International Journal of GEOMATE, March, 2022, Vol.22, Issue 91, pp.113-121
ISSN: 2186-2982 (P), 2186-2990 (O), Japan, DOI: https://doi.org/10.21660/2022.91.j2387
Geotechnique, Construction Materials and Environment

TRANSVERSE SALINITY DYNAMIC IN A SHALLOW TIDAL CHANNEL

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ABSTRACT: In this study, the variation in vertical and transverse salinity in stratification in the main branch of the Ōta River system was investigated over a tidal cycle at both spring tide and neap tide. The quantity and depth of stratification layers as well as the surface-to-bottom salinity difference deduced from the transverse distribution of salinity were used to evaluate the stratification/de-stratification degree in a shallow and narrow estuary. Two Compact Conductivity-Temperature-Depth sensor campaigns to collect salinity data sets were run across the river at two bridges, with a longitudinal distance of approximately 850 m between the two bridges. It was found that salt wedges moved from the left side to the right side of the riverbank during both neap flood tide and spring flood tide, resulting in lateral salinity distribution, likely due to the difference in the advection of the longitudinal salinity gradient, and the effects of cross-channel bathymetry variation. Besides, the stratification developed gradually from the surface to the bottom of the water column over the ebb tide period primarily caused by the tidal straining; tidal time scale, and freshwater discharge. The re-stratification process at the end of the flood phase is likely due to the combination of the lateral baroclinic pressure gradient and complex bathymetry.

Keywords: Tidal straining, Vertical salinity profile, Stratification layer depth, Salinity dynamic, Estuary

1. INTRODUCTION

Estuaries constitute a significant object of several physical processes that vary spatially and in time, including freshwater input, salinity intrusion, and tidal movement. Understanding these hydrodynamic conditions and their interactions is necessary for water resource management. Recently, several research studies on the salinity dynamic and its contributing factors contributed to the overall understanding of the estuarine dynamic. Previous researchers have tried to understand how the freshwater, saline water, and/or tide exchange creates a complex environment. However, most published studies on salinity dynamics were implemented using models for deep and wide estuaries [1 - 4], and fewer studies focused on shallow and narrow estuaries where the interaction between freshwater, saltwater, and tide often occurs very fast. Furthermore, transverse and vertical motions in estuaries are usually smaller than the horizontal dimension, but these dimensions have an essential role in the estuarine dynamic [2, 5]. The degree of stratification in single cross-sections is an important factor in the vertical mixing mechanism. Clarifying how a shallow and narrow estuary develops and breaks down the stratification will give a better understanding of the estuarine dynamic.

The Ōta diversion channel is the largest branch in the Ōta River system, which is the most crucial river in Hiroshima city and has a vital role in its transportation and ecosystem [6]. The Ōta River has an essential environment, with a coastal ecosystem for many tidal species. Changing salinity is one of the most important factors affecting the productivity of aquaculture and agriculture. Thus, understanding the salinity dynamics in the Ōta diversion channel is necessary.

Flow rate and salinity intrusion [7, 8], salt flux variability, and water circulation [9] were investigated to examine the estuarine dynamic in the Ōta River estuary. Soltaniasl [9] used the gradient Richardson number and a dimensionless variable named stratification parameter to evaluate the stratification variation in the Ōta River estuary during a high tide over four hours. Soltaniasl′s research contributed an important finding to the understanding of the stratification mechanism in the Ōta River estuary, specifically that during the later period of the spring tide, the tidal mixing contributes to decreasing the stratification over the whole cross-section. However, to fully understand the stratification process, and thus the lateral salinity dynamic, an evaluation of salinity distribution in the cross-sections during a complete tidal cycle is needed.

The present research investigated vertical and lateral salinity variations in stratification over a tidal cycle in both spring tide and neap tide. This research aims to examine how the saline water, freshwater, and tide mix and exchange through a dynamic process in the Ōta River floodway, the shallow and essential channel in the Ōta River system in Hiroshima city, Japan. This study is expected to answer important questions, including 1) How do the stratification layers form? 2) How and why do the stratification layer depths change? and 3) What are the main controls on stratification in this area? By discussing these questions, the transverse salinity dynamic in the Ōta channel will be clarified.

2. STUDY SITE

At about 9 km upstream from the estuary mouth, the Ōta River bifurcates into two main branches, the Ōta and Ōta diversion channels. In the present study, field observations were conducted in the second branch. The tide is mixed with a diurnal component and primary semidiurnal with the range of the tide at the river mouth varying from 1.2 m during the neap tide to about 4 m at the extreme spring tide. Fig.1 illustrates the location of the study site. It is located around 4.3 km upstream from the river mouth. Freshwater runoff is usually limited by the Gion sluice gates located 4.5 km upstream of the observation site. During the observation period, the Gion sluice gates worked under normal conditions in which only one of the three gates are opened slightly to make a cross-sectional area of the stream of 32m × 0.3m to spill the flow.

