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ADOMAIN DECOMPOSITION METHOD FOR SOLVING OF FREDHOLM INTEGRAL EQUATION OF THE SECONED KIND USING MATLAB

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ABSTRACT: Integral equation has been one of the essential tools for various areas of applied mathematics. The Fredholm integral equations can be derived from boundary value problems. In this paper, we are concerned with the application of the Adomian Decomposition Method by using MATLAB program for the Fredholm integral equation of the second kind. The new computational algorithm is applied directly without using any transformation, linearization, discretization or taking some restrictive assumptions. An exact solution of the illustrative example was successfully fund using the proposed method, and the results are compared with the results of the existing methods. The Adomain Decomposition will be obtained easily without linearizing the problem by implementing the Adomain Decomposition Method by using MATLAB program rather than the standard methods for the exact solutions. The concern will be on the determination of the solution $\boldsymbol{y(x)}$ of the Fredholm integral equations of the second kind. The results indicated that the method is very effective and simple.

Keywords: Fredholm integral equation of the second kind; Adomian Decomposition Method; MATLAB.

1. INTRODUCTION

In this paper, we consider the Fredholm integral equation of the second kind

$$y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)\,dt \qquad (1)$$

The unknown function $y(x)$, that will be determined, occurs inside and outside the integral sign. Th kernel $K(x,t)$ and the function $f(x)$ are given real-valued functions, and λ is aparameter.

In this paper, we present the computation of exact solution of Fredholm integral equation of the second kind using MATLAB.

2. ADOMIAN DECOMPOSITION METHOD

In this section, we use the technique of the Adomian Decomposition Method [4,9,13,14] . The Adomain Decomposition Method consists of decomposing the unknown function $y(x)$ of any equation into a sum of an infinite number of components defined by the decomposition series

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \qquad (2)$$

Or equivalenty

$$y(x) = y_0(x) + y_1(x) + y_2(x) + y_3(x) + \cdots \qquad (3)$$

Where the components $y_n(x), n \geq 0$ will be determined recurrently. The Adomain Decomposition Method concerns itself with finding the components $y_0(x), y_1(x), y_2(x), y_3(x), \cdots$ individually.

To establish the recurrence relation, we substitute (2) into the Fredholm integral equatin (1) to obtain

$$\sum_{n=0}^{\infty} y_n(x) = f(x) + \lambda \int_a^b K(x,t)\left(\sum_{n=0}^{\infty} y_n(t)\right) dt, \qquad (4)$$

or equivalenty

$$y_0(x) + y_1(x) + y_2(x) + y_3(x) + \cdots = f(x) + \lambda \int_a^b K(x,t)\,[\,y_0(t) + y_1(t) + y_2(t) \cdots]dt \qquad (5)$$

The zeroth component $y_0(x)$ is identified by all terms that are not included under the integral sign . (This means that the components $y_n(x), n \geq 0$ of the unknown function $y(x)$ are completely determined by setting the recurrence relation

$$y_0(x) = f(x), \qquad y_{n+1}(x) = \lambda \int_a^b K(x,t)y_n(t)\,dt, \quad n \geq 0 \qquad (6)$$

or equivalenty

$$y_0(x) = f(x),$$

$$y_1(x) = \lambda \int_a^b K(x,t)y_0(t)\,dt$$

$$y_2(x) = \lambda \int_a^b K(x,t)y_1(t)\,dt$$

$$y_3(x) = \lambda \int_a^b K(x,t) y_2(t)\, dt$$

$$y_4(x) = \lambda \int_a^b K(x,t) y_3(t)\, dt \qquad (7)$$

And so on for other component's.

In view of (7), the components $y_0(x), y_1(x), y_2(x), y_3(x), \cdots$ are completely determined. As a result, the solution $y(x)$ of the Fredholm integral equation (1) is readily obtained in a series form by using the series as assumption in (2).

3. EXAMPLE

3.1 Example 1. Consider the Fredholm integral equation of second kind

$$y(x) = \cos x + 2x + \int_0^{\pi} xty(\mathrm{t})dt \,.$$

Applying the Adomian Decomposition Method we find

$$\sum_{n=0}^{\infty} y_n(x) = \cos x + 2x + \int_0^{\pi} xt \sum_{n=0}^{\infty} y_n(t)\, dt \,.$$

