# STIFFNESS OF PILE GROUPS DURING LATERAL PUSHOVER

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**ABSTRACT:** A pushover analysis is carried out to study the performance of square precast piles supporting quasi-static lateral loading conditions as an approach to determine single piles, single pile groups, and multi pile groups failure criteria. The piles are modeled as embedded beam elements. The structure is modeled and analyzed using Plaxis3D. Pile stiffness, yielding, and hinge formation patterns in the piles during the increment of quasi-static lateral loading are simulated with Plaxis 3D. During displacement increment, stiffness changes of the single pile, pile groups during displacement increment. Piles and piles group stiffness ratio will be observed between pile stiffness at a certain displacement to its initial stiffness. Piles dan pile groups damage level criteria for a certain range of displacement are obtained from pile failure observation and pile stiffness reduction during displacement increment. This damage level is developed further to determine single pile and pile group damage criteria and performance levels during an earthquake.

Keywords: Performance-based analysis, Pushover analysis, Precast pile, Stiffness, Damage level

# 1. INTRODUCTION

This paper study applied lateral pushover to the pile foundation as prescribed displacement at the pile head or cap. Displacement multiplied by stiffness will produce force. Pile stiffness will change during the lateral displacement increment. Lateral pushover analysis was carried out for single piles, single pile groups, and multi-pile groups. Pile or pile group stiffness degradation correlates with a pile or pile failure criteria. Stiffness will be degraded to its initial stiffness according to its damage or failure criteria.

Based on the condition of stiffness degradation during lateral loading increment, the objective of this research is to get pile and pile group damage level criteria. In performance-based design, pile or pile group criteria are required to achieve a foundation performance level. In this paper, the lower structure is designed using performancebased principles, which have usually been employed for upper structures.

Research about structural damage states and their limit states which are assessed through pushover analysis has been studied in the "Seismic limit states for reinforced concrete bridge pile in sand" journal [6]. Soil pile interaction and pile lateral response were observed through pushover analysis.

Damage assessment of the pile-soil system consists of two types of failure which are a failure in loose sandy soil and Structural damages in the pile.

The NCHRP 440 bridge damage evaluation has five levels. As long as the core concrete is still intact and neither the transverse nor longitudinal steel has split or buckled, damage levels I through IV can be repaired. Damage level V would necessitate extensive repairs, possibly even the replacement of the entire column or bridge. The onset of damage in first-generation performance-based seismic design has typically been handled as discrete deformation limits based on strain (e.g., AASHTO SGS) or rotation (e.g., ASCE 41), which essentially quantifies each damage state deterministically, where the likelihood of damage increases from 0% to 100% the moment a damage limit is reached [3,10].

Unfortunately, there is a distribution of values for the beginning of damage rather than it being a discrete, deterministic quantity. In reality, predicting damage is a probabilistic issue rather than a deterministic one. Whether the reported deformation limits indicate lower bounds, the mean, or some other intermediate value for the commencement of damage is something that is not always evident in the codes and some literature. Due to this uncertainty, it is impossible to determine the data's dispersion and the precise placement of the limiting number within the statistical spread [5].

# 2. RESEARCH SIGNIFICANCE

In this study, the percentage of stiffness loss, the number of damaged piles, and the number of broken piles during pushover were used to determine damage levels. This research is significant to obtaining pile, pile group 1PC9, and multi pile group 9PC9 damage levels. This damage level is developed further to determine pile and pile group damage criteria during an earthquake. The damage level is also applicable as guidance to retrofit existing pile foundations to increase their performance level.

## 3. NUMERICAL MODEL

The assumed geotechnical conditions consist of two soil layers. The thickness of the upper soil layer is 12 m, overlying an 8 m thick, lower soil layer; the basic sketch is shown in figure 1. The undrained shear strength Su of the upper soil layer is varied to evaluate the effect of the strength on the behavior shear strength Su of the upper soil layer is varied to evaluate the effect of the strength on the behavior of laterally loaded pile groups, while the undrained soil modulus Eu is 150 x, Su. The Su values considered are 20 kPa, 40 kPa, 60 kPa, 80 kPa, 100 kPa and 200 kPa. The Su and Eu of the lower soil layer are 200 kPa and 50 MPa, respectively. Prescribed displacement was applied as cyclic loading from 0.003 m to 0.24 m. Cyclic loading was applied three times for each displacement increment [8].

