

ASSESSING THE ULTIMATE BEARING CAPACITY OF FOOTING IN A TWO-LAYERED CLAYEY SOIL SYSTEM USING THE RIGID PLASTIC FINITE ELEMENT METHOD

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ABSTRACT: Ultimate bearing capacity formulae for foundations are specified in the guideline published by the Architectural Institute of Japan for the design of building foundations. The rigid plastic finite element method was developed by Tamura and Ohtsuka to estimate the ultimate bearing capacity of footing. Unlike deformation analysis, this method employs limited soil constants; it uses only the strength parameters of cohesion c and friction angle ϕ as it considers the limit state directly and disregards the deformation of the building and ground. A series of rigid plastic finite element analyses were conducted to compare the ultimate vertical and inclined bearing capacities of spread foundations between the simulation results and the theoretical formula for a two-layered clayey soil system. The change in the failure mode of the ground was discussed using the geometrical ratio between the width of the footing and the height of the surface layer. The strength ratio of the surface and second ground layer was clearly shown to affect the formation of failure mode. The applicability of the rigid plastic finite element method for the assessment of the ultimate bearing capacity of a two-layered clayey soil system was successfully demonstrated.

Keywords: Two-layered clayey soil, Ultimate bearing capacity, Rigid plastic finite element method, Vertical and inclined load

1. INTRODUCTION

The calculation of the ultimate bearing capacity of the soil is important when designing a building [1]. The ultimate bearing capacity formulae for building foundations are specified in the guideline published by the Architectural Institute of Japan [2]. These formulae were based on experiments as well as theoretical considerations with regard to risk avoidance. However, the vertical bearing capacity of two-layered clayey soil has not been adequately investigated. In this research, the vertical bearing capacity of two-layered clayey soil was analyzed using numerical simulations. The bearing capacity under squeeze breakdown of clayey soils was last discussed at GEOMATE2017 [3]. In this case, while the soil was assumed to be two-layered, the lower clay layer was very stiff compared to its upper counterpart. Thus, shear failure occurs in the upper clay layer. In this research, it is assumed that the strengths of the upper and lower clay layers are relatively similar. Shear failure in both the upper and lower clay layers was considered. Moreover, the inclined bearing capacity was investigated in specific cases. First, the bearing capacity of the two-layered clayey soil was simulated [4, 5], and subsequently, the associated inclined bearing capacity was discussed. The analysis uses the rigid plastic finite element method (RPFEM), which was developed

separately by Tamura and Ohtsuka [6, 7]. This method was employed to estimate the ultimate bearing capacity of footing. The Drucker-Prager yield function was adopted as the soil constitutive equation, and associate and non-associate flow rules were introduced to establish the configuration relationship of the ultimate state. Hoshina et. al. [8-10] applied a higher order soil constitutive equation. Using this method, the structural safety assessment or calculation of soil bearing capacity was evaluated. A characteristic of this method is that, in contrast with deformation analysis, it applies limited soil constants; it uses only strength parameters such as cohesion and friction angle because it addresses the limit state directly by disregarding the deformation of the building and ground. Since the RPFEM uses the upper bound theorem of plastic theory, the end result is slightly larger than the true value.

2. THE CONSTITUTIVE EQUATION FOR RIGID PLASTIC FINITE ELEMENT METHOD

2.1 Outline of RPFEM

Tamura [6] developed the rigid plastic constitutive equation for frictional material. The Drucker-Prager's type yield function is expressed as follows.

$$f(\sigma) = \alpha I_1 + \sqrt{J_2} - k = 0 \quad (1)$$

where I_1 is the first invariant of stress σ_{ij} and $I_1 = \text{tr}(\sigma_{ij})$ in which extension stress is defined positive.

