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OFF-CENTER TETRAHEDRAL WEDGES

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ABSTRACT: Tetrahedral wedges in rock slopes can have symmetric and asymmetric shapes. Asymmetry is commonly recognized as differences in dip angles of the sliding planes. This form of asymmetry is known to influence the kinematic feasibility and stability of a wedge. Asymmetry also influences the appearance of wedges and therefore the ability to recognize wedges in rock faces and in data from rock masses. The relative orientation of the sliding planes in relation to the lower and upper slope faces is also part of the asymmetry system. In particular, the direction of the line of intersection which determines the sliding direction of two planes, relative to the direction of the slope of the rock face introduces an additional type of asymmetry. This form of asymmetry is defined here as off-center wedges. A detailed terminology for asymmetric tetrahedral wedges is presented and the implications for kinematic feasibility, sliding mechanism and stability analysis are discussed. A field example from Auckland New Zealand is presented.

Keywords: Wedge; Kinematic stability; Rock slope; Sliding

1. INTRODUCTION

The stability of a tetrahedral wedge exposed in a rock slope has been shown to be influenced by its shape in both static [1-3] and dynamic [4-6] conditions. Wang et al. [7] demonstrated limit-equilibrium and numerical solutions to wedge stability including comparison of symmetric and asymmetric examples. In a probabilistic analysis of wedge stability it was found that the reliability results were very sensitive to variations in the shape and therefore symmetry of the wedge [8].

Large amounts of discontinuity orientation data need to be investigated in field-based and synthetic rock mass characterization projects [9]. Stereographic methods form the initial step in wedge stability assessment in many rock engineering investigations [10-17]. In a summary of conventional stereographic methods, Wyllie and Mah [18] indicated that wedge-forming poles are restricted to inside the daylighting envelope of the slope face plane. However, the limits to wedge-forming poles has been shown to be more complex and defined by limits to the orientations of great circles on which wedge-forming pole pairs can lie [19].

Stereographic methods have also been used to distinguish between mechanisms of single-plane and two-plane sliding mode of wedges using the line of intersection [20] or the poles and daylighting envelope [21].

This paper is focussed on tetrahedral wedges defined by two sliding planes and an upper and lower slope face. The presence of tension cracks or basal sliding planes is not considered. Asymmetry is developed by the relationship between all four faces. The typical distributions of planes which can form kinematically feasible wedges are considered in terms of the shape and symmetry of a tetrahedral wedge. The typical shapes of feasible wedges are illustrated on a stereograph. Examples of applications of the approach are given for data from jointed and faulted sedimentary rock in Auckland, New Zealand. The feasibility and factor of safety are shown to be related to the shapes of the wedges, including the off-center asymmetry of some wedges.

2. TERMINOLOGY

2.1 Angle Between Sliding Planes

The angle between sliding planes is an important geometric parameter of wedges. The term dihedral angle can be used to specifically identify the angle measured in a plane perpendicular to the line of intersection of the planes. The terms thin and thick were applied by Hudson and Harrison [3] up to a dihedral angle of 90°. They did not extend their analysis beyond that value. When higher dihedral angles are considered the wedges become thinner with respect to the slope face. Therefore, the terms narrow (for thin) and open (for thick) are adopted. The term very open is applied to wedges where the two sliding planes are approaching parallelism with each other (Table 1). As wedges become increasingly open, they can approach the dip direction of the slope face and therefore approach the planar sliding case which represents a minimum stability for that dip angle. On a stereograph, that appears as poles to sliding planes approaching the opposite direction to the dip direction of the slope face. Lateral limits of planar sliding are typically accepted as +/-20º [18]. These limits are a general reference only and do not define a change in failure mechanism. As will be shown below, the change in mechanism from single plane to two-plane sliding varies according to the degree to which wedges are off-center.