The undrained friction angle of both soil layers is zero, and the groundwater is not modeled explicitly. The soil constitutive model used is the Mohr-Coulomb model. The assumed piles are 500 mm square piles, driven to the top of the lower soil layer; the length is then 12 m, and the pile tip may numerically rotate freely. The modulus of the piles is 30 GPa. In some analyses, the elastoplastic piles are assumed to have an ultimate bending moment capacity of 400 kNm. The piles are modeled numerically as embedded piles. [1]

Utilizing Plaxis 3D, the assessments were carried out utilizing the total stress analysis option. Tetrahedral elements with 10 nodes were used to model the soil components [9]. Using 3-node beam elements and the elastoplastic constitutive model, the embedded piles were modeled. While the pile cap and floor slab parts were modeled using 6-node thin shell elements, the tie-beam elements were modeled using 3-node beam elements. Using interface components with preset maximum capacities and additional nodes in the soil elements, the link between the soil and a pile were represented. A rigid link existed between a pile and the pile cap. The elements were all automatically created using Plaxis 3D; the nodes ranged in quantity from 127,043 to 169,061 while the soil elements ranged in number from 85,600 to 103,795 elements. the fully fixed boundary at the bottom of the soil, the free boundary at the top of the soil, and the normally fixed boundary at the sides of the soil in the X and Y directions. Pile groups and multiple pile groups layouts were shown in Fig.2 and Fig.3 [4].

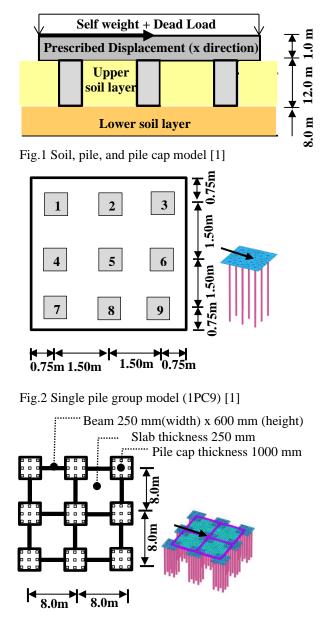


Fig.3 Multiple pile group model (9PC9) [1]

# 4. RESULT

Normalized lateral force and lateral displacement data for various undrained shear strengths At the ultimate bending moment, H\* is the average lateral force per pile. At the ultimate bending moment, U\* is the average displacement per pile. Normalized stiffness is calculated using normalized lateral and normalized displacement measurements [2].

Stiffness per initial stiffness in unnormalized condition compared for each pile, pile group, or multi pile group condition. Damage level criteria are determined based on stiffness degradation conditions and pile failure conditions such as pile in elastic, failure, or broken conditions.

Lateral displacement per pile dimension versus pile condition is plotted for various undrained shear strengths.

## 4.1 Single Pile

The normalized force-lateral displacement curves for single piles are shown in Fig.4.

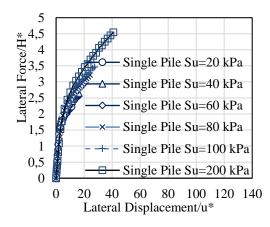


Fig.4 Normalized force – lateral displacement curves for single square pile

Normalized force vs. lateral displacement is depicted in Fig.4. The normalized curve will be converged as a straight line for undrained shear strength from 20 kPa to 200 kPa, showing the inelastic state and failure damage level. The normalized curve for undrained shear strength 20 kPa to 200 kPa will be divergence at the breaking damage threshold.

 Table 1 Damage level category for single pile

Damage	K/Ko	description		
level				
Ι	100%	The pile has no		
		failure, no stiffness		
		degradation		
II	37-84 % The pile has a			
		failure or broken,		
		Maximum bending		
		moment less than		
		400 kNm or equal		
		to 400 kNm		
III	10-16%	5% pile has broken,		
		Mmax > 400 kNm		

Note: K = Pile Stiffness, Ko = Initial Stiffness. Both K and Ko are Unnormalized stiffness.

Damage level 1, damage level 2, and damage level 3 are the three levels of pile stiffness

degradation for a single pile. During lateral displacement increment, each damage level is classified based on pile stiffness degradation and pile damage description.

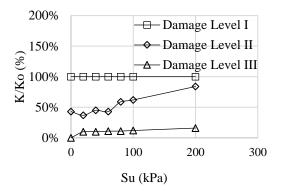


Fig.5 Undrained shear strength vs K/Ko for single square pile

Table 1 and Fig.5 present stiffness degradation for each damage level. In damage level I, pile in elastic condition, there is no stiffness degradation. In damage level II, pile in damaged condition, there is stiffness degradation. In damage level III, pile in broken condition, pile almost has no more stiffness.

