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DESIGNING A TRAPEZOIDAL MODULAR BLOCK WALL WITH NONLINEAR OPTIMIZATION

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ABSTRACT: A rectangular wall is better in terms of stability and ease of calculation than a trapezoidal wall. However, a trapezoidal wall is sometimes inevitable such as a retaining wall construction near rockface. FHWA provides simplified rules to design a trapezoidal wall. However, FHWA does not give an example to follow, and the rules need trial and error to implement. BS8006 gives an exact dimension of the block heights, but designing still needs to adjust the block widths. A 16-meters high modular block wall project near rock face in Thailand as an example to illustrate a calculation detail in external stability checking follow FHWA simplified rules and BS8006. The illustrations are trapezoidal walls with two zones, three zones, and four zones. Nonlinear optimization models are also used to minimize the wall base length to facilitate the construction instead of jacking the near rock face to build a rectangular wall. Optimization models also help to relax FHWA simplified rules and BS8006 guidelines. Using an optimization model can decrease the base length from 0.7H to 0.6H for a rectangular wall or even 0.5H for a rectangular wall with competent foundation soil. Optimization models can also achieve a base length down to 0.48H with a decrease in the cross-sectional area down to 0.92 for a three zones trapezoidal wall. A simple three zones wall with exact dimensions is also proposed in the competent foundation soil conditions.

Keywords: Nonlinear optimization, MSE wall, Trapezoidal wall, Uneven reinforcement lengths

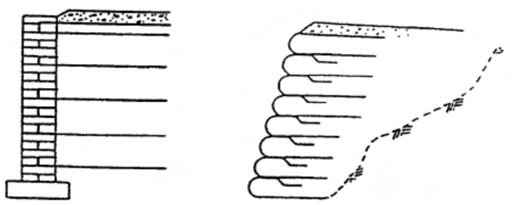
1. INTRODUCTION

A mechanically stabilized earth wall or MSE wall is a retaining wall that allows more displacement than a rigid reinforced concrete wall. There are two MSE wall types commonly used, which are segmental precast concrete panel (SPCP) and modular block wall (MBW), shown in Fig. 1.

SPCP MBW

Fig. 1 Segmental precast concrete panel (left) and modular block wall (right) (FHWA [1])

SPCP uses precast concrete panel as its facing unit and metal strips as its reinforcements, while MBW uses modular block as its facing unit and geogrid as its reinforcements. Reinforcements help to add tension because the tensile strength of soils is negligible. The MSE wall face can be set vertically, not like reinforced soil slope (RSS) with rigid facing units that can set its face slope up to 70 degrees, as shown in Fig. 2.



MSE Wall RSS

Fig. 2 MSE wall and RSS from FHWA [1]

Rectangular MSE walls are normally used but, in some circumstances, there is a need in designing a trapezoidal wall, such as a restriction in MSE wall base length due to the construction of an MSE wall near the rockface. Also, no clear examples and limited guidelines are helping in designing such trapezoidal walls. This paper illustrates examples, guidelines, and a simplified rule in designing a three zones MBW wall.

1.1 Designing a Rectangular MSE Wall

An MSE wall has to resist the failure modes in external stability, internal stability, and global stability. External stability requires this rigid body MSE wall to pass the safety factor in sliding, overturning and bearing capacity. Internal stability prevents the failure inside the MSE wall by setting safety factors in rupture and pullout. Global stability considers all other failures that may occur anywhere, especially outside the MSE wall.

Designing MSE deals with specifying the reinforcement length, reinforcement length spacings, and reinforced soil strength property. For a rectangular MSE wall, FHWA [1] and BS8006 [2] give a preliminary minimum reinforcement length (L) of 0.7H. FHWA also recommends L/H to be not greater than 1.1. Shored MSE wall [3] suggests that the reinforcement length can be as narrow as *0.3L/H.*

External stability checking for a rectangular MSE wall is simply to specify the reinforcement lengths to handle the sliding, overturning, and bearing capacity. Internal stability checking is slightly different for the SPCP wall and MBW wall. SPCP wall, which is an inextensible wall, adjusts the number of metal strips, spacings, and lengths to resist the rupture and pullout mainly from lateral earth with its trapezoidal (or bilinear) maximum tension force line. MBW wall, which is an extensible wall with less rigid reinforcements than SPCP wall, adjusts the geogrid tensile strengths with its triangular (or linear) maximum tension force line to resist the pullout. Global stability design is implemented by using either the limit equilibrium method (LEM) or finite element method (FEM).