J_2 is the second invariant of deviator stress S_{ij} and $J_2 = \frac{1}{2} S_{ij} S_{ij}$ and the coefficients, $\alpha = \frac{\tan \phi}{\sqrt{9+12 \tan^2 \phi}}$ and $k = \frac{3c}{\sqrt{9+12 \tan^2 \phi}}$ and the material constants corresponding to shear resistance angle and cohesion under the plane strain condition. The volumetric strain rate is expressed as follows:

$$\dot{\varepsilon}_v = \text{tr}(\dot{\varepsilon}) = \text{tr}\left(\lambda \frac{\partial f(\sigma)}{\partial \sigma}\right) = \text{tr}\left(\lambda \left(\alpha \mathbf{I} + \frac{s}{2\sqrt{J_2}}\right)\right) = \frac{3\alpha}{\sqrt{3\alpha^2 + \frac{1}{2}}} \dot{\varepsilon} \quad (2)$$

where λ is an indeterminate multiplier and $\mathbf{I}$ is the unit tensor. The strain rate $\dot{\varepsilon}$ which is purely plastic component should satisfy the volumetric constraint condition as follow:

$$h(\dot{\varepsilon}) = \dot{\varepsilon}_v - \frac{3\alpha}{\sqrt{3\alpha^2 + \frac{1}{2}}} \dot{\varepsilon} = \dot{\varepsilon}_v - \eta \dot{\varepsilon} = 0 \quad (3)$$

in which $\dot{\varepsilon}_v$ and $\dot{\varepsilon}$ indicate the volumetric strain rate and the norm of strain rate, respectively. The parameter η is defined in Eq. (3). The rigid plastic constitutive equation is expressed by Lagrangian method after Tamura as follows:

$$\sigma = \frac{k}{\sqrt{3\alpha^2 + \frac{1}{2}}} \frac{\dot{\varepsilon}}{\dot{\varepsilon}} + \beta \left(\mathbf{I} - \eta \frac{\dot{\varepsilon}}{\dot{\varepsilon}} \right) \quad (4)$$

where β represents a Lagrangian multiplier which indicates the equilibrating stress satisfying the yield function expressed by Eq. (1). Moreover, the constraint condition on the strain rate is introduced into the constitutive equation directly with the use of the penalty method [8-9], [11]. The stress-strain rate relation for the Drucker-Prager's yield function is expressed as follow:

$$\sigma = \frac{k}{\sqrt{3\alpha^2 + \frac{1}{2}}} \frac{\dot{\varepsilon}}{\dot{\varepsilon}} + \kappa (\dot{\varepsilon}_v - \eta \dot{\varepsilon}) \left(\mathbf{I} - \eta \frac{\dot{\varepsilon}}{\dot{\varepsilon}} \right) \quad (5)$$

where κ is a penalty constant. FEM with this constitutive equation provides the equivalent equation of the upper bound theorem in plasticity so that this method is called as RPFEM in this study. It is noted property of this constitutive equation that the relationship between stress and strain rate is specified. The norm of strain rate is substantially indeterminate since the limit state of the structure is focused. Stress is determined for the normalized strain rate using its norm. In order to determine the limit load coefficient

for the prescribed load, Hoshina et al. [9] introduced the constraint condition on external work into the equilibrium equation by using the penalty method. It reported the rational result was obtained by the developed method in comparison with the previous works. The use of the penalty method was profited computation time efficiency and obtaining a stable computational result.

2.2 Outline of high-order yield function

The high-order yield function can be expressed as follows:

$$f(\sigma) = a I_1 + (J_2)^n - b = 0 \quad (6)$$

where a , b , and n are material parameters. When $n = 0.5$, Eq. (6) corresponds to the DP model function. Assuming an associated flow rule, the strain rate and volumetric strain can be obtained by the Eqs. (6) and (7).

$$\dot{\varepsilon} = \lambda \frac{\partial f(\sigma)}{\partial \sigma} = \lambda \frac{\partial}{\partial \sigma} (a I_1 + (J_2)^n - b) = \lambda (a \mathbf{I} + n J_2^{n-1} \mathbf{s}) \quad (7)$$

$$\begin{aligned} \dot{\varepsilon}_v = \text{tr} \dot{\varepsilon} &= \text{tr} \left(\lambda (a \mathbf{I} + n J_2^{n-1} \mathbf{s}) \right) = 3a\lambda \\ &= \frac{3a}{\sqrt{3a^2 + 2n^2(b - a I_1)^{2-1/n}}} \dot{\varepsilon} \end{aligned} \quad (8)$$

where λ is the plastic multiplier. Based on Eqs. (6) - (8), the first stress invariant is expressed as follows:

$$I_1 = \frac{b}{a} - \frac{1}{a} \left\{ \frac{1}{2n^2} \left[\left(3a \frac{\dot{\varepsilon}}{\dot{\varepsilon}_v} \right)^2 - 3a^2 \right] \right\}^{\frac{n}{2n-1}} \quad (9)$$