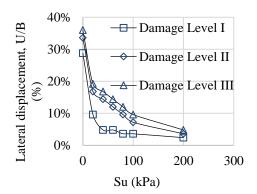


Fig.6 Undrained shear strength vs lateral displacement/pile dimension for single square pile

Figure 6 presents a correlation between undrained shear strength and lateral displacement per pile width. A lower undrained shear strength value will produce a larger lateral displacement per pile dimension value for each damage level I, II, and III.

A higher undrained shear strength value will produce a more brittle condition. The displacement between damage levels I, II, and III will be not too much different (Su= 200 kPa, 2.4% for damage level I until 5% for damage level IV). Lower undrained shear strength value will produce more ductile conditions. The displacement difference between damage levels I, II, and III will be larger (Su= 20 kPa, 9.6% for damage level I until 19% for damage level IV).

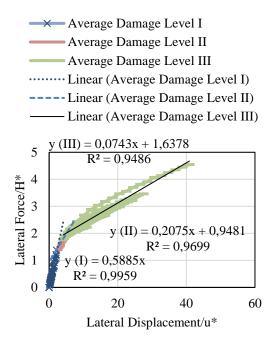


Fig.7 Normalized stiffness changes  $|K_1|$  for single square pile for each damage level

Figure 7 above shows normalized stiffness for each single pile damage level. Damage level I, II & III criteria are based on pile condition during displacement increment. Average damage level I is a condition where the pile has no failure and no stiffness degradation for Su = 20 kPa until Su = 200 kPa. Average damage level II is a condition where the pile has failed or broken and stiffness degradation happens for Su = 20 kPa until Su = 200 kPa. Average damage level II is a condition where the pile has failed or broken and stiffness degradation happens for Su = 20 kPa until Su = 200 kPa. Average damage level III is a condition where the pile has broken and almost no stiffness for Su = 20 kPa until Su = 200 kPa until Su = 200 kPa.

$$|K_1| = \frac{\{\text{Lateral Force}/H^*\}}{\{\text{Lateral Displacement}/u^*\}}$$
(1)

where:  $|K_1| = \text{normalized stiffness}$   $H^* = \text{lateral forces at Mmax 400 kNm}$   $u^* = \text{deflection at pile head at Mmax 400 kNm}$ Elastic / Damage Level I: Lateral Displacement/u<sup>\*</sup> <= 3

$$|K_1| \ge 0.59$$
 (2)

Non-Elastic / Damage Level II: 3 < Lateral Displacement/u\* <= 3

$$0.2 \le |K_1| < 0.59 \tag{3}$$

Non Elastic / Damage Level III: Lateral Displacement/u\* > 4

$$0.07 \leq |K_1| < 0.2 \tag{4}$$

Eq. 2 to Eq. 4 show that in damage level I normalized stiffness is still 100 % of its initial normalized stiffness. In damage level II, normalized stiffness will decrease from 100 % to 30 % of its initial normalized stiffness. In damage level III, normalized stiffness will decrease from 30% of its initial normalized stiffness to 12 % of its initial normalized stiffness.

#### 4.2 Single Pile Group 1PC9

The normalized force-lateral displacement curves for single pile groups are shown in Fig.8.

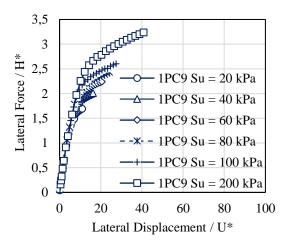


Fig.8 Normalized force – lateral displacement curves for 1PC9

Normalized force vs. lateral displacement is depicted in Fig.8. The normalized curve for undrained shear strength 20 kPa to 200 kPa will be convergent as a straight line for undrained shear strength 20 kPa to 200 kPa, indicating an inelastic condition until some piles fail and minor broken condition. The normalized curve for undrained shear strength from 20 kPa to 200 kPa will be divergence in broken conditions.

Damage level 1, damage level 2, damage level 3, and damage level 4 are the different levels of pile group stiffness deterioration for single pile group 1PC9. During lateral displacement increment, each damage level is classified based on pile group stiffness degradation and pile group damage description.