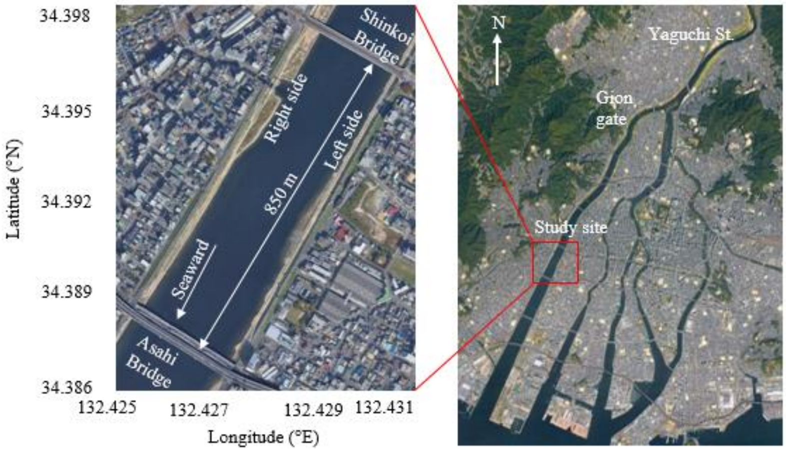

Fig.1 The location of the field observation at the Ōta River Estuary

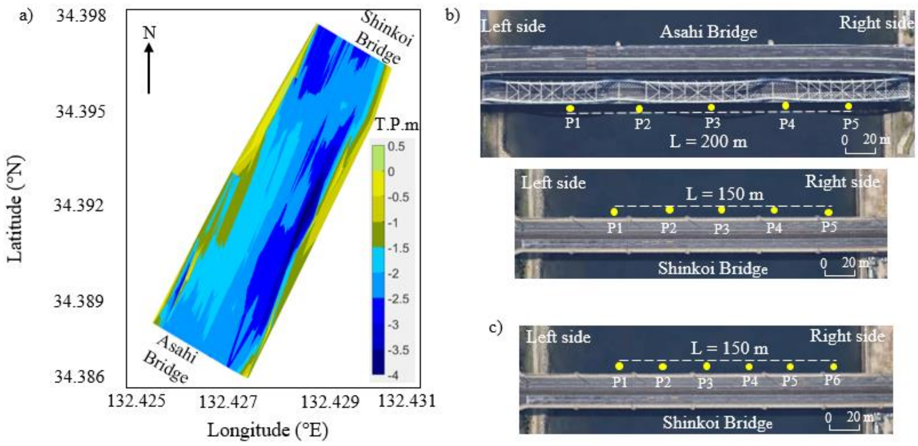

Fig.2 a) The bathymetry distribution of the observation site; b) and c) the transverse lines of CTD measurement for the first and second campaign respectively, yellow symbols indicating the locations of the data collection points

3. METHODS AND DATA DESCRIPTION

The vertical salinity profile was estimated from the data set of salinity concentration collected from two CTD campaigns. The first campaign was conducted from 14:00 to 17:00 on September 5th, 2018, during neap tide. This work was done simultaneously along two bridges, Asahi Bridge and Shinkoi Bridge, with a distance between those two bridges of approximately 850 m (Fig.1). In this campaign, the vertical salinity distributions were collected every thirty minutes at five points along the 150 m length of Shinkoi Bridge and the 200 m length of Asahi Bridge, respectively (Fig.2).

The second campaign was operated on October 2nd, 2020, from 8:00 to 23:30, during spring tide at Shinkoi Bridge (Fig.1), in which CTD ran at six points along the 150 m length of the bridge every thirty minutes (Fig.2).

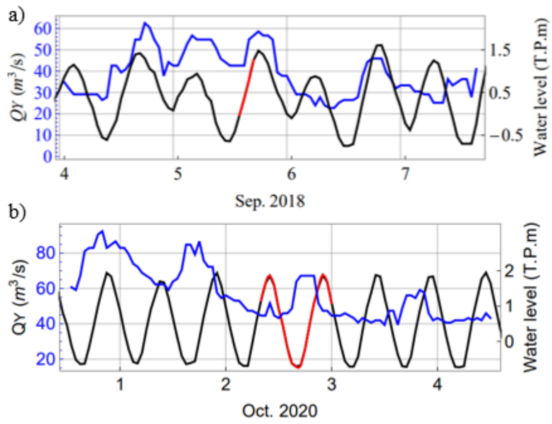

Fig.3 Temporal variations of freshwater discharge at Yaguchi Gauging Station (blue line), the water level at the observation site (black line), and the studied periods (red line) for a) the first campaign and b) the second campaign.