1.2 Designing an Uneven Length Reinforcement MSE Wall

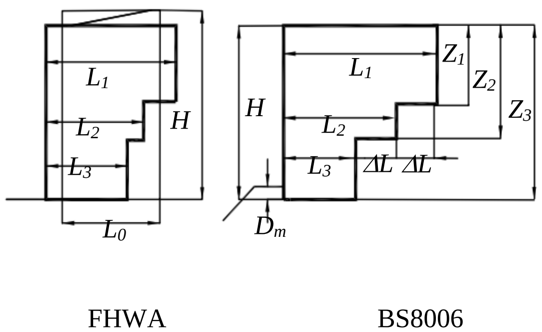


Fig. 3 Dimensioning uneven reinforcement lengths wall guided by FHWA [1] and BS0006 [2]

Uneven length reinforcement MSE wall is needed difficult to widen the base and shored MSE wall is due to the limited right of way base, near rock face that is also not an optional. FHWA gives simplified rules used in external stability checking for uneven length reinforcement MSE walls as follow: 1) the wall is represented by a rectangular block (L_o, H) having the same total height and the same cross-sectional area as the trapezoidal section for external stability calculations, as shown in Fig. 3 (left). 2) minimum base length (L_3) is 0.4H, with the difference in length in each zone being less than 0.15H. 3) Internal stability is designed using the same methods as the rectangular MSE wall. 4) Global stability analysis is also needed in the design. 5) Trapezoidal wall is considered given a rock base or competent foundation soil.

BS8006 gives more specific trapezoidal wall dimensions. Fig. 3 (right) shows the three zones wall where $Z_1 = 0.5H$, $Z_2 = 0.75H$, $Z_3 = H$, $L_1 \geq 0.4H$, $\Delta L \leq 0.15H$. D_m is wall embedment length. For the two zones wall, BS8006 recommends minimum reinforcement lengths at the top at 0.7H and at the bottom at 0.4H or 3 meters. BS8006 also recommends $0.55 \leq L/H \leq 0.75$ and $0.125 \leq S_v/H \leq 0.222$ where S_v is reinforcement vertical spacing.

1.3 Nonlinear Optimization Techniques in Designing Uneven Length Reinforcement MSE Wall

Optimization techniques have been applied popularly in civil engineering for both practitioners and researchers. The techniques are well suited to handle problems on hand that demand solutions under limitations. Uneven MSE wall design, which is a retaining wall design, is also an optimization problem that seeks a safe and the most economical solution under its narrowest base limitation.

Numerous optimization techniques have been applied to retaining wall designs, such as Dungca [4], which applies linear optimization of soil mixes in designing vertical cut-off walls. Optimization is also applied in some other areas, such as Araki [5] in a buried pipe with geogrid-gabion, Bala [6] in mix design optimization of asphalt mixtures, and Bernardo [7] in optimization of concrete compressive strength.

Narrow retaining walls near rock faces are addressed in some studies. Chen [8] pays attention to its translational mode. Leshchinsky [9] developed a design chart for narrow SPCP walls. Dai [10] illustrated a project of uneven reinforcement lengths of trapezoidal walls, which is a shored MSE wall 8 meters high and 4 meters base width.

1.4 Aim, Limitations, and Significance of the Study

From these studies, there is no example showing a design of uneven reinforcement lengths MSE wall. There exist guidelines such as BS8006 and the simplified rules provided by FHWA to design this unconventional trapezoidal wall geometry. Hence, this study aims to provide an example in designing this uneven reinforcement lengths MSE wall.

A 16-meter-high retaining wall project in Thailand is chosen to illustrate the design. Four different geometries of MSE walls are proposed. The first wall is a rectangular MBW wall. The second wall is a 2-block trapezoidal MBW wall. The third wall is a 3-block trapezoidal MBW wall. The fourth wall is a 4-block trapezoidal MBW wall.

Nonlinear optimization using GAMS (General Algebraic Modeling Software) program with NLP solver [11] is also used to dimension these MBW walls. The optimization models mainly follow the simplified rules from FHWA and BS8006. The nonlinear optimization models consider both external and internal stability checking. Internal stability can be adjusted by varying the geogrid reinforcement lengths and spacings for each block. Global stability is also addressed in this study.

With the necessity in the lack of clear guideline and example in designing a trapezoidal MSE wall needed in restricted wall base, this study help illustrate and give guideline in designing this uneven reinforcement length MBW wall. The optimization models proposed also give an approach in dimensioning the 2-block, 3-block, and 4-block trapezoidal MBW walls. A simple 3-block wall with exact dimensions is also proposed.

