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COMPARISON OF THE SHEAR STRENGTH OF UNSATURATED SANDY SOILS AT OPTIMAL AND RESIDUAL MOISTURE CONTENTS

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ABSTRACT: The shear strength of soil is a key input parameter required for the satisfactory geotechnical design of any structure. The existence of most residual soils in unsaturated conditions highlights the importance of studying the shear strength of unsaturated soils. In this study, a modified triaxial apparatus was used to examine the mechanical behavior of 5 types of silica sands with mean grain sizes varying from 0.15 to 1.5 mm. The experimental test results of soil shear strength determination were compared with the shear strength estimated through analytical methods. To study the mechanical behavior of test soils at optimal and residual moisture contents, the test samples were initially prepared at the optimal moisture content. To study the effect of the initial moisture content on the mechanical behavior of test soils at the residual moisture content, the test samples were initially prepared at the optimal and residual moisture contents. Residual moisture content samples exhibited higher shear strength compared to optimal moisture content samples. Furthermore, the residual moisture content samples prepared at the optimal moisture content exhibited stronger mechanical behavior compared to the sample prepared at the residual moisture content. The analytical methods employed in this study were not able to accurately predict the shear strength of test soils at residual moisture content.

Keywords: Shear strength, Unsaturated soil, Suction, Moisture content, Residual zone

1. INTRODUCTION

Accurate determination of the shear strength of soil is necessary for the safe geotechnical design of structures, including embankment slopes, foundations and retaining walls. Soil shear strength is not an index property of the soil; rather, it changes with the change in water content, i.e., the degree of saturation of the soil. For saturated conditions, soil shear strength is conventionally described using the Mohr–Coulomb failure envelope and the effective stress equation proposed by Terzaghi as Eq. (1).

$$\tau' = c' + (\sigma - u_w)tan\varphi \quad (1)$$

Equation (1) satisfactorily works for the prediction of the shear strength of soils for saturated conditions. However, many soils present in nature remain in unsaturated conditions above the groundwater table. When the soil is unsaturated and the pores of the soil are partially filled with water, the pore water generates a negative pressure. This negative pore water pressure has also been termed suction and is defined as the difference between pore air and pore water pressure.

Meaningful interpretation of the shear strength of soil in unsaturated conditions requires at least two out of three independent stress state variables known as $(\sigma - u_a)$, $(\sigma - u_w)$ and $(u_a - u_w)$ [1,2]. The suction has a positive influence on the shear strength of soil, which has been observed by many researchers [3]-[8]. However, the effect of suction is conventionally ignored for practical geotechnical engineering design, and to remain conservative, it is assumed that soil is in a saturated condition [9].

In this study, efforts have been made to study the practical implication of incorporating the influence of suction in predicting the shear strength of soil for geotechnical engineering design problems without compromising the safety requirement of a structure.

Studies have been conducted in the past to obtain the effective stress parameter for unsaturated soils. Bishop [10] also offered an expression to describe the effective stress for unsaturated soils as in Eq. (2).

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w) \quad (2)$$

where the χ parameter is correlated with the degree of saturation. Khaili and Khabaz [11] gave a unique relationship for the parameter χ, which is related to the suction and the air entry value of the soil. The author concluded that the shear strength of the soil increased linearly with an increase in suction up to the air entry value of the soil, and a further increase in suction caused an increase in shear strength at a decreased rate. However, generally, due to limited practical implications and involved complexities, Bishop's concept of describing effective stress for unsaturated soils is not widely used.

To address this issue, [12] proposed Eq. (3) for the shear strength of unsaturated soils considering the independent stress variable approach.

$$\tau = c' + (\sigma - u_a)tan\varphi' + (u_a - u_w)tan\varphi^b \quad (3)$$

where φ^b is related to the increase in shear strength of soil with the corresponding increase in suction. In other words, the shear strength of unsaturated soil increases linearly with the increase in suction at the rate of $tan\varphi^b$.