2.2 Wedge Symmetry

The symmetry of a tetrahedral wedge must be considered in two ways. First, the symmetry of the two sliding planes relative to each other. Second, the symmetry of the sliding planes relative to the slope face also needs to be considered. These aspects of symmetry can be considered to be related to the sliding plane dip angles and dip directions, respectively. If the two sliding planes have the same dip angle then a plane halfway between them would be vertical. If the two sliding planes have the different dip angles then a plane halfway between them would be dipping. Hudson and Harrison [3] refer to this as upright and inclined, respectively. For simplicity, the dip angle of each plane will be compared and will be termed equal or unequal. An arbitrary 10º difference in angle of dip of the planes (irrespective of their dip directions) is applied here to separate the two terms.

Regarding the influence of dip direction, if a wedge is oriented such that intersection line of the two planes is the same as the dip direction of the slope face that will be described here as a centered wedge. Other sliding directions will be referred to as off-center. Therefore, the symmetry of the tetrahedral wedge is a combination of its centered or off-center characteristics and its equal versus unequal characteristics (Table 1). A symmetrical wedge must be both centered and equal.

A range of examples of cetrered wedges are shown on a stereograph and as block diagrams in Fig. 1. Note that the intersection line can be directly observed as the pole to the great circle on which the wedge-forming poles lie [19]. Therefore, the great circle on which the poles of wedge-forming planes lie can be used to recognize all the symmetry characteristics of the wedge.

2.3 Off-center Wedges

Wedge-forming planes with poles lying in other locations of the stereograph other than on a centered great circle are also capable of forming a wedge. Such poles will typically lie on a great circle which has a different strike to the strike of the slope face. An off-center wedge great circle example has a pole (equivalent to line of intersection of wedges) trending northeast (Fig. 2a). For simplicity, a southeast trending example is not shown, as the principle can be applied in a mirror image for comparison. On this basis, a wide range of wedge-

Table 1 Terminology used to describe features of tetrahedral wedges observable on a stereographic representation

Application	Term	Comment/ Quantification
Describes the (dihedral) angle between two sliding planes forming a wedge	Narrow	<60º (>120º pitch on stereograph between poles on opposite sides of the slope face)
	Open	60º-120º (60º-120º pitch angle on stereograph)
	Very open	>120º (<60º pitch on stereograph between poles on opposite sides of the slope face)
Describes the overall shape of a tetrahedral wedge (defined by four planes)	Symmetric	Only possible for centered, equal wedges
	Asymmetric or Non-symmetric	Asymmetry of a tetrahedral wedge can be due to off-centered orientation of the great circle joining the sliding plane poles and/or unequal dip of sliding planes
Describes the direction of the sliding plane intersection relative to the slope direction	Centered	Great circle joining sliding plane poles dips 180º from slope face direction
	Off-center	Great circle joining sliding plane poles dips 180º +/- <30º from slope face direction
	Highly off-center	Great circle joining sliding plane poles dips 180º +/- >30º from slope face direction
Describes the difference in dip of the wedge sliding planes	Equal	Difference of dip <10º
	Unequal	Difference of dip >10º