γ_b, ϕ_b

2. WALL GEOMETRY β AND PARAMETERS

2.1 Wall Geometry

Geometry for nonlinear optimization models is shown in Fig. 4. Those models are the rectangular (1-block) wall, the trapezoidal walls with two zones (2-block), three zones (3-block), and four zones (4-block). Those walls are MBW walls, not the SPCP wall because its resistant zone is triangular (linear) according to FHWA.

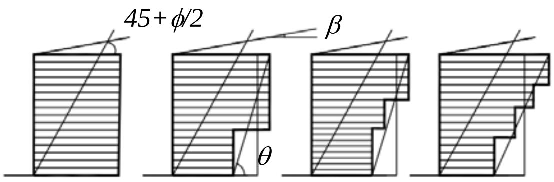


Fig. 4 Wall geometry for the three walls

2.2 Parameters

2.2.1 Materials

The reinforced soil density, γ_r, is 18 kN/m^3
The reinforced soil friction angle, ϕ_r, is 32 degrees
The back soil density, γ_b, is 18 kN/m^3
The back soil friction angle, ϕ_b, is 30 degrees
The foundation soil density, γ_f, is 18 kN/m^3
The foundation soil friction angle, ϕ_f, is 35 degrees
The soil cohesion, c, is 0 kPa.

2.2.2 Bearing capacity parameters

The bearing capacity parameters are defined by the following equations from Meyerhof [12].
For the foundation soil, N_{qf} = 33.3, N_{cf} = 46.1, $N_{\gamma f}$ = 37.2. For the reinforced soil, N_{qr} = 23.2, N_{cr} = 31.7, $N_{\gamma r}$ = 22.

2.2.3 Other parameters

The wall height, H, is 16 meters.
The surcharge, q, is 10 kN/m^2.
The retained soil backslope, β, is 0 degrees.
The active earth pressure coefficient is from Rankine [12]. K_{ab} for back soil is 0.31, and K_{ar} for reinforced soil is 0.33.

3. RECTANGULAR MBW WALL

3.1 1-Block Wall Geometry

The geometry of the rectangular MBW wall or 1-block wall, as well as its force diagram, is shown in Fig. 5.

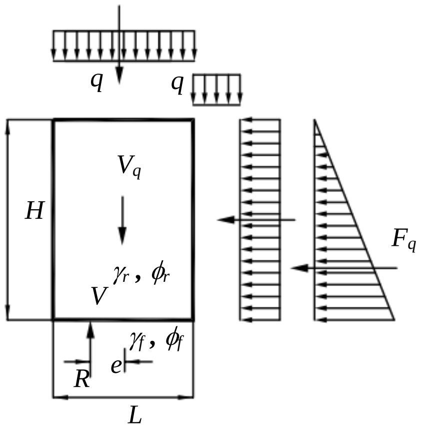


Fig. 5 1-block rectangular wall force diagram

3.2 FHWA Design Criteria

The additional notations are as follows:
V is the vertical load in kN/m.
V_q is the vertical surcharge load in kN/m.
F is the horizontal load in kN/m.
F_q is the horizontal surcharge load in kN/m.
P_R is the resisting force in kN/m.
P_D is the driving force in kN/m.
M is the overturning moment in kN-m/m.
M_R is the resisting moment kN-m/m
M_{RBP} is the resisting moment in the bearing pressure in kN-m/m.
L' is the effective bearing width in m.
σ_V is the maximum bearing pressure in kN/m^2.
σ_{utl} is the ultimate bearing capacity of the foundation soil in kN/m^2.

The design calculation details from FHWA [1] are as follows.

The factor of safety - sliding

$$FSS = \frac{P_R}{P_D} = \frac{Vtan\emptyset_f}{F+F_q} = \frac{\gamma_b HLtan\emptyset_f}{{}^1/_2\gamma_b H^2K_{ab}+qHK_{ab}} \quad (1)$$

The factor of safety - overturning

$$FSO = \frac{M}{M_{RO}} = \frac{V(L/2)}{F(H/3)+F_q(H/2)} \quad (2)$$

Eccentricity (m)

$$e = \frac{L}{2} - \frac{M_{RBP}-M}{V+V_q} = \frac{L}{2} - \frac{\gamma_b HL\left(\frac{L}{2}\right)-F\left(\frac{H}{3}\right)-F_q\left(\frac{H}{2}\right)}{\gamma_b HL+qL} \quad (3)$$

The factor of safety–bearing capacity

$$FSB = \frac{\sigma_{ult}}{\sigma_V} = \frac{0.5L'\gamma_f N_{\gamma f}}{(V+V_q)/L'} = \frac{0.5(L-2e)\gamma_f N_{\gamma f}}{(\gamma_b HL+qL)/(L-2e)} \quad (4)$$

Under FHWA design criteria for external stability checking, $L \geq 0.7H$, $FSS \geq 1.5$, $FSO \geq 2.0$, $e \leq L/6$ and $FSB \geq 2.5$. FHWA also recommends global stability factor of safety ≥ 1.3.