Ho and Fredlund [13], among many researchers, experimentally determined the unsaturated shear strength of soils, supported the idea of the increase in shear strength with the increase in suction, and determined φ^b. Their experimental results showed a linear failure surface in the shear stress and matric suction plane, which supports the linear increase in shear strength with an increase in suction.

Gan [14] performed direct shear testing for unsaturated soils and observed the nonlinearity of the failure surface drawn in the shear stress versus the matric suction plane. The author showed that the increase in shear strength with suction in terms of φ^b is not constant, and φ^b decreases beyond a certain value of suction.

A series of direct shear tests on unsaturated soils was conducted [15]. The results of a direct shear test showed that the strength of the tested soils increases almost linearly with increasing suction if the soil remains saturated. When the soil becomes unsaturated, the rate of increase in shear strength with an increase in suction decreases, and the shear strength decreases to a constant value after reaching a peak value with a further increase in suction. The same trend has been explained by [16] for sandy soils. This phenomenon cannot be explained by the equation proposed by [12], even when considering the nonlinearity of φ^b.

Lu and Likos [17] defined effective stress in terms of suction stress as in Eq. (4), which depends upon the forces transferred through the soil grain and the interparticle forces.

$$\sigma' = (\sigma - u_a) - \sigma_s \quad (4)$$

Based on the concept of suction stress, the author proposed a suction stress curve for coarse- and fine-grained soil. The shear strength equation for unsaturated soils defined in terms of suction stress can describe the nonlinear failure surface in the shear stress versus metric suction plane and the shear strength decrement of unsaturated soils after reaching the peak value beyond certain suction, as observed by [15].

As discussed above, the unsaturated shear strength of the soil depends upon the suction. Furthermore, a unique relationship exists between the water content of the soil and the suction, which is called the soil–water characteristic curve (SWCC) (Fig. 1). Considering the value of suction and corresponding water content, the SWCC can be divided into three zones. The soil remains saturated with low suction values in the first zone. The second zone is called the transition zone, which starts from the air entry value until the start of the residual zone. In this zone, the water content of soil responds to changes in suction. In the third, residual zone, the water content of the soil does not decrease much with the increase in suction. Considering the suction as a common variable between the shear strength function and the SWCC, researchers [18-20] have developed some relation for predicting the shear strength of soil by employing the SWCC directly, but [21] discussed the uncertainties involved in the prediction of soil shear strength in unsaturated conditions using the SWCC.

2. RESEARCH SIGNIFICANCE

The work presented herein specifically aims to investigate the mechanical behavior of soil at different moisture contents and the effect of the initial moisture content on the mechanical behavior of soil. The purpose is to study the possible positive impact of suction on the soil shear strength, which can be incorporated into the geotechnical design of a structure. Incorporating the unsaturated shear strength of soil will allow the designer to take advantage of additional shear strength available due to suction more than the shear strength available in a saturated condition.

3. TEST MATERIALS

Different silica sands, i.e., silica nos. 3 to 7, were used for laboratory experiments. Grain size analysis of the test materials showed that the mean grain sizes (D50) for silica nos. 3, 4, 5, 6, and 7 were 1.5, 0.9, 0.6, 0.3, and 0.15 mm, respectively (Fig. 2). All these soils are uniformly graded soils with coefficients of uniformity ranging from 1.33 to 2.00.

The maximum dry density and optimal moisture content of the test materials were determined by the standard Proctor test. The results show that the maximum dry density decreases with the decrease in the mean particle size of the silica sand. The maximum dry density of silica no. 3 to 7 ranges from 1.59 to 1.44 g/cm3 with optimal water content ranging from 10.5 to 21.8%. The moisture density curves thus obtained are presented in Fig. 3. Index properties of the test materials are tabulated in Table 1.