Broup			
Damage level	K/Ko	description	
Ι	100%	The pile has no failure	
		and no stiffness	
		degradation.	
II	82-	some piles have failed. 2	
	93%	until 9 piles have failed	
		but Mmax < 400 kNm	
III	63-	some piles have failed	
	71%	and broken. 9 piles have	
		failed but Mmax < 400	
		kNm or 1 until 4 piles	
		have broken and Mmax	
		>= 400 kNm	
IV	15-	some piles until all piles	
	20%	have broken. 1 until 9	
		piles have broken and	
		Mmax > 400 kNm	

Table 2 Damage level category for single pile group

Note: K = 1PC9 Stiffness, Ko = 1PC9 Initial Stiffness. Both K and Ko are Unnormalized stiffness.

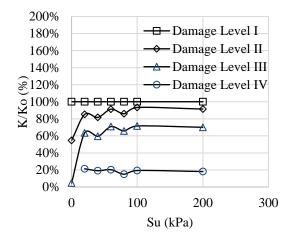


Fig.9 Undrained shear strength vs K/Ko for 1PC9

Table 2 and Fig.9 present stiffness degradation for each damage level. In damage level I, pile in elastic condition, there is no stiffness degradation. In damage level II where some piles are in damage conditions, there are small changes in stiffness degradation. In damage level III, some piles have failure and broken condition, there are large changes in stiffness degradation. In damage level IV, some piles have broken, pile stiffness 15% -20% of its initial stiffness.

Figure 10 presents a correlation between undrained shear strength and lateral displacement per pile dimension. A lower undrained shear strength value will produce a larger lateral displacement per pile dimension value for each damage level I, II, III, and IV.

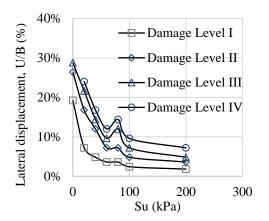


Fig.10 Undrained shear strength vs lateral displacement/pile dimension for 1PC9

A higher undrained shear strength value will produce more brittle conditions. The displacement between damage levels I, II, III, and IV will be not too much different (Su= 200 kPa, 1.8% for damage level I until 7% for damage level IV). Lower undrained shear strength value will produce more ductile conditions. Displacement difference between damage levels I, II, III, and IV will be larger (Su = 20 kPa, 7.2% for damage level I until 24% for damage level IV).

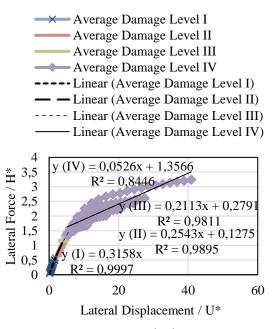


Fig.11 Normalized stiffness  $|K_1|$  changes for 1PC9 for each damage level

Figure 11 above shows normalized stiffness for each pile in the 1PC9 damage level. Damage level

I, II, III & IV criteria based on piles in 1PC9 condition during displacement increment. Average damage level I is a condition where all piles in 1PC9 have no failure and no stiffness degradation for Su = 20 kPa until Su = 200 kPa. Average damage level II is a condition where some piles have failed and small stiffness degradation happens for Su = 20 kPa until Su = 200 kPa. Average damage level III is a condition where all piles have a failure or some broken and large stiffness degradation happens for Su = 20 kPa until Su = 200 kPa. Average damage level IV is a condition where some piles have broken or all piles broken and almost no stiffness for Su = 20 kPa until Su = 200 kPa. Elastic / Damage Level I:

Lateral Displacement/u\* <= 2

$$|K_1| \ge 0.32 \tag{5}$$

Non-Elastic / Damage Level II: 2 < Lateral Displacement/u\* <= 4

$$0.25 \le |K_1| < 0.32 \tag{6}$$

Non Elastic / Damage Level III: 4 < Lateral Displacement/u\* <= 5.5

$$0.2 \le |K_1| < 0.25 \tag{7}$$

Non Elastic / Damage Level IV: Lateral Displacement/u\* > 5.5

$$0.05 \le |K_1| < 0.2 \tag{8}$$

Eq. 5 to Eq. 8 show that in damage level I normalized stiffness is still 100 % of its initial normalized stiffness. In damage level II, normalized stiffness will decrease from 100 % to 78 % of its initial normalized stiffness. In damage level III, normalized stiffness will decrease from 78% of its initial normalized stiffness to 62 % of its initial normalized stiffness. In damage level IV, normalized stiffness will decrease from 62% of its initial normalized stiffness to 16 % of its initial normalized stiffness.