3.3 1-Block Wall Optimization Models

The optimization model for 10 meters high 1-block wall is done by relaxing the design criteria of $L \geq 0.7H$ and trying to minimize L. Note that the constraint $L \geq 0.3H$ is added to guarantee the optimality.

1-block L/6 optimization model (1-block L/6)

Minimize L
Subject to *FSS in* (1) ≥ 1.5
FSO in (2) ≥ 2.0
e in (3) $\leq L/6$
FSB in (4) ≥ 2.5
$L \geq 0.3H$
(5)

FHWA also recommends maximum allowable eccentricity for competent foundation soil at L/4. Another 1-block optimization model for competent foundation soil is developed by replacing e in Eq. (3) < L/6 in Eq. (5) with e in Eq. (3) < L/4.Using an optimization model can decrease the base length from $0.7H$ to $0.6H$ for a rectangular wall or even $0.5H$ for a rectangular wall with competent foundation soil.

3.4 1-Block Wall Output Comparison

The output of the optimization model in (5) is shown in Table 1, as well as the criteria provided by FHWA using $L = 0.7H$. The output in Table 1 shows that the restrained design criteria for this rectangular wall design are its eccentricity. In the 1-block L/6 model in (5), eccentricity (e) at 1.59 reaches the maximum allowable eccentricity at L/6 (1.59 = 9.54/6). This e < L/4 optimization model for competent foundation base gives L at 8.01 or L/H at 0.5 with the binding constraint of FSB at 2.5.

Table 1 1-block wall output

Model	*0.7H*	*e ≤ L/6*	*e ≤ L/4*	*FHWA*
H, m	16	16	16	
L, m	11.2	9.54	8.01	
L/H	0.7	0.60	0.50	0.7
FSS	2.75	2.34	1.97	1.5
FSO	3.99	2.90	2.04	2.0
e, m	1.36	1.59*	1.89	
L/6, m	1.87	1.59	1.34	
L/4, m	2.8	2.39	2.00	
FSB	7.22	4.76	2.50*	2.50

Note: Model 0.7H uses L/H at 0.7. $e \leq L/6$ is the optimization model in (5). $e \leq L/4$ is the optimization model for a competent foundation. FHWA model is the minimum design criteria recommended by FHWA. * is the binding constraint.

4. 2- BLOCK TRAPAZOIDAL MBW WALL

4.1 2-Block Wall Geometry

The geometry of the 2-block trapezoidal MBW wall or 2-block wall and its force diagram is shown in Fig. 6.

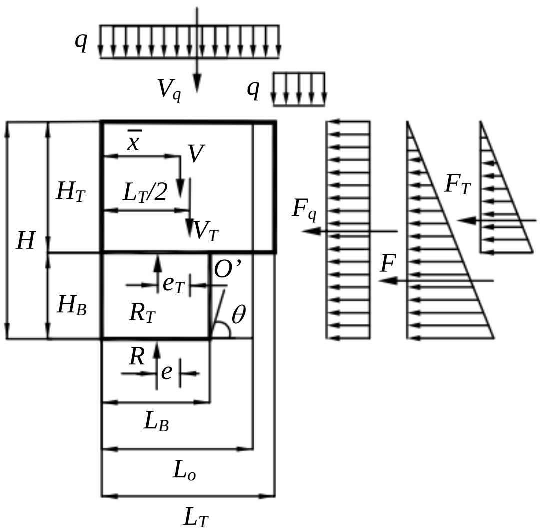


Fig. 6 2-block trapezoidal wall force diagram

4.2 FHWA Simplified Rules Optimization Models (FHWA Model)

Simplified rules from FHWA are applied to design this uneven reinforcement length wall. The equal sectional area can be stated as:

$$L_B H_B + L_T H_T = L_o H \quad (6)$$

where the minimum L_o is 0.7H.

This simplified rule is simply to guarantee that the trapezoidal wall gains a little less sliding factor of safety in Eq. (11) than the original rectangular wall. The decrease in the trapezoidal factor of safety is due to the extra length (L_T – L_o) of the surcharge load (q).