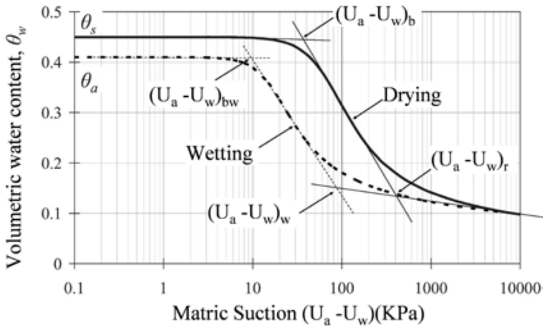

Fig. 1 SWCC curve zones [22]

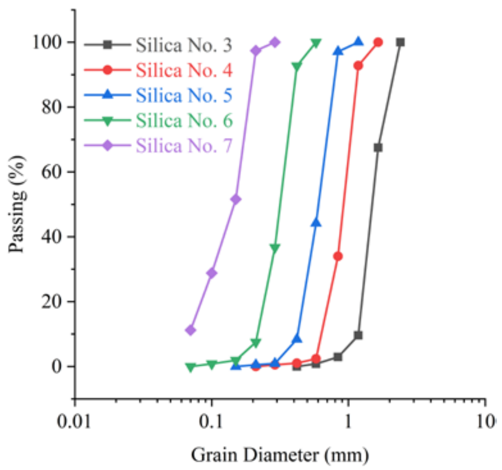

Fig. 2 Grain size distribution of the test material

5. RESEARCH METHODOLOGY

5.1 Apparatus

A modified triaxial apparatus was used to study the mechanical properties of unsaturated silica sands. The apparatus was equipped with a system to measure pore water pressure, pore air pressure, cell pressure, axial and volumetric strain, axial load, and infiltration/drainage of pore water. The details of the triaxial apparatus are shown in Fig. 4.

The pedestal was equipped with a high air entry value (100 kPa) 7 mm thick ceramic disk for pore water drainage/infiltration from the test sample without allowing the flow of air. An electric transducer was also attached to this system to measure the pore water pressure.

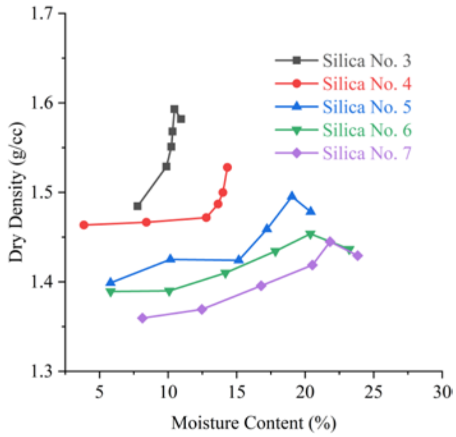

Fig. 3 Moisture density curve for the test material

An external weight balance was also attached to measure the quantity of pore water drainage/infiltration. The supply, control, and measuring system for pore air pressure was provided in the top cap, which included a remotely controlled switch and an electric transducer. A PTFE filter was fixed at the bottom of the top cap to restrict the flow of water to the air supply and measurement system. A low-capacity differential transducer (LCDPT) was attached to the double-cell assembly to measure the volume change in the sample. The axial load application and measuring system consisted of a servomotor jack unit and a load cell. The axial strain was measured with an external arrangement that consisted of an LVDT.

Table 1 Index properties of test materials

	Silica No. 3	Silica No. 4	Silica No. 5	Silica No. 6	Silica No. 7
Mean Grain Size D50 (mm)	1.50	0.90	0.60	0.30	0.15
Sand Content (%)	100	100	100	100	90
Fine Content (%)	0	0	0	0	10
Coefficient of Uniformity (Cu)	1.33	1.54	1.44	1.59	2.00
Maximum Dry Density (g/cm^3)	1.59	1.53	1.50	1.45	1.44
Optimal Moisture Content (%)	10.5	14.3	19.0	20.1	21.8