To determine the components of $\boldsymbol{y(x)}$, we use the recurrence relation

$$y_0(x) = \cos x + 2x,$$

$$y_{n+1}(x) = \int_0^{\pi} xt\, y_n(\mathrm{t})dt, \qquad n \geq 0.$$

This in turn gives

$$y_0(x) = \cos x + 2x,$$

$$y_1(x) = \int_0^{\pi} xt\, y_0(\mathrm{t})dt = \frac{2x(\pi^3 - 3)}{3},$$

$$y_2(x) = \int_0^{\pi} xty_1(\mathrm{t})dt = \frac{2627688692113193x\pi^3}{422212465065984},$$

$$y_3(x) = \int_0^{\pi} xty_2(\mathrm{t})dt = \frac{565797518176089x\pi^3}{8796093022208},$$

$$y_4(x) = \int_0^{\pi} xty_3(\mathrm{t})dt = \frac{1461939532799565x\pi^3}{2199023255552},$$

$$y_5(x) = \int_0^{\pi} xt\, y_4(\mathrm{t})dt = \frac{5666162705481445x\pi^3}{824633720832},$$

$$y_6(x) = \int_0^{\pi} xt\, y_5(\mathrm{t})dt = \frac{228758604898117x\pi^3}{3221225472},$$

$$y_7(x) = \int_0^{\pi} xt\, y_6(\mathrm{t})dt = \frac{1773238149117601x\pi^3}{2415919104},$$

$$y_8(x) = \int_0^{\pi} xt(\mathrm{t})dt = \frac{2290896361316795x\pi^3}{301989888},$$

$$y_9(x) = \int_0^{\pi} xty_8(\mathrm{t})dt = \frac{138734700048595x\pi^3}{1769472},$$

$$y_{10}(x) = \int_0^{\pi} xt\, y_9(\mathrm{t})dt = \frac{1433882164955047x\pi^3}{1769472},$$

$$y_{11}(x) = \int_0^{\pi} xty_{10}(\mathrm{t})dt = \frac{694677298961617x\pi^3}{82944},$$

$$y_{12}(x) = \int_0^{\pi} xty_{11}(\mathrm{t})dt = \frac{1346209783445453x\pi^3}{15552}.$$

And so on. Using (2) gives the series solution

$$y(x) = \cos x + 2x + \frac{2x(\pi^3 - 3)}{3} + \frac{2627688692113193x\pi^3}{422212465065984} + \frac{565797518176089x\pi^3}{8796093022208} + \frac{1461939532799565x\pi^3}{2199023255552} + \frac{5666162705481445x\pi^3}{824633720832} + \frac{228758604898117x\pi^3}{3221225472} + \frac{1773238149117601x\pi^3}{2415919104} + \frac{2290896361316795x\pi^3}{301989888} + \frac{138734700048595x\pi^3}{1769472} + \frac{1433882164955047x\pi^3}{1769472} + \frac{694677298961617x\pi^3}{82944} + \frac{1346209783445453x\pi^3}{15552} + \cdots$$

$$y(x) = \cos x + 2x + \frac{2x(\pi^3 - 3)}{3} + \frac{3277455530293170328219378537x\pi^3}{34199209670344704} + \cdots$$

Then the gives the exact solution

$$y(x) = \frac{33873897660118575396366621689x}{11399736556781568} + \cos x$$

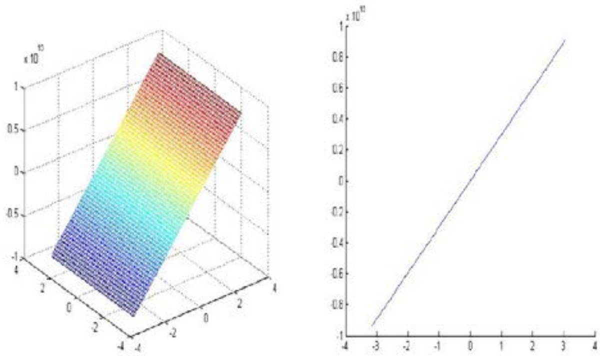


Fig. 1 Plot 3D and 2D of the exact solutions result of Fredholm integral equation for example 1.

3.2 Example 2. Consider the Fredholm integral equation of second kind

$$y(x) = e^{x+2} - 2\int_0^1 e^{x+t} y(t)dt\,.$$

Applying the Adomian Decomposition Method we find

$$\sum_{n=0}^{\infty} y_n(x) = e^{x+2} - 2\int_0^1 e^{x+t} \sum_{n=0}^{\infty} y_n(t)\, dt\,.$$

To determine the components of $\boldsymbol{y(x)}$, we use the recurrence relation

$$y_0(x) = e^{x+2},$$
$$y_{n+1}(x) = -2\int_0^1 e^{x+t}\, y_n(t)dt, \quad n \ge 0.$$