FHWA simplified rule also limits the different zone lengths to be less than 0.15H. This rule can be stated as:

Different zone length

$$L_T - L_B \leq 0.15H \quad (7)$$

Another FHWA simplified rule is the minimum base width, or base reinforcement length (L_B) is *0.4H*, stated as:

Minimum base length

$$L_B \geq 0.4H \quad (8)$$

Accompanying with FHWA simplified rules in Eq. (6), Eq. (7), and Eq. (8) to specify the base layer height (H_B) and the top layer height (H_T) in Fig. 6, one more relationship between the top layer height (H_T) and the top layer width (L_T - L_B) is needed. This relationship can be set through the top rear slope angle (θ_T) in Fig. 6. BS8006 in Fig. 6 for 3-block wall also implies that the upper block height should be greater than the lower block height and the different zone lengths as Z_1 = 0.5H, Z_2 = 0.75H, Z_3 = H, L_1 > 0.4H, ΔL< 0.15H. The rear slope is from 1.67V1H to 3.33V1H.

The rear slope can be set due to actual site restrictions, such as to avoid jacking the rock face at the back of the wall. This paper proposes the constraint in balancing the moment equilibrium at point *O'* in Fig. 6 that the geogrid holds enough strength to hold the right-wing of the top block to not fall due to the insufficient bearing capacity of the foundation soil, stated as follows:

Top block height

$$K_{ar}\frac{1}{2}\gamma_r H_T{}^2\left(\frac{H_T}{3}\right) + K_{ar} q H_T \frac{H_T}{2} \geq \gamma_r H_T (L_T - L_B)\frac{L_T - L_B}{2} + q(L_T - L_B)\frac{L_T - L_B}{2} \quad (9)$$

Internal stability is also in consideration. FHWA requires the reinforcement lengths at all layers in the resistant zone to be at least 1 meter, stated as follows.

Internal stability

$$L_B \geq H_B \tan(45 + \emptyset_r/2) + 1 \quad (10)$$

An optimization model for a 2-block wall following FHWA simplified rules for external stability to minimize the base width (L_B) is set as follows:

2-block FHWA optimization model (2-block FHWA)

Minimize	*Base length (L_B)*
Subject to	*FSS in Eq. (1) of top block ≥ 1.5*
	FSO in Eq. (2) of top block ≥ 2.0
	e_T in Eq. (3) of top block ≤ L_T/6
	FSB in Eq. (4) top block ≥ 2.5
	FSS in Eq. (1) of 2-block ≥ 1.5
	FSO in Eq. (2) of 2-block ≥ 2.0
	e in Eq. (3) of 2-block ≤ $\bar{x}/3$
	FSB in Eq. (4) 2-block ≥ 2.5
	Equal areas in Eq. (6)
	Different zone lengths in Eq. (7)
	Min base in Eq. (8), L_B ≥ 0.4H
	Top block height in Eq. (9)
	Internal stability in Eq. (10)
	L_T ≥ 0.7H
	L_T - L_B ≥ H/20

(11)

In Eq. (11), 2-block means the whole block, including the top block and the bottom block. $\bar{x}$ in Eq. (11) shown in Fig. 6 is the center of mass for the whole block. Meyerhof's effective bearing length for the 2-block wall is replaced $L' = L - 2e$ with $L' = \bar{x} - 2e$. For top-block wall, $L'_T = L_T - 2e_T$.

The minimum different zone length L_T - L_B ≥ *H/20* and the minimum bottom block height H_B ≥ *0.15H* are added to set lower bounds for this nonlinear optimization model to avoid the optimality and convergence problems. H/20 in the minimum different zone constraint is set following FHWA in designing superimposed MSE walls that if the difference in the offset for the upper wall and the lower wall is less than H/20, the two-tier wall can be treated as one wall. The bottom block height can be adjusted lower or higher according to the rock face at the back of the wall.

4.3 BS8006 Optimization Model (BS Model)

BS8006 recommends the top block reinforcement length to be at least 0.7H, the bottom reinforcement to be at least 0.4H, and the different zone lengths to be less than 0.15H. Equal areas constraint is not imposed as FHWA. The 2-block BS optimization model is similar to the 2-block FHWA model in Eq. (11) except that the equal areas constraint in Eq. (6) is relaxed.

4.4 Relaxing Minimum Top Length Model (Proposed Model)

In decreasing the base length, the minimum top length at 0.7H is relaxed. Omitting $L_T \geq 0.7H$ still guarantees the wall stability since both external stability (sliding, overturning, bearing, and eccentricity) and internal stability in Eq. (10) is checked at both the top block and the whole 2-block wall.