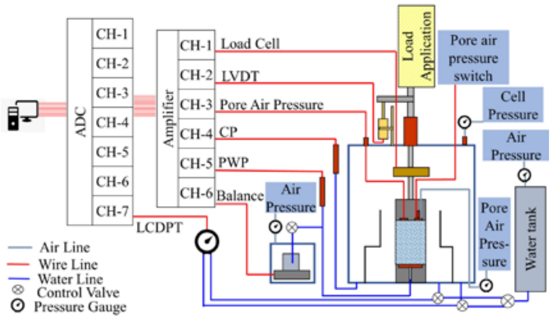

Fig. 4 Schematic diagram of the modified triaxial apparatus

Cell pressure was controlled and measured by means of a continuous supply of compressed air to the partially water-filled triaxial cell and an electric transducer, respectively. A set of amplifiers was used to collect the data from all the transducers and convey the data to the analog-to-digital converter (ADC) board except for the LCDPT, which directly transmitted data to the ADC.

A computer program was used to receive and record all data from ADC in a data file. The same program was used to control the axial loading system through a digital-to-analog converter and a load control box. Cell pressure and pore air pressure were controlled manually using an air pressure regulator.

5.2 Experimental Procedure

The base pedestal containing the ceramic disk was placed underwater in a vacuum chamber for saturation for 24 hours under a negative pressure of 101.3 kPa. Then, the pedestal was removed from the vacuum chamber and immediately fixed in the base plate of the triaxial apparatus. After fixing the outer cell and top plate of the triaxial apparatus, de-aired water was poured into the triaxial cell to inundate the pedestal. Cell pressure greater than 100 kPa was applied to backflow the water through the pedestal to saturate the water lines connecting the pedestal to the external water balance and the porewater pressure transducer. Fig. 5 shows the procedure adopted for the saturation of the ceramic disk.

The wet temping method was used to prepare the test soil sample (50 × 100 mm) in five equal layers, directly on the pedestal using a metallic mold and rubber membrane. The initial moisture content for sample preparation was maintained as per the test conditions. The test conditions will be discussed later in this paper. The negative pore water pressure generated by the unsaturated soil sample prepared on the pedestal was recorded by the pore water pressure transducer.

The axis translation technique [23] provides the basis for maintaining suction at the required level by controlling the pore air pressure without making the pore water pressure negative. The same technique was used in this study to achieve and maintain the required suction. The pore air pressure was increased up to the required level, which caused a decrease in the initially generated negative pore water pressure, keeping the suction constant. An increase in pore air pressure was performed along with the simultaneous increase in the cell pressure to avoid any change in the test sample volume. After that, the drainage valve was opened to atmospheric pressure, and the test sample was isotopically consolidated at the desired consolidation pressure before monotonic shearing.

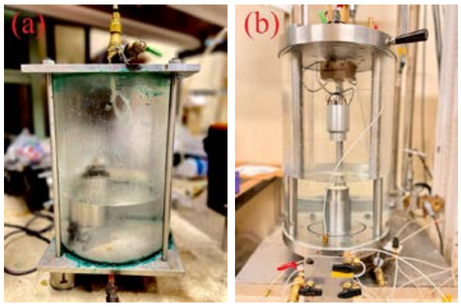

Fig. 5 Pedestal saturation. (a) pedestal in a vacuum chamber. (b) pedestal in the triaxial cell

To study the mechanical properties of soil at residual moisture content for the sample initially prepared at optimal water content, suction was increased up to the required level by the axis translation technique after the completion of the consolidation process. Under these conditions, drainage was allowed from the soil until no further water was draining out from the sample. Approximately 24 hours were allowed for the drainage process to complete before applying deviatoric stress on the soil specimen. The drainage valve remained open during and after the consolidation process until the completion of shearing.

5.3 Experimental Program

In this study, an experimental program is devised to study and compare the mechanical behavior of soil at optimal and residual moisture contents and the effect of the initial moisture content on the mechanical behavior of soil at the residual moisture content. The nomenclature, along with the description of the experimental program, is provided in Table 2.