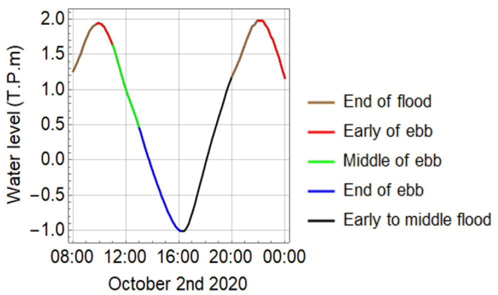

Fig.4 Seven stages of the second campaign

The more extended campaign was divided into seven stages for further clarification, including the end of the flood of the first tide, early in the ebb of the first tide, middle of the ebb, the end of the ebb, early to middle of the flood, the end of the flood of the second tide, and early in the ebb of the second tide (Fig.4). The stage classification is based primarily on the pattern of the cross-section salinity distribution. During the observation period, the single cross-sections which have similar patterns of vertical salinity profile will be grouped into the same stage. For the present research, the quantity and the depth of stratification layers, and the surface-to-bottom salinity difference were used to evaluate the stratification/de-stratification degree.

4. RESULTS AND DISCUSSION

Results from the two campaigns reflect the different range of stratification conditions observed between spring tide and neap tide as well as between ebb and flood tide conditions in the Ōta River. Stratification was highly developed during ebb tide periods while de-stratification occurred during higher flood tides. The stratification degree was stronger during neap tide than during spring tide.

4.1 Spring Tide (the first campaign)

4.1.1 The ebb stages

The salinity distribution in the cross-section of the ebb stages during spring tide, including early in the ebb of the first tide, early in the ebb of the second tide, the middle of the ebb, and the end of the ebb are shown in Figs.5, 6, 7 and 8, respectively. Overall, the Figures of ebb stages revealed a complete stratification, with the stratification degree increasing sharply from the start to the end of the ebb tide.

The early part of the ebb stages

Fig.5 shows five significant stratification layers at the beginning of the stage, with corresponding salinity ranges from 26 −31 psu (Fig.5a). There was a sharp difference in depth of these stratification layers; the saltiest layer (above 30 psu) had the highest depth of approximately 2 m, while the total depths of the remaining layers were approximately 1 m. The number of these significant layers increased, and thus their depths decreased at the end of the stage, however, the depth of the saltiest layer was always higher than the total depths of the remaining layers. A similar phenomenon was also found in the early part of the ebb of the second tide (Fig.6). This phenomenon suggested that at the beginning of ebb tide, the stratification occurred more strongly at the surface than in the middle and bottom parts of the water column.

The mid ebb and the end of ebb stages

The stratification continues to rise during these stages. The end of the ebb stage presented the highest number of stratification layers over the observation period (Figs.8c, 8d), which suggests that the strongest stratification happened at the end of the ebb tide period. In the middle of the ebb stage, the depth of the saltiest layer (above 29 psu) was still highest, approximately 1.5 m (Fig.7a), and sank gradually during the first half of the stage (Figs.7b, 7c). However, it fell more rapidly during the last half of the stage (Figs.7d, 7e, 7f).

The last figure of the middle stage and the first one of the end ebb stage also show an equal depth for all significant stratification layers, approximately 0.2 m (Figs.7f, 8a). After that, the depth of the layers that were in the middle of the water column increased twofold while the depth of the layers located on the bottom decreased rapidly (Figs.8b, c, d, e). During the end of the ebb and early stage of flood tide (Fig.8f), the depths of the significant stratification layers become almost equal again. The depth variation of the significant stratification layers suggests that the stratification developed gradually from the surface to the bottom of the water column over the ebb tide period. The following parts will discuss the contribution of the factors and mechanisms that may control the stratification process in the Ōta River estuary.

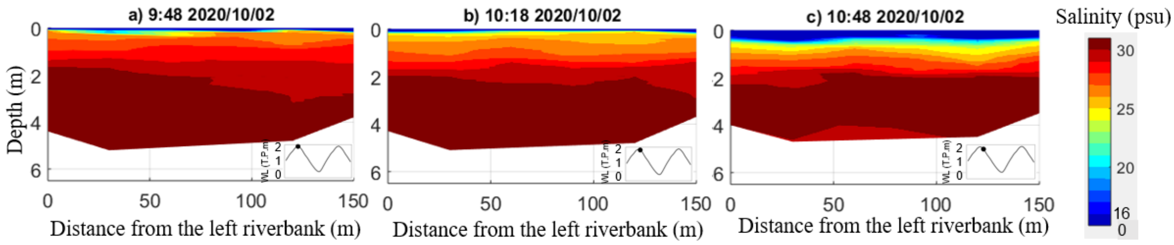

Fig.5 Salinity distribution in the cross-section of the early ebb of the first tide stage during spring tide