4.5 Comparison of 2-Block Wall Models

Table 2 2-block wall output

Description	FHWA	BS	Proposed
Height, H (m)	16	16	16
Base L_B (m)	10.19	9.30	8.37
Top L_T (m)	12.59	11.2*	10.77
$L_T - L_B$ (m)	2.4*	2.4*	2.4*
Top H_T (m)	6.71*	6.71*	6.71*
L_B /H	0.64	0.55	0.52
L_T /H	0.79	0.7*	0.67
$(L_T - L_B)/H$	0.15*	0.15*	0.15*
H_T/H	0.42*	0.42*	0.42*
$FSO_{RW} H_T$	1.0*	1.0*	1.0*
2-block FSS	2.75	2.41	2.30
2-block FSO	4.04	3.11	2.84
2-block e (m)	1.33	1.52	1.59*
2-block al. e (m)	1.88	1.65	1.59
2-block FSB	5.68	3.79	3.21
Top FSS_T	6.04	5.4	5.2
Top FSO	25.4	20.1	18.6
Top e_T (m)	0.23	0.26	0.27
Top al. e (m)	2.10	1.87	1.79
Top FSB_T	18.4	16.2	15.5
Area (m^2)	179.2*	156.9	150.0
Area/($0.7H^2$)	1.0*	0.88	0.84

Note: 2-block means the whole block. Top means the top block. al .e means allowable eccentricity. * is the binding constraint.

The 2-block proposed model is similar to the 2-block FHWA in Eq. (11) and BS8006 model except that the equal areas constraint in Eq. (6) and minimum top reinforcement length at 0.7H are relaxed. Wall stability is guaranteed through external and internal stability of the top block and the whole 2-blocks.

The optimized result for the three optimized models for 2-blocks walls is shown in Table 1. The different zone lengths and top block height are the binding constraints for all three models. The additional binding constraint for the FHWA model in Eq. (11) is the equal areas constraint. The additional constraint for the BS model is the top reinforcement length. The additional constraint for the Proposed model is the eccentricity.

The top block external stability is sliding, overturning, and bearing capacity are not the critical constraints in the design. This can be seen from its factors of safety and eccentricity. Practically, top block stability can be ignored in the design.

5. 3-BLOCK TRAPAZOIDAL MBW WALL

The 3-block MBW wall uses the same design parameters as the 2-block MBW wall. Its force diagram is similar to Fig. 6 of the 2-block wall, except that there is one middle between the top and the base block. The 3-block optimization models in FHWA, BS and Proposed models are also similar to those 2-block optimization models except that there are three sets of external stability checking for the top block, the top two blocks, and the whole three blocks.

The three optimized models for the three blocks are not shown. The output is shown in Table 3. Remind that the FHWA model has equal areas constraint. BS has exact value of block heights that $Z_1 = 0.5H$, $Z_2 = 0.75H$, $Z_3 = H$. Proposed model relaxed the equal areas constraint in FHWA model.

For the FHWA and the Proposed models, lower bounds for all heights are set to be at least 0.15H to guarantee that the optimization models will not force these values to be zero. Upper bounds for different heights are adjusted that there is no infeasibility. The upper bounds and lower bounds are used to avoid optimality and convergence problems, but these bounds are not the binding constraints in Table 3.

For all three models in Table 3, the same binding constraints are the different zone lengths and top block heights (in both middle block and top block). FHWA has one more binding constraint in equal areas. Both BS and the Proposed model have more binding constraints in FSB. Interestingly, the preset heights by BS met the top block height constraint in (9) with the overturning factor of safety of the right-wing top block at 1.0. That is shown in $FSO_{RW} H_T$ of 1.0 in Table 2.

With relaxing equal areas constraint in the FHWA model, the Proposed model performed better in the minimum base length (L_B) and area. The area in the proposed model can be reduced to 0.84 compared with 0.88 in the BS model and 1.0 in the FHWA model. Similar to the 2-block model, there is no need for external stability checking for the top blocks. Only the whole block's external stability checking is needed.

6. 4-BLOCK TRAPAZOIDAL PROPOSED MODEL AND COMPARISION

The 4-block optimization model also relaxes the FHWA equal areas constraint. The binding constraints are all different lengths (the second block, the third block, the top block) at *0.15H* and the block heights. An upper bound of second top block height is used as a binding constraint to avoid infeasibility in violating the block height constraint in (26) for this 4-block Proposed model result. Table 4 compares only the proposed model for rectangular 1-block, trapezoidal 2-block, 3-block, and 4-block optimized models.

From Table 4, 4-block performs better than 3-block, 2-block, and 1-block models in base length (L_B) with the trade-off in increasing top length (L_T). In terms of the area, 1-block and 2-block are better than 3-block and 4-block because they have fewer constraints to achieve. For trapezoidal 2-block, 3-block, and 4-block, foundation soil bearing capacity (*FSB*) is a binding constraint. This follows the recommendation by FHWA that the trapezoidal wall should be considered under competent foundation soil conditions.