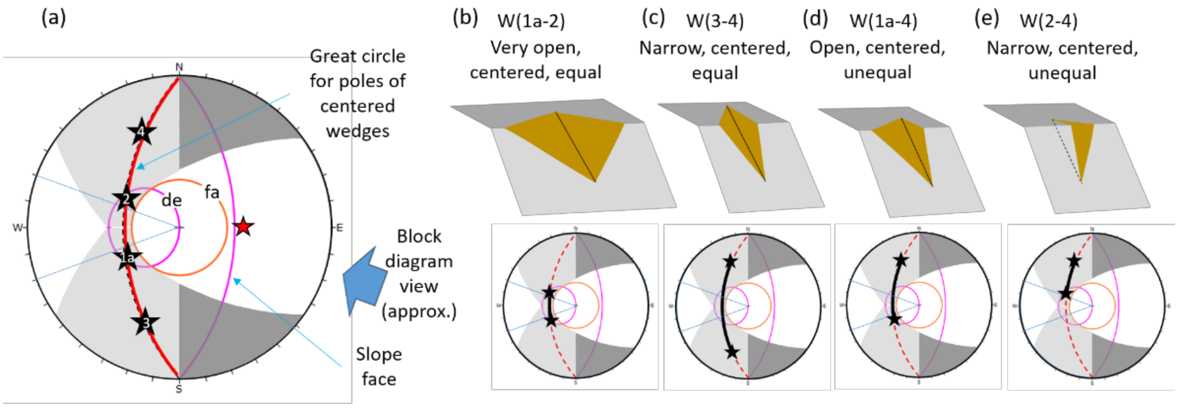

Fig.1. a) Stereograph (lower hemisphere, equal angle) of an east-dipping slope face showing the daylight envelope (de) and a 35º friction circle (fc). Radial lines mark a +/-20º lateral limit for planar sliding. Gray areas indicate where poles to feasible wedge sliding planes can be located. The dark gray areas indicate that the poles represent planes dipping toward the slope face. A great circle representing centered wedges is shown with example pole locations (black stars). All wedges formed by planes with poles on this great circle share an intersection line which is the pole to the great circle (red star). (b-e) The example wedge-forming pole combinations are illustrated as block diagrams and stereographs with the recommended descriptive terminology.