Finally, the rigid plastic equation was given as follows:

$$\begin{aligned} \sigma &= \frac{3a}{n} \left\{ \frac{1}{2n^2} \left[\left(3a \frac{\dot{\varepsilon}}{\dot{\varepsilon}_v} \right)^2 - 3a^2 \right] \right\}^{\frac{1-n}{2n-1}} \frac{\dot{\varepsilon}}{\dot{\varepsilon}_v} \\ &+ \left(\frac{b}{3a} - \frac{1}{3a} \left[\frac{1}{2n^2} \left(3a \frac{\dot{\varepsilon}}{\dot{\varepsilon}_v} \right)^2 - 3a^2 \right]^{\frac{n}{2n-1}} \right. \\ &\quad \left. - \frac{a}{n} \left[\frac{1}{2n^2} \left(3a \frac{\dot{\varepsilon}}{\dot{\varepsilon}_v} \right)^2 - 3a^2 \right]^{\frac{1-n}{2n-1}} \right) \mathbf{I} \end{aligned} \quad (10)$$

3. SIMULATIONS OF VERTICAL BEARING CAPACITY OF TWO-LAYERED CLAYEY SOIL

3.1 Review of the bearing capacity of two-layered clayey soil

This research considers two key parameters. The first is B/H , where B is the foundation width and H is the height of the upper clayey soil. The other is c_{u2}/c_{u1} , where c_{u1} and c_{u2} are the shear strength of the upper and lower clayey soil, respectively. The vertical bearing capacity of two-layered clayey soil was reported by Vesic [12] as follows.

1. $c_{u1} > c_{u2}$: Calculate the vertical bearing capacity by applying the distributed fracture mode.
2. $c_{u1} \ll c_{u2}$: Calculate the vertical bearing capacity by squeeze breakdown of clayey soils.
3. $c_{u1} < c_{u2}$: Calculate the vertical bearing capacity by interpolation between the results from the above two methods.

In the case of $c_{u1} < c_{u2} < \infty$, the vertical bearing capacity is calculated as follows.

$$q_f = c_{u1}N_m + \gamma_t D \quad (11)$$

q_f (kPa) is the vertical bearing capacity, γ_t (kN/m³) is the unit weight, and N_m is as seen in Fig. 1. D (m) is the penetration depth.

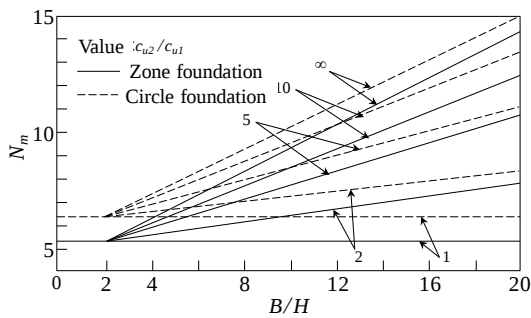


Fig. 1 Bearing capacity of two-layered clayey soils (after Yamaguchi [13])

3.2 Numerical conditions

Figure 2 shows the numerical meshes for a plain strain condition. Fig. 2(a) shows that the mesh is 200 m wide, and the depth of the upper layer is 15 m while that of its lower layer is 35 m. The foundation width was set to 30 m ($B/H = 2$). Fig. 2(b) shows that the mesh is 390 m wide, and the depths of the upper and lower layers are 5 m and 95 m, respectively. The foundations widths were thus set to 18, 30, 42, 50, 78, and 102 m ($B/H = 3.6-20.4$).

In the RPFEM, the numerical mesh only needs to be set within the failure mode. The required conditions for the numerical meshes are satisfied with Fig. 2. The strength of the upper clay layer was set to $c_{u1} = 10$ kPa, while that of the lower clay layer was set to $c_{u2} = 12, 15, 20, 30, 50, 75$, and 100 kPa. The strength of the foundation was set to $c = 100,000$ (kPa), assuming a rigid foundation.

