the random variables

Variable	Distribution Type	mean	std. dev.
c	Normal	0.80%	0.3
С	Normal	281.7	0.3
fc	Normal	21	0.3

The results of the simulation show a logarithmic relationship between time and carbonation depth (Fig. 3). Attention is directed to the upper right of

the graph with  $e_{c1}$  and RH < 70% which shows the highest carbonation depth from 122.9 mm to 131 mm at t = 100 years. The bottom right of the graph with RH < 70% and  $e_{c4}$  had the lowest results of carbonation depth of 58.9 mm. This is due to the wet and dry cycles that tend to vary the amount of CO<sub>2</sub> that penetrated into the structure.





Fig.5 Bridge beam bending capacity over time

The compressive strengths of concrete as time progresses were estimated considering an increasing carbonation depth as simulated earlier (see Fig. 4). These results correspond to a reduction in the bending capacity of the beam. Since all the variables in Eq. 14 have been calculated, the bending capacity of the RC beam is calculated as shown in Fig. 5. Exposure class 1 produces the lowest bending capacity of 398.08 kN-m, followed by exposure class 3 with a bending capacity of 398.58 kN - m. Case 1 with a bending capacity of 398.74 kN - m, and finally exposure class 4 with a bending capacity of 399.1 kN - m. Having a mean bending strength of 401.11 kN - m computed from the simulation, the RC beam lost 0.75% of its bending capacity solely due to carbonation. Figure 6 shows the plot between the resistance and the load at t = 25 years. The 45 degree line of each sub-graph is the limit state (R = S) for each exposure class and RH value.



Fig.6  $\boldsymbol{R}$  vs.  $\boldsymbol{S}$  (t = 25 years)

In Figure 7 the different probabilities of failure for all four cases under a uniformly distributed load of 14.175 kN/m are shown. As can be seen from this figure, exposure class 1 had the highest  $P_f$  in 100 years, at 0.032. This is followed by exposure class 3 with  $P_f = 0.007$ , followed by RH > 70% with  $P_f = 0.004$  and finally exposure class 4 had the lowest  $P_f = 0.001$ .

These flexural probabilities of failure were based on the carbonation equations coupled with Monte Carlo simulations and the severe case (exposure class) will be the case considered in the particle filter. The next step is to estimate the present state of the bridge RC beams conditioned on observations obtained from the rebound hammer tests. The succeeding section updates the  $P_f$  using observation data from rebound hammer tests and particle filter.



#### 3.2 Observation updating process

In the observation updating process, on-site rebound hammer tests on the underside of the RC beams of Dungon bridge. A total of 150 rebound hammer tests were conducted. The results are shown in Figure 8 which has a mean is 54.84 with variance of 46.55. These statistics are then used in particle filter.



Figure 8. Dungon Bridge rebound hammer data

With data available from hammer tests (for t = 33 years) the  $p_f$  is then updated. Figure 9 shows the result before updating and after updating. The model predicted a  $p_f$  equal to 0.003 but site observations show that this is under-estimated by around 0.001.



Figure 9. Updated  $p_f$  of Dungon Bridge

# 4. CONCLUSION

Carbonation is one of several environmental factors that contribute to the deterioration of RC beams. This paper emphasizes the estimation of the flexural reliability of RC bridge beams subjected to carbonation using Monte Carlo simulation and particle filter. In general, the results show that the time-variant probability of flexure failure of RC beams due to carbonation can be estimated using an appropriate state equation model. The present state therefore, in terms of the probability of failure,  $P_f$ , is 003. However the predicted probability of failure at any given time tcan under-estimate or over-estimate the present state of the RC beam. Using an appropriate observation equation, particle filter and the results from rebound hammer tests the predicted probability of failure can be updated. In this case at t = 33 years,  $p_f = 0.004$ .

# 5. ACKNOWLEDGEMENTS

The authors would like to acknowledge the assistance of Mr. Dyland Moises and Engr. Bryan Peroy during the site inspections. We also

acknowledge the help extended by the Department of Public Works and Highways (DPWH) office of Iloilo City during the bridge inspection.6. REFERENCES

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