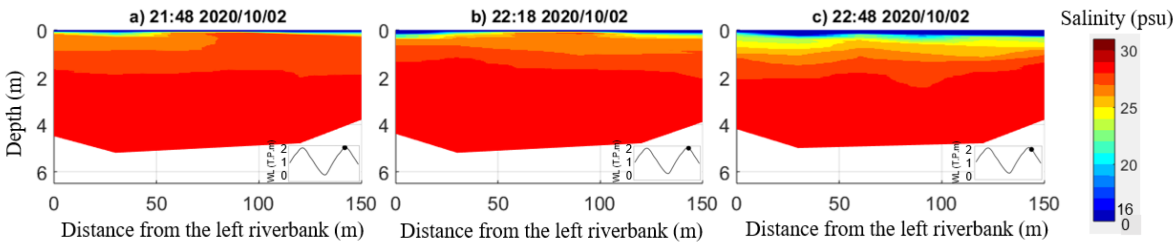

Fig.6 Salinity distribution in the cross-section of the early ebb of the second tide stage during spring tide

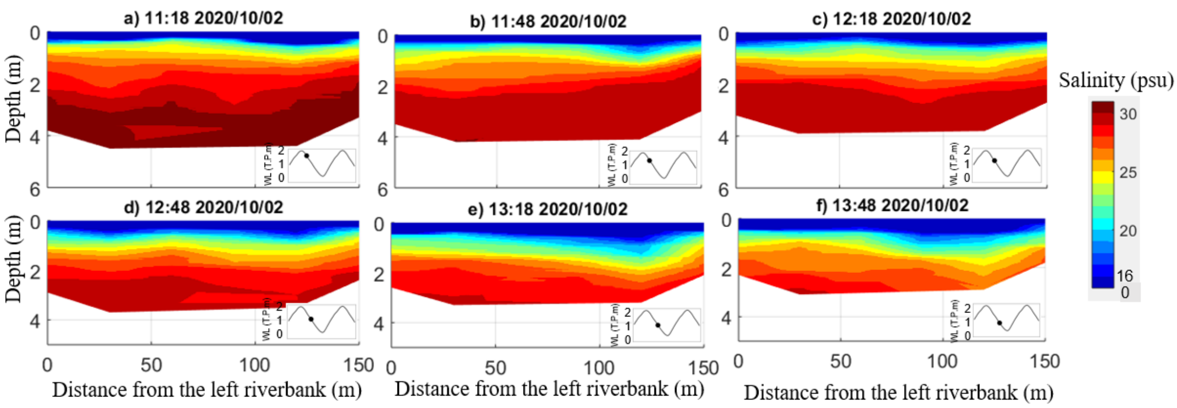

Fig.7 Salinity distribution in the cross-section of the middle ebb stage during spring tide

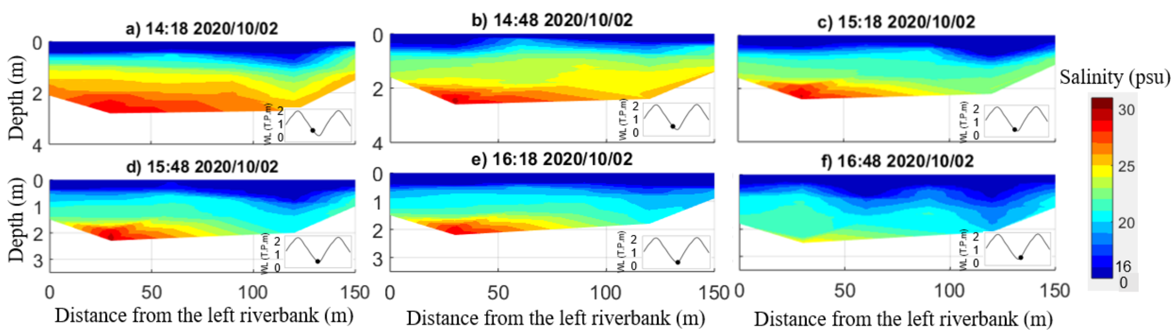

Fig.8 Salinity distribution in the cross-section of the end ebb stage during spring tide

The effect of the tidal straining

The process in which the oscillatory vertical shear of tidal currents interacts with horizontal density gradients creating a straining is termed tidal straining [10]. The stratification is created by shearing the faster and lighter layer of the water column over the saltier and slower-moving water of the next lower layer. During ebb tides, the surface water that carries fresh water from upstream is often lighter and will move faster toward the river mouth and overtake the lower layers that are saltier and heavier. Soltaniasl [9] already confirms the phenomenon in the Ōta floodway by showing the contour plots of velocity and salinity at cross-section during ebb tide. This process will continue to happen between two layers that are close, resulting in a stable structure and suppressing turbulent mixing.