#### 4.3 Multiple Pile Group 9PC9

The normalized force-lateral displacement curves for multiple pile groups are shown in Fig.12.

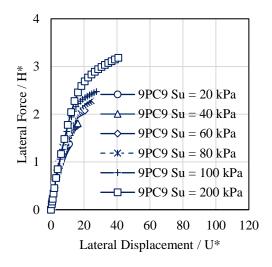


Fig.12 Normalized force – lateral displacement curves for 9PC9

Normalized force vs. lateral displacement is depicted in Fig.12. The normalized curve for undrained shear strength 20 kPa to 200 kPa will be convergent as a straight line for undrained shear strength 20 kPa to 200 kPa, indicating inelastic condition until some piles fail and collapse. The normalized curve for undrained shear strength 20 kPa until 200 kPa will be divergence if some piles have broken until all piles have broken.

Table 5 Damage level category for 9PC9 pile gloup				
Damage	K/Ko	description		
level				
Ι	100%	The pile has no failure, no		
		stiffness degradation		
II	89-97%	some piles have failed and		
		broken. 2 until 66 piles have		
		failed and Mmax < 400 kNm.		
		6 until 13 piles have broken		
		and Mmax $\geq 400$ kNm.		
III	63-76%	some piles have failed and		
		broken. 30 until 62 piles have		
		failed and Mmax < 400 kNm,		
		11 until 48 piles have broken		
		and Mmax $\geq 400$ kNm.		
IV	26-37%	some piles have broken until		
		all piles have broken. 1 until		
		58 piles have failure and		
		Mmax < 400 kNm. 19 until		
		81 piles have broken and		
		$Mmax \ge 400 \text{ kNm}.$		

Note: K = 9PC9 Stiffness, Ko = 9PC9 Initial Stiffness. Both K and Ko are Unnormalized stiffness.

Damage level 1, damage level 2, damage level 3, and damage level 4 are the different levels of pile group stiffness deterioration for nine pile group

1PC9. During lateral displacement increment, each condition was classified based on nine pile group 1PC9 stiffness degradation and nine pile group 1PC9 damage descriptions.

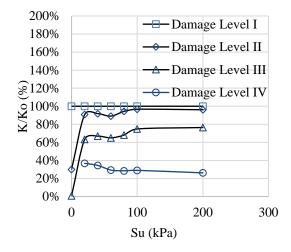


Fig.13 Undrained shear strength vs K/Ko for 9PC9

Table 3 and Fig.13 present stiffness degradation for each condition. In damage level I, pile in elastic condition, there is no stiffness degradation. In damage level II where some piles are in damaged condition, there are small changes in stiffness degradation. In damage level III, some piles have failure and broken condition, there are larger changes in stiffness degradation. In damage level IV, some piles have broken, pile almost has no more stiffness.

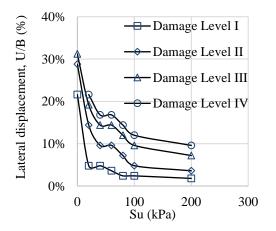


Fig.14 Lateral displacement / Pile dimension vs undrained shear strength for 9PC9

Figure 14 presents a correlation between undrained shear strength and lateral displacement

per pile dimension. A lower undrained shear strength value will produce a larger lateral displacement per pile dimension value for each damage level I, II, III, and IV.

A higher undrained shear strength value will produce a more brittle condition. The displacement between damage levels I, II, III, and IV will be not too much different (Su= 200 kPa, 1.8% for damage level I until 9.6% for damage level IV). Lower undrained shear strength value will produce more ductile conditions. Displacement difference between damage levels I, II, III, and IV will be larger (Su= 20 kPa, 4.8% for damage level I until 21.6% for damage level IV).

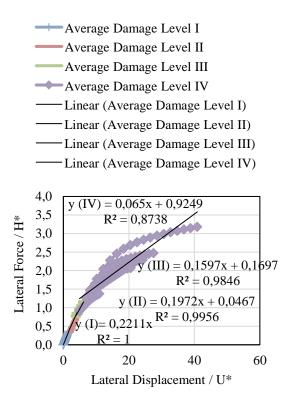


Fig.15 Normalized stiffness  $|K_1|$  changes for 9PC9 for each damage level

Figure 15 above shows normalized stiffness for each 9PC9 damage level. Damage level I, II, III & IV criteria based on piles in 9PC9 damage level during displacement increment. Average damage level I is a condition where all piles in 9PC9 have no failure and no stiffness degradation happens for Su = 20 kPa until Su = 200 kPa. Average damage level II is a condition where some piles have a failure or broken and small stiffness degradation happens for Su = 20 kPa until Su = 200 kPa. Average damage level III is a condition where most piles have a failure or broken and large stiffness degradation happens for Su = 20 kPa until Su = 200 kPa. Average damage level IV is a condition where some piles until all piles have broken and piles have small stiffness for Su = 20 kPa until Su = 200 kPa. Elastic / Damage Level I: Lateral Displacement/u\* <= 1.8