The authors suggest using three zones block is the best option since the 4-block needs to use a subjective upper bound of the second top height to avoid infeasibility. A simple rule recommended by the authors is to combine both the FHWA simplified rules in equal areas and BS8006 guidelines resulted in a simple three zones block with $Z_1 = 0.5H$, $Z_2 = 0.75H$, $Z_3 = H$, $\Delta L = 0.15H$, $L_1 = 0.5125H$. The dimension of this three zones wall is simple to use with sufficient stability under competent foundation soil conditions.

7. COMPARISON OF GLOBAL STABILITY

Global stability checking using LEM for rectangular MBW wall of the model in (21), the Proposed models of 2-block, 3-block, and 4-block are shown in Fig. 7-10. Parameters used in Fig. 7–10 are described in Table 5. Geogrid tensile strengths using vertical spacings (S_v) of 0.5 meters at 80 kN/m are derived from the maximum force of the bottom geogrid (T_{max}) with the factor of safety in rupture at 1.5, hence $1.5T_{max}$ = 68.7 kN/m where

$$1.5T_{max} = 1.5K_{ar}(\gamma_r H + q)S_v \quad (11)$$

Table 3 3-block wall output

Description	FHWA	BS	Proposed
Height, H (m)	16	16	16
Base L_B (m)	8.46	7.66	7.66
Top L_T (m)	13.26	12.46	12.46
L_B/H	0.53	0.48	0.48
$(L_M - L_B)/H$	0.15*	0.15*	0.15*
$(L_T - L_M)/H$	0.15*	0.15*	0.15*
L_T/H	0..83	0.78	0.78
H_B/H	0.28	0.25	0.28
H_M/H	0.30*	0.25	0.30*
H_T/H	0.42*	0.5	0.42*
FSO_{RW}-H_M	1.0*	1.0**	1.0*
FSO_{RW}-H_T	1.0*	1.39	1.0*
3-block *FSS*	2.75	2.62	2.66
3-block *FSO*	4.12	3.75	3.82
3-block *e* (m)	1.31	1.39	1.36
3-block al. *e* (m)	1.92	1.84	1.84
FSB	3.39	2.5*	2.5*
Area (m^2)	179.2*	170.6	166.4
Area/($0.7H^2$)	1.0*	0.95	0.93

Note: 3-block means the whole block. * is the binding constraint. *M* implies middle block. *T* implies top block.

Table 4 Comparison of the Proposed model output

Description	1	2	3	4
Height H (m)	16	16	16	16
Base L_B (m)	9.54	8.37	7.76	7.62
Top L_T (m)	9.54	10.8	12.5	13.9
L_B/H	0.6	0.52	0.48	0.48
L_T/H	0.6	0.67	0.78	0.87
FSS	2.34	2.30	2.66	2.79
FSO	2.90	2.84	3.82	4.26
e (m)	1.59*	1.59*	1.36	1.28
al *e* (m)	1.59	1.59	1.84	1.96
FSB	4.76	2.5*	2.5*	2.5*
Area (m^2)	152.6	150.0	165.5	181.9
Area/($0.7H^2$)	0.85	0.84	0.92	1.02

Note: 1 is the rectangular wall in (5). 2 is the 2-block wall in (11). 3 is the 3-block wall. 4 is the 4-block wall.