Contribution from the tidal time scale

Razaz [11] showed that the Ōta channel is an ebb-dominant estuary, with an asymmetry in ebb-tide and flood-tide velocities and the highest velocities occurring during the ebb tide. Some previous studies have been conducted on the influence of the tidal time scale on the variation of stratification. Jay [12] stated that the tidal asymmetry during a spring-neap tidal cycle plays an important role in stratification variability. The combination of two kinds of pressure gradients, barotropic and baroclinic strengthens stratification during ebb tides and de-stratification during flood tides. Simpson [10] also found that this asymmetry creates a vertical shear, and thus a straining; this phenomenon was described as tidal straining. During ebb tides, the surface current will accelerate and strain the horizontal salinity gradient, which might increase the stratification. Furthermore, the combination of strong advection and straining during ebb tides may also enhance the stratification process.

The effect of the water temperature

Generally, the vertical variation of temperature will affect vertical gradients of salinity due to the vertical transference of heat [10], thus, affecting the salinity dynamics also. However, the surface-to-bottom temperature differences at the study site are not significant in both CTD campaigns, under 1.5 °C (Figs. 9 and 10). Thus, the effect of temperature variation in the vertical direction on salinity dynamics is not significant compared to other factors mentioned above.

The time delays of the salinity peaks

A time delay of salinity maximum was found during slack water periods, with the range from approximately 30 minutes to 1 hour. The first high water slack occurred at approximately 9:48 (Fig.5a) while the depth-averaged salinity reached its maximum value approximately 30 minutes after that (Fig.5b). Similar time delays of salinity peaks also were found in the early ebb of the second tide stage, and in the end ebb stage (Figs.6, 8). During the end ebb stage, the low water slack happened between 15:48 and 16:18 (Figs.8d, 8e), while the lowest salinity occurred around 16:48 (Fig.8f). Similarly, the second-high water slack occurred at about 21:48 (Fig.6a), and the maximum salinity happened 30 minutes later, around 22:18 (Fig.6b). Dyer [13] stated that the density current effects, changes in water depth, and the differences in vertical velocities likely cause these delays in the time of the salinity peak. In the Ōta River estuary, another reason for that phenomenon is the effects of the freshwater flows from the Yaguchi Gauging station located upstream [14, 15].

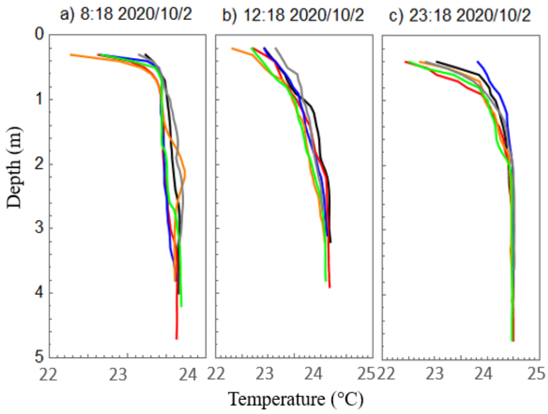

Fig.9 Vertical profile of temperature at the cross-section estimated by CTD of the second campaign, P1 (black line), P2 (red line), P3 (blue line), P4 (orange line), P5 (green line), and P6 (gray).

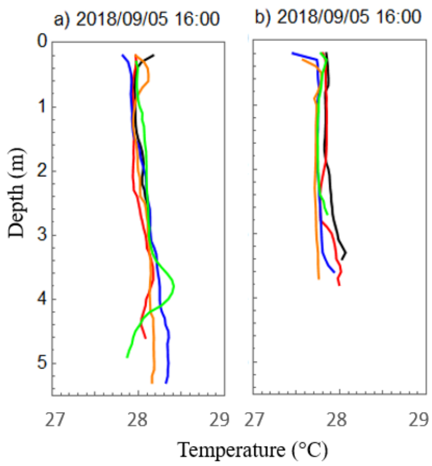

Fig.10 Vertical profile of temperature at the cross-section estimated by CTD of the first campaign at a) Shinkoi Bridge, and b) Asahi Bridge. P1 (black line), P2 (red line), P3 (blue line), P4 (orange line), P5 (green line).