The test sample was prepared at the optimal moisture content for test series (1) and (2). The stress path applied on the sample for test series (1) and (2) is shown in Figs. 6(a) and 6(b). The suction was maintained at the initial suction observed after sample preparation using the axis translation technique shown in point A. The test sample underwent isotropic consolidation of 50 kPa from point A to B. Deviatoric stress was applied just after consolidation for test series (1), shown as points B to C in Fig. 6(a). For test series (2), after consolidating the test sample, the suction was increased beyond the residual suction of the soil being tested (details of the maximum suction applied are presented later in this section) using the axis translation technique, shown as points B to C in Fig. 6(b). Sufficient time was allowed for the sample to fully drain. The drained water was collected in an external balance, and the soil was considered fully drained when no more increase in water was recorded. After that, the shearing stress was applied, as shown in points C to D in Fig. 6(b). The water content of the tested sample was determined using the oven drying method after the completion of the test and recorded as the residual moisture content of the soil.

Table 2 Details of the Experimental program

Test Series	Name	Description
1	OMC-OMC	Shear strength of soil sample prepared at optimal moisture content
2	OMC-RMC	Shear strength of soil at residual moisture content for test sample prepared at optimal moisture content
3	RMC-RMC	Shear strength of soil at residual moisture content for test sample prepared at residual moisture content

For carrying out test series (3), the test sample was prepared at residual moisture content. The suction was increased equal to the maximum suction applied in test series (2) using the axis translation technique shown as points A to B in Fig. 6(c). After that, the sample undergoes isotropic consolidation and experiences monotonic shearing, shown as points B to C and C to D, respectively, in Fig. 6 (c).

SWCC for silica no. 7 experimentally determined by [24] and the author found 5 kPa as the residual suction for silica no. 7. Gallage and Uchimura [25] found that residual suction tends to decrease when the D10 of the soil increases. The grain size distribution of silica nos. 3 to 6 shows that silica no. 7 has the smallest grain size at D10, so it is believed that the residual suction of silica nos. 3 to 6 will also be less than the residual suction of silica no. 7.

Keeping in mind the above and considering the limitation AEV value of the pedestal ceramic disk, a suction value of 25 kPa was selected to achieve the residual moisture content of the test sample initially prepared at optimal moisture content, and the same suction was maintained for the test sample that was initially prepared at residual moisture content.

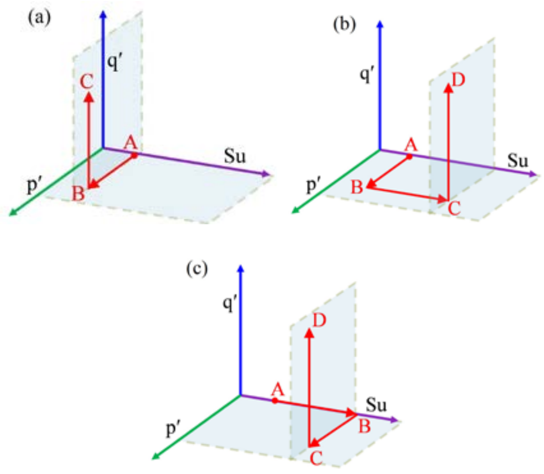

Fig. 6 Stress path for triaxial compression tests

6. DISCUSSION OF TEST RESULTS

Three test series of triaxial compression tests were carried out on test samples prepared at a 90% compaction ratio with an initial moisture content equal to the optimal or residual moisture content, depending upon the test condition. All the samples were isotopically consolidated at a mean effective stress of 50 kPa.

6.1 Effect of the Moisture Content (during shearing) on Soil Shear Strength

The results of the triaxial compression test carried out for various silica sands show that the peak deviatoric stress for monotonically sheared OMC-OMC samples ranges from 156 kPa to 144 kPa with the volumetric strain ranging from 3.5% to 1.5%. The test results of the OMC-RMC samples show that the peak deviatoric stress ranges from 177 kPa to 165 kPa with volumetric strains ranging from 3.7% to 2%.