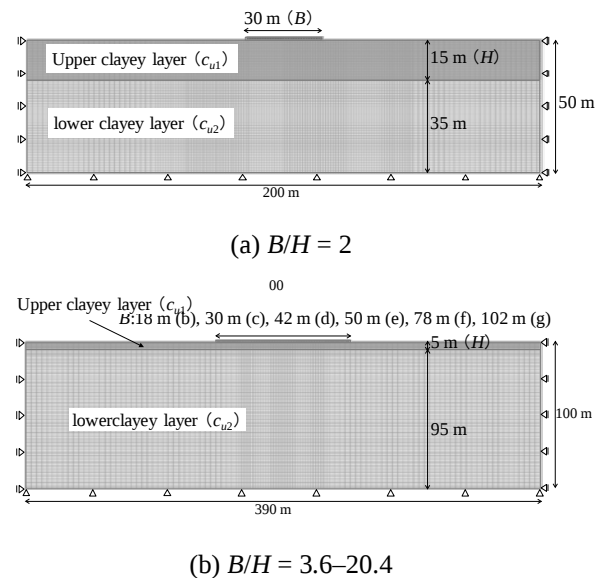


Fig. 2 Numerical meshes

3.3 Numerical results

Numerical simulations were conducted for $c_{u1} < c_{u2} < \infty$. Table 1 shows the results of c_{u2}/c_{u1} for different values of B/H . Figure 3 shows the relationship between N_m and B/H as per Fig. 1. In Fig. 1, N_m increases as B/H increases. Moreover, the larger the value of c_{u2}/c_{u1} , the larger the slope of B/H with respect to N_m . Figure 3 shows that as B/H increases, N_m also increases. However, for small values of c_{u2}/c_{u1} , the increase in N_m is not observed, and N_m approaches a constant value. This tendency becomes increasingly prominent as c_{u2}/c_{u1} decreases.

Table 1 Variation in N_m as per c_{u2}/c_{u1} and B/H

	c_{u2}/c_{u1}						
B/H	1.2	1.5	2.0	3.0	5.0	7.5	10.0
2.00	5.55	5.48	5.40	5.39	5.39	5.39	5.39
3.60	5.94	6.49	6.45	6.35	6.34	6.34	6.34
6.00	6.12	7.09	8.05	7.81	7.74	7.74	7.74
8.40	6.28	7.36	8.84	9.50	9.27	9.20	9.20
10.00	6.25	7.46	9.25	10.88	10.23	10.20	10.19
15.60	6.33	7.69	9.74	12.94	14.12	13.86	13.83
20.40	6.27	7.66	9.84	13.55	18.15	17.06	16.87

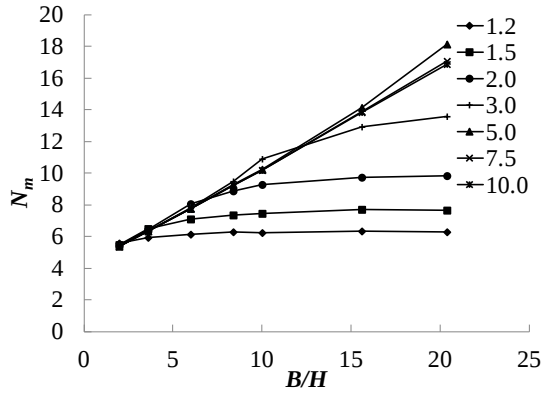

 Fig. 3 The relationship between N_m and B/H

Figure 4 shows the shear strain contours at failure for $B/H = 3.6$ and 15.6 , and $c_{u2}/c_{u1} = 3$. The red line marks the boundary between the upper and lower clayey soil layer. When $B/H = 3.6$, the squeeze breakdown mode of clayey soils was observed. On the other hand, when $B/H = 15.6$, the whole failure mode, including the lower clayey layer, was observed. When B/H is relatively small, the squeeze breakdown of clayey soils occurs because the strength of the lower clayey layer is higher than that of the upper layer. On the other hand, when B/H is larger, the whole failure occurs. In the case of squeeze breakdown of clayey soils as B/H increases, N_m increases linearly, as shown in Fig. 1. Conversely, it is clear that when the whole failure occurs, even if B/H increases, the increase in N_m will cease.