4.1.2 The flood stages

The plots of the surface and bottom salinity of the flood stages during spring tide, including the end of the first flood tide, the end of the second flood tide, and the early to middle flood are shown in Figs.11, 12, and 13, respectively. These figures revealed the main trend of the salinity dynamic during flood tide, which is that the salt wedges moved from the left side to the right side of the river, resulting in the lateral distribution of salinity. The sectional-cross salinity was not symmetric, as the water was fresher on the right side than on the left side. This might be because of the influence of bathymetry, which will be discussed in the following parts. The plots of salinity distribution over the cross-section during flood stages also show a high de-stratification rate, essentially for the early flood stage.

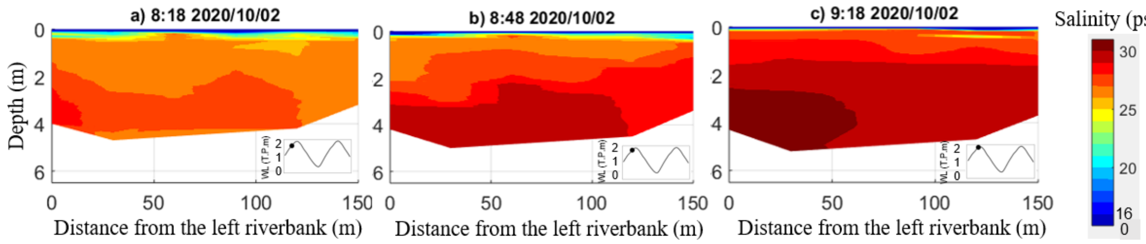

Fig.11 Salinity distribution in the cross-section of the end of the first flood tide stage during spring tide

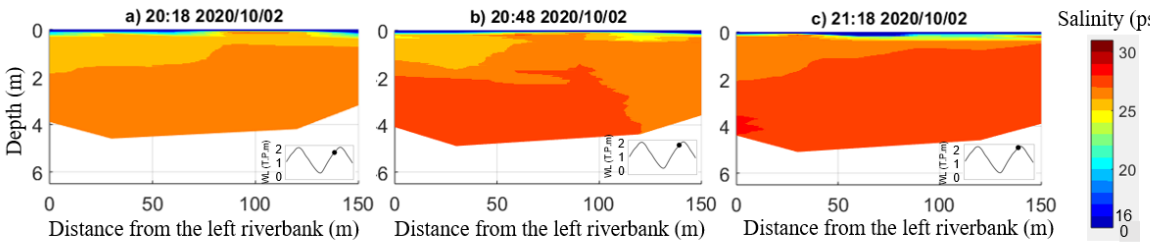

Fig.12 Salinity distribution in the cross-section at the end of the second flood tide stage during spring tide

The effect of freshwater discharge on stratification

Although the water depths at the two peaks are similar (Fig.3b), the highest salinity during the first tide was approximately 2 psu higher than during the second tide (Figs.11c, 12c). The lateral salinity gradient in the first tide was also higher than that in the second tide. Besides, the number of significant stratification layers during the first tide ranged from 5 – 15 (Fig.5), higher than that during the second tide of 3 – 10 (Fig.6). This is because, during the second tide, freshwater inflow recorded at Yaguchi Gauging Station almost doubled during the first tide, from approximately 40 m^3/s to approximately 70 m^3/s (Fig.3). Comparing the vertical salinity profile between the two stages of the end of the flood, the significant influence of the freshwater flow on the stratification process is evident. Thus, the salinity dynamic process in the Ōta River estuary is affected significantly by freshwater input upstream.

Furthermore, a re-stratification appeared late in the flood tide. During this period, tidal currents decelerated leading to a rapid diminishment of turbulent mixing. The force deduced from the lateral baroclinic tends to become stronger and overtakes the turbulent mixing, enhancing the vertical stratification. The reason for this phenomenon has been discussed in previous studies with the term “lateral straining” [16, 17]. The variation of cross-section bathymetry will also cause the vertical shear that leads to the stratification. Thus, the stratification development during flood tides in the Ōta River estuary might be due to a combination of the lateral baroclinic pressure gradient and complex bathymetry.

The early to middle flood stage

This stage showed a complete lateral salinity distribution. The lateral salinity gradient was highest at the beginning of the stage (Fig.13a), close to the previous ebb, decreasing during the mid flood and increasing again during the late flood (Fig.11f). The difference in the advection of the longitudinal salinity gradient during flood tide is claimed to intensify the lateral baroclinic forcing, thus increasing the lateral salinity gradient.