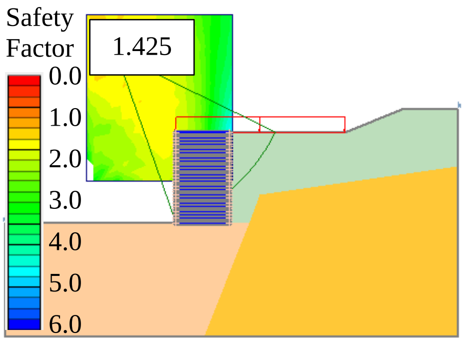


Fig. 7 Rectangular wall LEM global stability

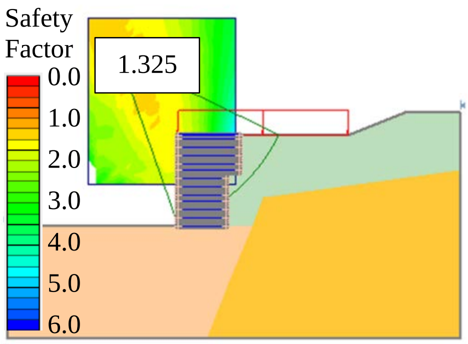


Fig. 8 2-block wall LEM global stability

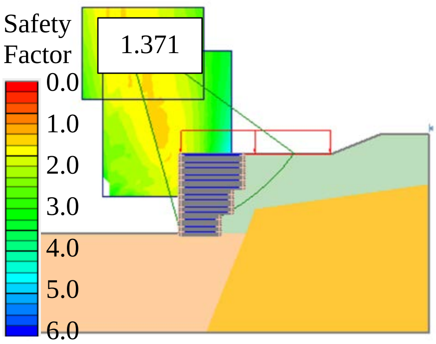


Fig. 9 3-block Proposed model LEM global stability

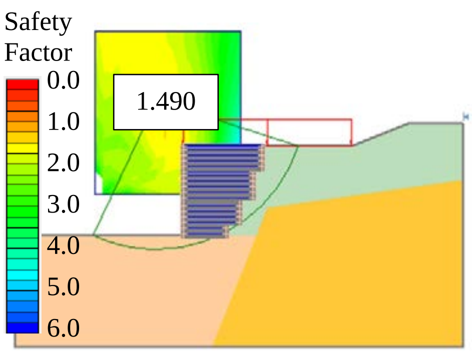


Fig. 10 4-block Proposed model LEM global stability

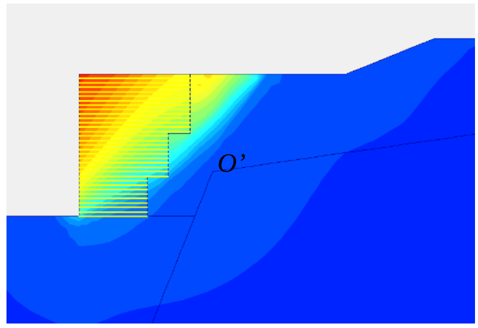


Fig. 11 FEM global stability with Top block height constraint

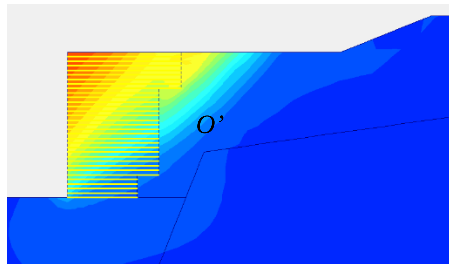


Fig. 12 FEM global stability without Top block height constraint

Table 5 Parameters used for LEM global stability checking

Material	Color	Parameter
Rock	Yellow	$\gamma = 18$ kN/m^3, $\phi = 40$
Retained	Green	$\gamma = 18$ kN/m^3, $\phi = 30$
Foundation	Brown	$\gamma = 18$ kN/m^3, $\phi = 35$
Backfill	Orange	$\gamma = 18$ kN/m^3, $\phi = 32$
Geogrid	Blue	Tensile 80kN/m @0.5m

Table 6 Parameters used for FEM global stability checking

Material	Bck	Rft	Fnd	Rock
Model	MC	MC	MC	MC
γ-unsat, kPa	18	18	18	22
γ-sat, kPa	20	20	20	-
C, kPa	1	1	1	10
ϕ	30	32	35	35
υ	0.3	0.3	0.3	0.3
k, m/day	0.01	1	0.1	-
E, MPa	15	20	30	30

Note: Bck is back the soil. Rft is reinforced soil. Fnd is foundation soil. Soil type is drained. Rock type is non-porous. EA fro geogrid is 1600 kN/m

LEM global stability of rectangular, 2-block, 3-block and 4-block are 1.425, 1.345, 1.371, and 1.490. All are greater than the 1.3 required by FHWA.

FEM global stability for 3-block Proposed model is also shown in Fig. 11 where the deformation is smooth at point O' that imposed that top block height constraint. Fig. 12 shows the proposed model that intentionally took the top block height constraint out of the model. Displacement above point O' in Fig. 12 is not smooth since there is no balance in moment equilibrium between the geogrid strength and the right-wing weight. The information of FEM parameters is shown in Table 6. EA for geogrid is 800 kN/m which is from 80 kN/m divided by 0.1 strain to generate that 80 kN/m.

8. CONCLUSIONS

Optimization techniques can help design MSE walls with economic and safe solutions. The reinforcement length can be decreased from 0.7*H* to 0.6H for or even 0.5H for a rectangular wall with competent foundation soil. For a trapezoidal wall, the three zones wall seems to be the best option in terms of the minimum base length and optimality difficulty. The three zones wall proposed by this study can reduce the base length ratio (L_B/H) to 0.48 and reduce the cross-sectional area down to 0.92. from the area of 1.0 of rectangular walls with the reinforcement ratio of 0.7*L/H*. A simple three zones wall without optimization following BS8006 guideline with 0.5125*H* base length is also recommended. In the solution sensitivity, these recommended *L/H* ratios need to be rechecked if any design parameters are changed.

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