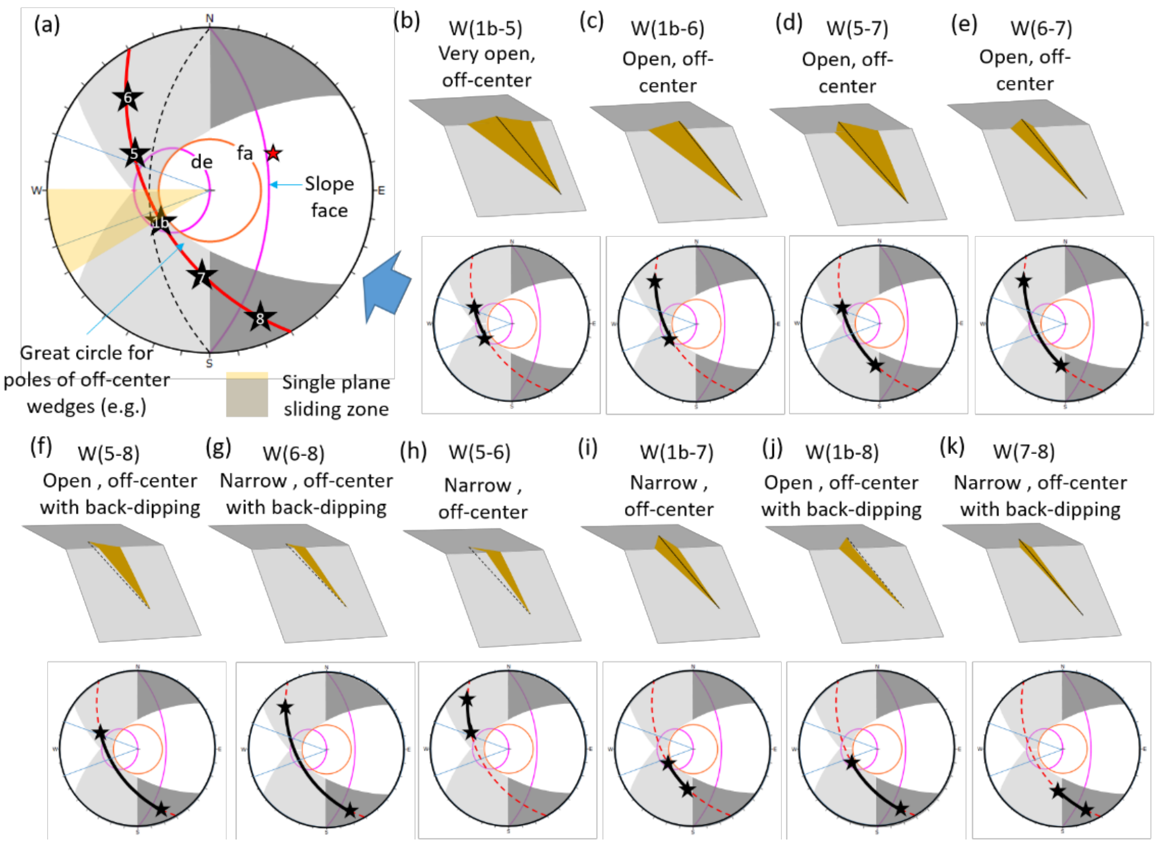

Fig.2. a) Stereograph (lower hemisphere, equal angle) of an east-dipping slope face showing the daylight envelope (de) and a 35º friction circle (fc). Radial lines mark a +/-20º lateral limit for planar sliding. Gray areas indicate where poles to feasible wedge sliding planes can be located. The dark gray areas indicate that the poles represent planes dipping toward the slope face. An example off-center great circle is shown with example pole locations (stars) along that great circle. (b-k) The wedge-forming pole combinations are illustrated as block diagrams and stereographs with the recommended descriptive terminology

forming possibilities can be considered for example, the eight general locations of poles shown as numbered stars on Fig. 1 and Fig. 2. Note that location 1a and 1b represent the approximate intersection of the two great circles from Fig. 1 and Fig. 2.

Location 1 (1a and 1b) represents a pole close to the lateral limit of planar sliding near where the centered and off-center great circles intersect. Location 2 is symmetrical to Location 1a. Locations 3 and 4 occur nearer the primitive circle (outer edge of the stereograph) on the centered great circle (Fig. 1). Locations 5 and 6 occur on the opposite side of the stereograph from Location 1b, between the lateral limit and the primitive circle on the off-center great circle (Fig. 2). Location 7 and 8 occur on the same side of the stereograph as Location 1b nearer the primitive circle on the off-center great circle. Location 8 is distinguished by dipping back into the slope face. The numbering of these locations is entirely arbitrary and they have been selected for the purpose of illustrating general examples of the shapes of kinematically feasible wedges according to the position of poles on a stereograph.

Any combination of poles from great circle distribution e.g. any pair of poles from Locations 1a-4 or Locations 1b-8 will form a wedge with the same intersection plunge (red star on Fig. 1 and 2). The shape of the wedge will differ greatly for each pair of planes. For example, the wedge formed by poles at Locations 1 and 2 are very open whereas a wedge formed by poles at locations 3 and 4 would be narrow.

2.4 Wedge Sliding Mode

The lateral limits of planar sliding are recognized as an arbitrary range used for practical purposes [18]. The occurrence of single plane sliding wedges and two-plane sliding wedges depends on other geometric relationships [20,21]. For centered wedges, the single plane sliding mode cannot occur. Even two planes one degree either side of the plane perpendicular to the slope face will form a two-plane sliding pair, in the centered case.

Where a pair of planes (poles) are joined by an off-center great circle the zone of single plane sliding is defined by the difference between the strikes of the slope face and the off-center great circle. The example in Fig. 2 has a 30° difference in strike between these two features and therefore the range of the single plane sliding zone is 30° as shown on Fig. 2a. A wedge formed by a pair of poles with one of those poles being in that zone will have the mechanism of single plane sliding on that plane. A wedge formed by a pair of poles both of which are outside that zone will have the mechanism of two plane sliding.

Identifying the wedge sliding mode requires information on the joint orientations (Fig. 3). The sliding mode will influence the actual sliding direction and therefore is an important consideration in wedge stability analysis.

Wedges are often of irregular shape and orientation which influences their stability (Fig. 4). When multiple planes are present in a rock mass they can combine in various ways to form wedges. For example, a basement wall could have two different wedges that share one of the wedge planes (Fig. 5). When viewed on a stereograph it can be shown that there is a zone on the stereograph in which planes of single-plane sliding occur (Fig.6).