Another reason for the lateral salinity gradient is the effect of significant bathymetry variation at the study site. As shown in Fig.2, the left riverbank is deeper than the right riverbank on the middle of the channel segment, and the right side is deeper than the left side at the areas close to the two bridges, resulting in meandering in the observation site. During the flood tide, the meandering tends to force bottom water that is slow-moving and saltier toward the deeper area of the channel while at the surface, river flow tends toward straight. This lateral shear created a remarkable lateral salinity gradient during the early and middle floods. Previous studies [18, 19] have reported similar findings and stated that the transverse shear generates a lateral salinity gradient.

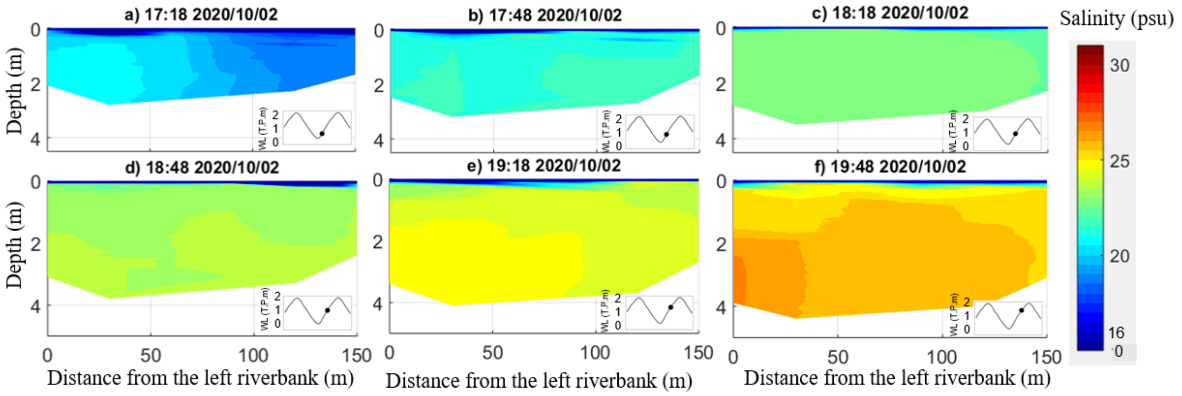

Fig.13 Salinity distribution in the cross-section of the early to mid flood stage during spring tide

Also, the comparison between the transverse salinity distribution of this stage and other stages, including the end of ebb and the end of second flood stages, shows that the stratification and de-stratification respond quickly to tidal changing. In Fig.8f, the cross-section salinity distribution showed a high stratification with the stratification layer depths equal at the end of the ebb tide, then changing quickly to lateral distribution after 30 minutes at the early stage of flood tide (Fig.13a). Similarly, the re-stratification happened at the end of the flood tide, close to the high water level (Fig.12a) after the previous lateral distribution stage (Fig.13f). To evaluate fully this rapid change, a consideration of the spatial and temporal variabilities is necessary for future research. In this case, the initial reason might be attributed to the shallow depth of the estuary.

4.2 Neap Tide (the second campaign)

The salinity distributions measured using CTD in the mid flood tide during neap tide at the upriver and the downriver sections are shown in Figs.14 and 15, respectively. In general, during neap flood tide, cross-section salinity distribution at the downstream and upstream areas was similar, with stratification developed during the middle of the flood tide, and was weakened during the end of the flood tide. There was a slight difference in the de-stratification process between the two tidal conditions close to the end of the flood tide, with the de-stratification sharper at the downstream (Figs.15e, 15f) than upstream area (Figs.14d, 14e).

In general, the surface to the bottom distribution of salinity at the locations along the estuary at different distances from the river mouth was expected to be different because of several factors, such as the longitudinal salinity gradient and the bathymetry effect. However, it seems that the distance of 850 m between the two locations in this study was insufficient to reveal the expected difference. A study in vertical salinity distribution at a location along the Ōta River estuary with a long-distance apart is needed in the future.