A comparison of deviatoric stress and volumetric strain vs. axial strain for the respective OMC-OMC and OMC-RMC samples shows similar behavior for silica no. 3 to silica no. 7. Figs. 7 and 8 show the relationship between deviatoric stress vs. axial strain and volumetric strain vs. axial strain, respectively, for silica no. 3 and silica no. 7 (OMC-OMC and OMC-RMC samples).

The peak deviatoric stress and volumetric strain of the respective OMC-OMC and OMC-RMC samples were compared for various silica sands.

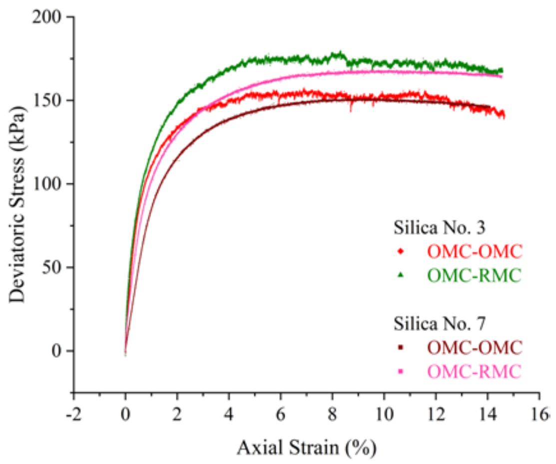

Fig. 7 Deviatoric stress vs. axial strain for Silica No. 3 & 7, OMC-OMC and OMC-RMC samples

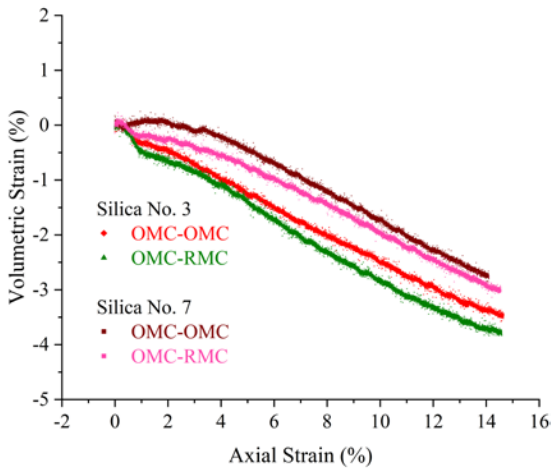

Fig. 8 Volumetric strain vs. axial strain for Silica No. 3 & 7, OMC-OMC and OMC-RMC samples

This comparison showed that the peak deviatoric stress of the various silica sands is higher on the order of 16 to 33 kPa for the respective OMC-RMC samples. Furthermore, the volumetric strain is also 0.2% to 0.5% higher for the respective OMC-RMC samples. The comparison of the peak deviatoric stress of the OMC-OMC and OMC-RMC samples is shown in Fig. 9.

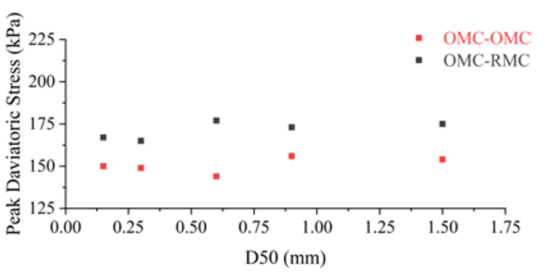

Fig. 9 Comparison of peak deviatoric stress for the OMC-OMC and OMC-RMC samples

6.2 Effect of Moisture Content (at Compaction) on Soil Shear Strength

The effect of the initial moisture content on the soil shear strength at the residual moisture content was investigated by comparing the test results of the OMC-RMC and RMC-RMC samples. The test results show that the peak deviatoric stress for the RMC-RMC samples was 2 to 17 kPa lower than the peak deviatoric stress of the respective OMC-RMC samples. The test results showed the influence of the initial water content on the soil shear strength at the residual moisture content. Figs. 10 and 11 show the deviatoric stress vs. axial strain and volumetric strain vs. axial strain, respectively, for silica nos. 3 and 7 (OMC-RMC and RMC-RMC samples).