$$|K_1| \ge 0.22 \tag{9}$$

Non Elastic / Damage Level II: 1.8 < Lateral Displacement/u\* <= 3.8

$$0.19 \le |K_1| < 0.22 \tag{10}$$

Non Elastic / Damage Level III: 3.8 < Lateral Displacement/u\* <= 6

 $0.16 \le |K_1| < 0.19 \tag{11}$ 

Non Elastic / Damage Level IV: Lateral Displacement/u\* > 6

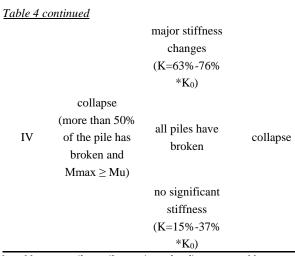
$$0.06 \le |K_1| < 0.16 \tag{12}$$

Eq. 9 to Eq. 12 show that in damage level I normalized stiffness is still 100 % of its initial normalized stiffness. In damage level II, normalized stiffness will decrease from 100 % to 86 % of its initial normalized stiffness. In damage level III, normalized stiffness will decrease from 86 % of its initial normalized stiffness to 73 % of its initial normalized stiffness. In damage level IV, normalized stiffness will decrease from 73% of its initial normalized stiffness to 27 % of its initial normalized stiffness.

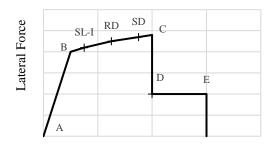
#### 5. DISCUSSION

Damage classification and performance level in Table 4 below are similar to damage classification and performance level in NCHRP 440. The percentage of stiffness loss, the number of damaged piles, and the number of broken piles during pushover were used to determine damage levels [7]. If we compare with Bridge Performance Level NCHRP 440 shown in Fig. 16 below, Table 4 shows that Damage Level I is equal to SL-1: Service Level Immediate. Damage Level 2 is equal to RD: Repairable Damage. Damage Level 3 is equal to SD: Significant Damage

PC9			
Damage level	damage classification	damage description (damage	performance
		measures)	level
Single P	Pile		
Ι	no	no damage, no stiffness changes	fully operational
Π	minor damage (damage but Mmax < Mu)	(K=100 %*K <sub>0</sub> ) minor crack at the pile head	life safety
		minor stiffness changes (K=37%-84% *K <sub>0</sub> )	
III	major damage (broken and Mmax ≥ Mu)	Pile broken no significant stiffness (K=10%-16% *K <sub>0</sub> )	collapse
1PC9 &	9PC9 (pile group)		
Ι	no	no damage, no stiffness changes (K=100 % *K <sub>0</sub> )	fully operational
II	minor damage	damage at some piles head	limited damage
	((1%-100%) piles in pile group have damaged but Mmax < Mu and less than 10% of the pile has broken with Mmax ≥ Mu)	minor stiffness changes (K=82%-97% *K <sub>0</sub> )	
III	major damage (less than 50% of the pile has broken and Mmax ≥ Mu)	Some piles have broken	life safety



Note:  $Mmax = pile \text{ or } piles \text{ maximum bending moment. } Mu = pile \text{ or } piles ultimate bending moment. } K= Stiffness, K_0 = Initial Stiffness. Both K and Ko are Unnormalized stiffness.$ 



Lateral Displacement

Fig.16 Bridge Performance level [5].

### 6. CONCLUSIONS

From the analysis for single pile, single pile group 1PC9, and multiple pile group 9PC9, we can conclude that stiffness degradation correlates with pile damage level and performance level. Pushover analysis is used to obtain pile, single pile group 1PC9, and multi pile group 9PC9 performance level and damage level for each displacement increment.

The performance-based design is commonly used for upper structures and this research tries to apply performance-based design to pile foundations this performance level is also applicable as guidance to retrofit existing pile foundations to increase their performance level.

# 7. ACKNOWLEDGEMENT

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## 659/UN2.RST/HKP.05.00/2020).

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