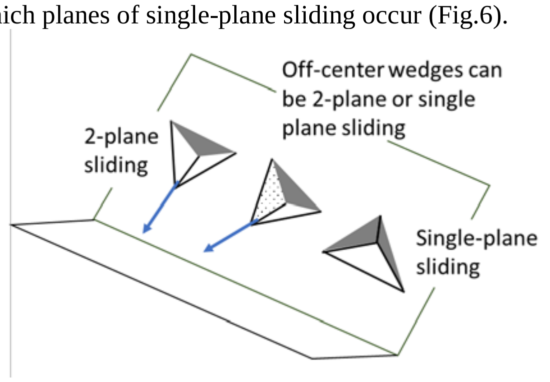

Fig.3 Wedges of various sliding modes

Fig.4 Wedge in a rock slope at Kicking Horse Canyon near Golden, British Columbia (used with permission, Brandon Thomas, Greely Rock Ltd)

3. FIELD EXAMPLE: AUCKLAND, NEW ZEALAND

The rocks in the field area are interbedded sandstone and siltstone of the Miocene age Waitemata Group. Weathered rock and soil approximately 2 m thick is present at the top of the cliff. Coastal retreat in sedimentary rocks north of Auckland, New Zealand is controlled by bedding, joints and faults [22]. A report on a fatal rockfall at Rothesay Bay, in the northern suburbs of Auckland New Zealand by Hancox [23] concluded that the failure occurred by joint-controlled sandstone block failure related to erosional undercutting of a sandstone bed with heavy rain two weeks prior and a low to moderate earthquake 16 hours prior was likely to have influenced the timing.

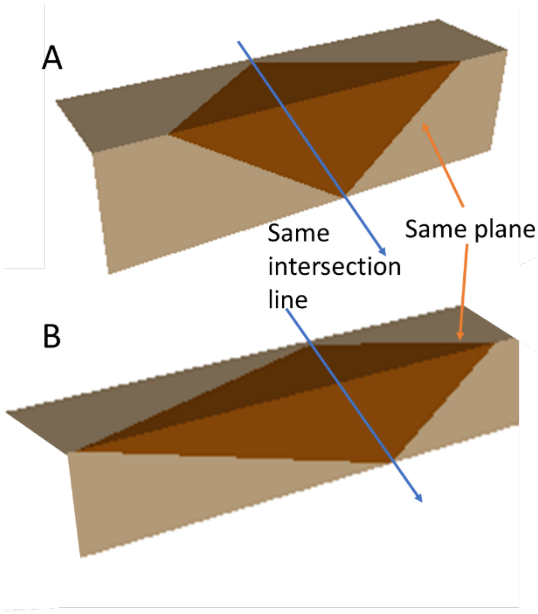

Fig.5 Wedges sharing one of the planes and also having the same intersection line

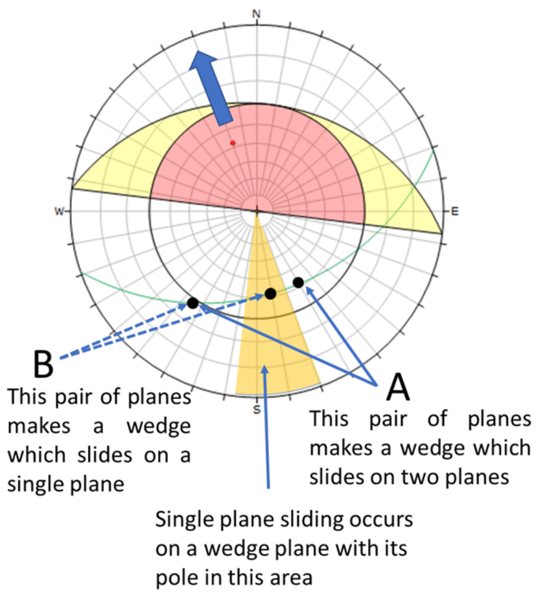

Fig.6 Stereographic representation of the wedges in Fig. 5. Large arrow shows intersection line direction. Red zone is the critical area for kinematically feasible wedge sliding.