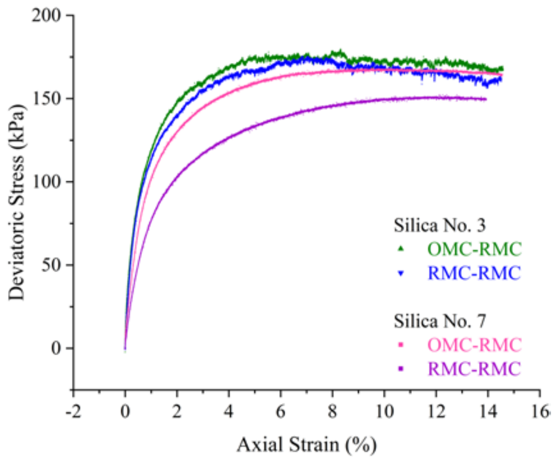

Fig. 10 Deviatoric stress vs. axial strain for Silica No. 3 & 7 OMC-RMC and RMC-RMC samples

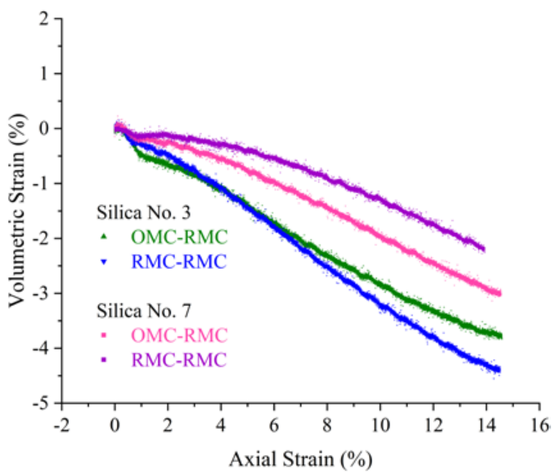

Fig. 11 Volumetric strain vs. axial strain for Silica No. 3 & 7 OMC-RMC and RMC-RMC samples

The test results showed that silica no. 3 to silica no. 6 RMC-RMC samples undergo 0.7% to 2.0% more volumetric strain than the respective OMC-RMC samples. In contrast, the silica no. 7 RMC-

RMC sample undergoes 0.75% less volumetric strain than the respective OMC-RMC sample.

The effect of the initial moisture content on the shear strength of soil at residual moisture content can be observed in Fig. 12, which includes the comparison of peak deviatoric stress for OMC-RMC and RMC -RMC samples for various silica sands.

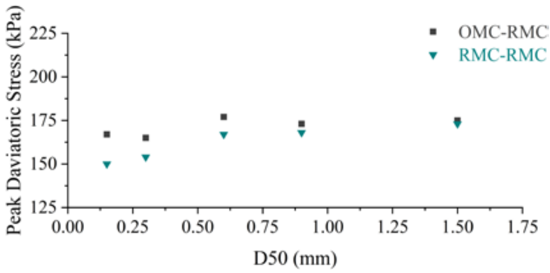

Fig. 12 Comparison of peak deviatoric stress for the OMC-RMC and RMC-RMC samples

The difference in peak deviatoric stress for the respective OMC-RMC and RMC-RMC samples is pronounced with a decrease in the soil mean grain size. This is because the amount of water present in the soil at the time of compaction influences soil particle lubrication. Soil compacted at optimal moisture content results in better particle-to-particle connection, which results in higher shear strength, as observed in this experimental study.