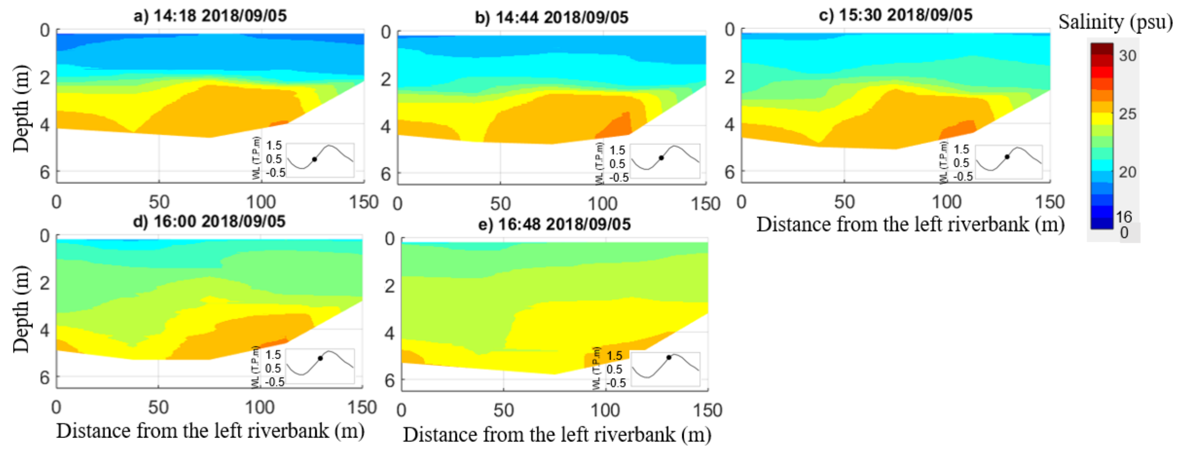

Fig.14 Salinity distribution in the cross-section at the upstream area (Shinkoi Bridge) during neap tide

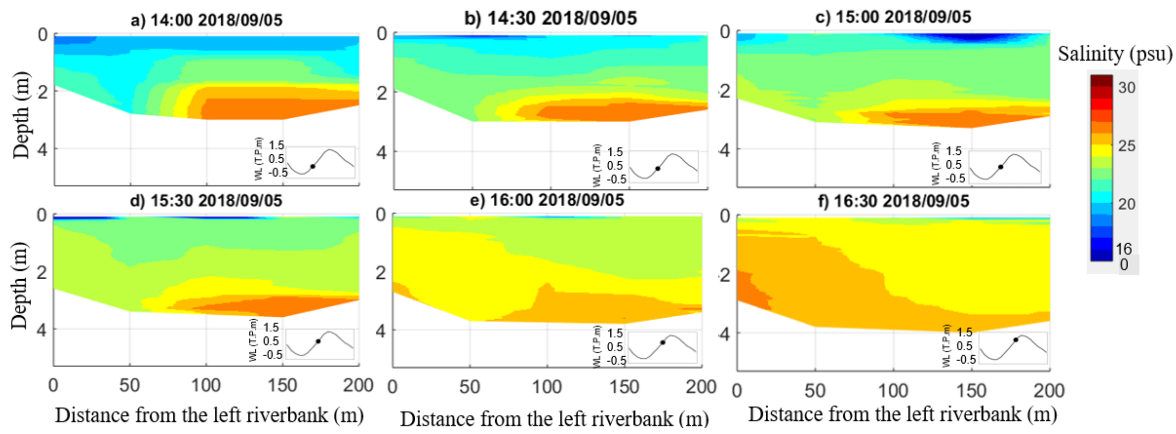

Fig.15 Salinity distribution in the cross-section at the downstream area (Asahi Bridge) during neap tide

Also, the comparison between figures of surface-to-bottom salinity of mid-flood during the two periods (December 5th 2018 and October 2nd 2020) presents a remarkable differencing e it hehe ratification process between neap and spring conditions, with stratification increasing significantly more a larger during the neap tide than during spring tide. During spring tide, salinity distribution was almost lateral (Fig.13), while during neap tide the surface-to-bottom salinity difference was 5 − 8psu (Figs.14, 15), or 20% − 32% (mean of 25 psu). At the middle of the neap flood tide (Figs.14a, b, c, and Figs.15a, b, c, d), the number of significant stratification layers was seven over a water depth of 4 m, indicating a well-stratified water column, then decreased and thus the significant stratification layer depths increased, corresponding to the increase in the water depth. On the neap tide, the tidal range of the estuary decreased and the currents were moving more slowly than at the spring tide. Thus, the vertical mixing reduces and instantaneous mixing energy reaches the minimum value over a spring-neap tidal cycle that leads to stronger stratification during the neap tide periods than during the spring tide periods. Stratification between neap and spring conditions was significantly different, which suggests that the salinity dynamic of these two tidal periods might be different.

5. CONCLUSION

The stratification developed gradually from the surface to the bottom of the water column over the ebb tide period primarily due to the tidal straining; tidal time scale and freshwater discharge. The re-stratification process at the end of the flood phase was likely due to the combination of the lateral baroclinic pressure gradient and complex bathymetry. The quick response of stratification and de-stratification to tidal changing seems related to the shallow water condition. Salt wedges moved from the left side to the right side of the riverbank during both neap flood tide and spring flood tide, resulting in lateral salinity distribution, likely due to the difference in the advection of the longitudinal salinity gradient, the effects of cross-channel bathymetry variation. The slight variation in stratification between the downstream and upstream areas during the neap flood tide might be due to the longitudinal salinity gradient and bathymetry variation.

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