4. DISCUSSION

The common practice of assessing wedge kinematic stability according to the line of intersection of the sliding planes fails to provide information on the shape of the wedges.

Table 2 Structural readings from Rothesay Bay, Auckland, New Zealand

Dip	Dip Dir.	Structure	Notes
78	013	Joint set 1	N=41, SD=12.4
86	100	Joint set 2	N=40, SD=9.5
81	310	Fault 1a	
54	140	Fault 1b	
79	320	Fault 2a	
69	140	Fault 2b	
68	110	Fault 3	
49	290	Fault 4a	
55	076	Fault 4b	

Slope face orientation 70-063, height 30 m. N=number of points, SD=Angular standard deviation based on Fischer dispersion

Table 3 Potential wedges, Rothesay Bay, Auckland, New Zealand

Wedge	Wedge angle (°)	FoS	Slide Planes	Fig.
W(J1-J2)	95	NW		8a
W(J1-F1b)	68	1.0524	1&2	8b
W(J1-F2b)	61	0.9515	1&2	8b
W(J1-F3)	88	0.386	1&2	8b
W(J1-F4b)	59	0.4043	slide on F4b	8c
W(F1b-F4b)	53	0.4043	slide on F4b	8d
W(J2-F1a)	32	NW		8e
W(J2-F2a)	43	NW		8e
W(J2-F4b)	38	2.5318	1&2	8e
W(F2a-F4b)	95	0.9825	1&2	8g
W(F3-F4b)	32	0.9316	1&2	8f
W(F2b-F4b)	59	0.4043	slide on F4b	8c
W(F1a-F3)	36	2.9526	1&2	8g
W(F2a-F3)	44	1.966	1&2	8g

Fig. 7 Location maps (a-b) and satellite image (c) of the rocky cliffs south of Rothesay Bay, Auckland, New Zealand (Google Earth). Star shows approximate location of rockfall reported by Hancox [29]. Dashed line shows the strike of bedding on the rock platform. (d) Cliffs approximately 30 m high (person circled for scale) and slope of approximately 70° (dashed line). (e) Typical jointing pattern on the surface of a bed (backpack is approximately 30 cm wide. (f) Location near top of cliff where a joint-block in a bed has fallen from the cliff (arrow). (g) Joint surface dipping out of the slope face. (h) Conjugate faults (dashed lines). (i & j) A sea cave formed in the hangingwall of a fault