Second, when the soil is compacted at the optimal moisture content and then drained to the residual moisture content, a small capillary bridge remains between the soil particles at the residual moisture content [26]. These small capillary bridges contribute to the strength of the soil. As coarser soil particles are more affected by gravitational forces, the effect of the initial moisture content on the soil shear strength tends to decrease from silica no. 7 to 3.

6.3 Soil Shear Strength Comparison for Experimental and Analytical Methods

The shear strength of soil generally has a direct relation with the particle size of the test material for saturated soils [27,28]. In this study, it was observed that the peak shear strength for sands at the optimal moisture content does not show a clear relation with the particle size. Each test sample was prepared at the optimal moisture content, causing different initial suctions (due to differences in the particle size of sands and degree of saturation), which influences the shear strength of the test soils [29].

Various approaches adopted in the past to determine the shear strength of unsaturated soils, as discussed in the literature review section, include the effective stress approach [10] either using parameter χ [11] or the suction stress concept [5] and the independent stress variable approach [12] using φ^b.

Parameter χ is related to the degree of saturation, and suction stress σ_s defined by [5] also depends upon the degree of saturation. Vanapalli [9] gave Eq. 5, which proposes a relationship between the degree of saturation and $tan\varphi^b$ as follows.

$$\tau = c' + (\sigma - u_a)tan\varphi' + (S^k)(u_a - u_w)tan\varphi' \quad (5)$$

where S is the degree of saturation and k is unity for nonplastic soils. Considering this approach, the deviatoric stress at the residual moisture content was predicted for various silica sands. Analytically predicted values of the deviatoric stress were also compared with experimentally determined deviatoric stress of the same silica sand at residual moisture content. The comparison shows that the predicted values of deviatoric stress for the samples at residual moisture content are well below the experimental values (Fig. 13). Equation 5 considers the product of the degree of saturation and the suction to predict the strength increment due to an increase in suction. However, beyond the residual suction, the soil contains residual moisture content with the degree of saturation approaching zero. That is why Equation 5 failed to predict the accurate increment in deviatoric stress at residual moisture content.

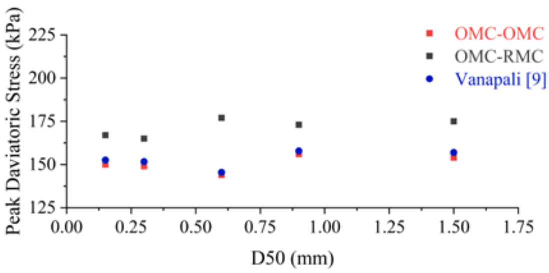

Fig. 13 Comparison of peak deviatoric stress for silica sands (experimental and analytical)

7. CONCLUSIONS

A modified triaxial apparatus was used in this study to explore the mechanical behavior of unsaturated silica sand for various combinations of initial and final moisture content conditions. This study aimed to study the practical implication of the unsaturated shear strength of soil. For that purpose, triaxial compression tests were carried out on silica sand nos. 3 to 7 with mean grain sizes ranging from 1.5 to 0.15 mm.

The shear strength of silica nos. 3 to 7 was determined at the optimal moisture content for the sample initially prepared at the optimal moisture content. The shear strength of the same soils was also determined at residual moisture content for the sample initially prepared at optimal and residual moisture content. The comparison of the test results shows the following:

1. The soil shear strength remains higher at the residual moisture content than at the optimal moisture content when the sample is prepared at the optimal moisture content.
2. The soil shear strength at residual moisture content is higher for the sample initially prepared at optimal moisture content compared with the shear strength of the soil sample prepared at residual moisture content.
3. Experimental determination of the shear strength of soil at residual moisture content gives higher shear strength compared to the shear strength predicted from the analytical method for the same soil. Furthermore, the analytical methods do not incorporate the effect of the initial moisture content on the shear strength of unsaturated soil.

8. ACKNOWLEDGMENTS

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