This in turn gives

$$y_0(x) = e^{x+2},$$
$$y_1(x) = -2\int_0^1 e^{x+t}\, y_0(t)dt = -2e^{x+2}\left(\frac{e^2}{2} - \frac{1}{2}\right),$$

$$y_2(x) = -2\int_0^1 e^{x+t}\, y_1(t)dt$$
$$= \frac{3596718833299159}{281474976710656} e^{x+2}\left(\frac{e^2-1}{2}\right),$$

$$y_3(x) = -2\int_0^1 e^{x+t}\, y_2(t)dt$$
$$= \frac{-23896921143600823608451547612389 e^x(e^2-1)}{79228162514264337593543950336},$$

$$y_4(x) = -2\int_0^1 e^{x+t}\, y_3(t)dt$$
$$= \frac{1192802888892091002535651796753 e^x(e^2-1)}{618970019642690137449562112},$$

$$y_5(x) = -2\int_0^1 e^{x+t}\, y_4(t)dt$$
$$= \frac{-7620884572098113352043020692037 e^x(e^2-1)}{618970019642690137449562112},$$

$$y_6(x) = -2\int_0^1 e^{x+t}\, y_5(t)dt$$
$$= \frac{24345129527304974066926436076553 e^x(e^2-1)}{309485009821345068724781056},$$

$$y_7(x) = -2\int_0^1 e^{x+t}\, y_6(t)dt$$
$$= \frac{-19442799785710562748951626477373 e^x(e^2-1)}{38685626227668133590597632},$$

$$y_8(x) = -2\int_0^1 e^{x+t}\, y_7(t)dt$$
$$= \frac{31055284637795404994033548487189 e^x(e^2-1)}{9671406556917033397649408},$$

$$y_9(x) = -2\int_0^1 e^{x+t}\, y_8(t)dt$$
$$= \frac{-24801744464891759411050052979325 e^x(e^2-1)}{1208925819614629174706176},$$

$$y_{10}(x) = -2\int_0^1 e^{x+t}\, y_9(t)dt$$
$$= \frac{19807467092192025406293026624207 e^x(e^2-1)}{151115727451828646838272},$$

$$y_{11}(x) = -2\int_0^1 e^{x+t}\, y_{10}(t)dt$$
$$= \frac{-15818877303717202044491673925583 e^x(e^2-1)}{18889465931478580854784},$$

$$y_{12}(x) = -2\int_0^1 e^{x+t}\, y_{11}(t)dt$$
$$= \frac{25266923628887507229618429573899 e^x(e^2-1)}{4722366482869645213696}.$$

And so on. Using (2) gives the series solution

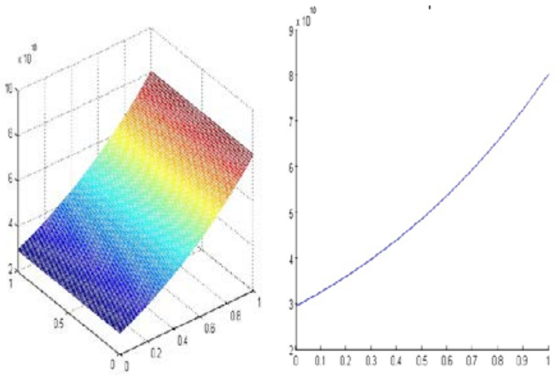
$$\begin{aligned} y(x) &= e^{x+2} - 2e^{x+2}\left(\frac{e^2}{2} - \frac{1}{2}\right) + \frac{3596718833299159}{281474976710656} e^{x+2}\left(\frac{e^2-1}{2}\right) \\
&- \frac{23896921143600823608451547612389 e^x(e^2-1)}{79228162514264337593543950336} \\
&+ \frac{1192802888892091002535651796753 e^x(e^2-1)}{618970019642690137449562112} \\
&- \frac{7620884572098113352043020692037 e^x(e^2-1)}{618970019642690137449562112} \\
&+ \frac{24345129527304974066926436076553 e^x(e^2-1)}{309485009821345068724781056} \\
&- \frac{19442799785710562748951626477373 e^x(e^2-1)}{38685626227668133590597632} \\
&+ \frac{31055284637795404994033548487189 e^x(e^2-1)}{9671406556917033397649408} \\
&- \frac{24801744464891759411050052979325 e^x(e^2-1)}{1208925819614629174706176} \\
&+ \frac{19807467092192025406293026624207 e^x(e^2-1)}{151115727451828646838272} \\
&- \frac{15818877303717202044491673925583 e^x(e^2-1)}{18889465931478580854784} \\
&+ \frac{25266923628887507229618429573899 e^x(e^2-1)}{4722366482869645213696} \\
&+ \cdots \end{aligned}$$

$$\begin{aligned} y(x) &= e^{x+2} - \frac{3596718833299159}{562949953421312} e^{x+2} \\
&+ \frac{2341837215339010984094056314041165497573}{79228162514264337593543950336} e^{x+2} \\
&+ \cdots \end{aligned}$$

Then it gives the exact solution as

$$y(x) = e^x$$

Fig.2 Plot 3D and 2D of the exact solutions result of Fredholm integral equation for example 2



4. CONCLUSION

The aim of this paper is to introduce a new computational algorithm using MATLAB for solving Fredholm integral equations of the second kind using Adomain Decomposition Method without any linearization, discritization, transformation, or taking some restrictive assumptions. The computations associated with two examples were performed using MATLAB. This method proved to be an accurate and efficient technique.

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