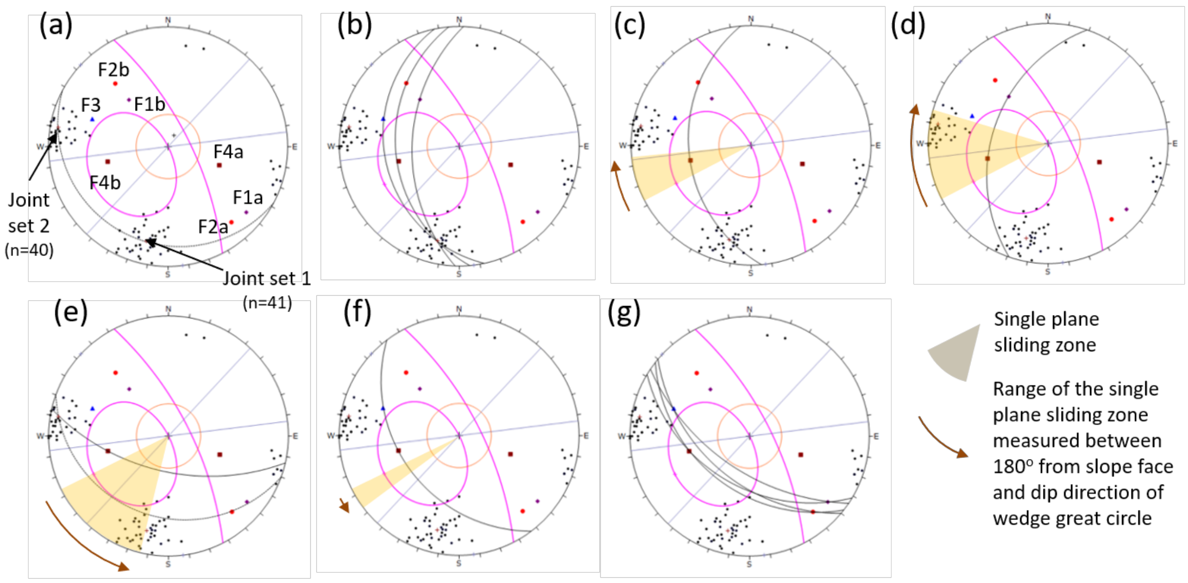

Fig. 8 Stereographic representations (equal angle, lower hemisphere) of structural data from the coastal cliff south of Rothesay Bay, Auckland. (a) Stereograph with joint sets and faults labelled F1-4 with conjugate pairs labelled a and b. The northeast facing slope is shown as a great circle and its daylight window (pink lines). The great circle (black line) between joints does not pass through the daylight window indicating stability. (b) Wedges between joint set 1 and east-southeast-dipping faults. (c,d) Single plane sliding on F4b forming a wedge with joint set 1 and faults. (e-g) Wedges formed by F4b, joint set 2 and other faults

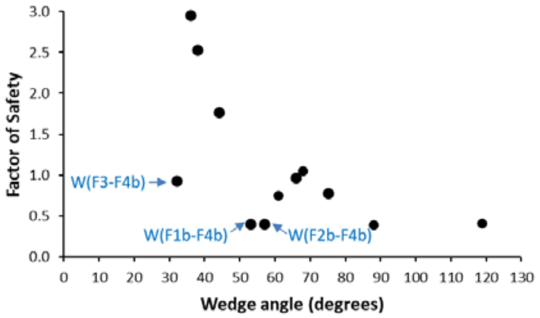

Fig.9 Stability and shape of wedges in Table 3. Anomalous low FoS cases are labelled

Bedding dips very gently to the west (into the cliff face). Joints are close-spaced and clustered in two main sets with vertical to steep easterly dipping and northeasterly dipping orientations (Fig. 7, Table 2). Faults are also present with moderate to steep dips. Kinematic analysis shows that wedges can form with combinations of joints with faults and faults with other faults (Fig.8, Table 3).

The stability of combinations of joints and faults capable of forming wedges have been analyzed using the software SWedge (Version 7, Rocscience). The factor of safety is closely related to the wedge shape as defined by the wedge angle (Fig. 9). For most cases, as the wedge angle increases the factor of safety decreases.

Wedges with lower factors of safety than the general trend (Fig. 9) were found to be those wedges with one or more of the planes close to the planar sliding direction. These wedges were highly asymmetrical in their orientation. This situation occurs in both single plane and two-plane sliding cases.

The method of assessing kinematic stability from the poles of sliding planes allows the shape of the wedges to be observed concurrently. This approach allows symmetrical and non-symmetrical wedges to be distinguished on the stereograph. Symmetry of wedges is observable according to the two factors: (1) the dip directions of sliding planes relative to the slope face and (2) the similarity of the dip angle of the sliding planes.

5. CONCLUSIONS

The relationship between wedge sliding compared to a rock slope face dip direction is described here as centered or off-center. This feature can be observed by the position of the poles of the sliding planes along their shared great circle. Only off-center wedges can slide with a single plane sliding mechanism.

The shapes of wedges are defined in terms of narrow, open and very open. This property can also be readily observed on a stereograph using the circle method of wedge analysis. The further apart are the poles along the great circle, the more narrow is the wedge (i.e. lower wedge angle). The closer the poles are located to each other the more open the wedge is (i.e. higher wedge angle).

The more open a wedge is, the lower the factor of safety tends to be.

6. ACKNOWLEDGMENTS

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