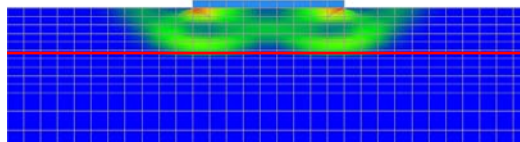
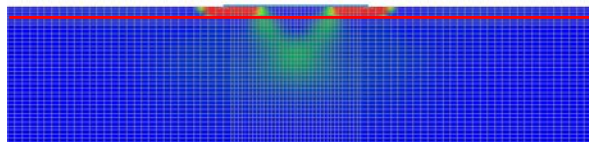

 (a) $B/H = 3.6, c_{u2}/c_{u1} = 3$

 (b) $B/H = 15.6, c_{u2}/c_{u1} = 3$

Fig. 4 Shear strain contours

Figure 5 shows the shear strain contours at failure for $B/H = 10.0$ and 15.6 , and $c_{u2}/c_{u1} = 7.5$ and 10.0 . All failure modes show a squeeze breakdown of clayey soils, and the vertical bearing capacity was calculated using the following Eq. (12).

$$q_f = 4.14c_u + \frac{c_u B}{2H} \quad (12)$$

where q_f , c_u , B , and H are the ultimate bearing capacity of squeeze breakdown (kN/m^2), undrained strength (kN/m^2), foundation width (m), and height of the clay layer (m), respectively. The bearing capacity during squeeze breakdown of clayey soils is a function of c_u (c_{u1} : the strength of the upper clayey layer) and B/H , as seen in Eq. (12). When the strength of the clay was constant, the bearing capacity of squeeze breakdown in clayey soils follows a linear relationship with B/H .

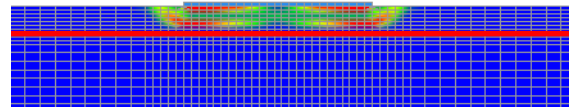
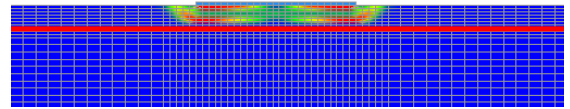
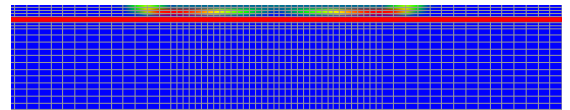
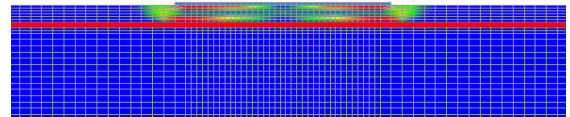

 (a) $B/H = 10.0, c_{u2}/c_{u1} = 7.5$

 (b) $B/H = 10.0, c_{u2}/c_{u1} = 10.0$

 (c) $B/H = 15.6, c_{u2}/c_{u1} = 7.5$

 (d) $B/H = 15.6, c_{u2}/c_{u1} = 10.0$

Fig. 5 Shear strain contours

3.4 Remarks

Vesic [12] concluded the following. When the strength of the lower clayey soil reaches ∞ , squeeze breakdown occurs, and the vertical bearing capacity increases linearly in relation to B/H . When the ratio between the upper and lower clayey soil strength (c_{u2}/c_{u1}) exceeds 10.0 , whole failure (including the lower clayey layer) takes place. However, using the RPFEM, the threshold for the squeeze breakdown of clayey soils and whole failure, including that of the lower clayey layer, is 3 ($c_{u2}/c_{u1} > 3.0$). Comparing Fig. 1 and 3 with regard to the slope between N_m and B/H , the formula proposed by Vesic [12] provides a different result, that is, lower than the numerical results of this study. This can be attributed to the factor of safety in design. However, when $c_{u2}/c_{u1} >$

3.0, the vertical bearing capacity corresponds to the upper limit for B/H , and thus, it is clear that the method proposed by Vesic [12] for bearing capacity calculations may result in a dangerous design.

4. SIMULATIONS OF THE VERTICAL BEARING CAPACITY OF TWO-LAYERED CLAYEY SOIL WITH AN INCLINED LOAD

The simulations for bearing capacity were also performed with an inclined load. They were conducted for three inclined angles (10, 20, and 30°) at $B/H = 2.0$ and 20.0, and $c_{u2}/c_{u1} = 1.2$ and 10.0. The numerical mesh and conditions are the same as those in Fig. 2.

4.1 Numerical results

Figure 6 shows the relationship between the inclined angle and N_m at $B/H = 2.0$ and $c_{u2}/c_{u1} = 1.2$ and 10.0. The theoretical bearing capacity (Th) was calculated following Eq. (3), as proposed by AIJ [2] for $c_u = 1.0$ (upper clayey layer strength) and 1.2 (lower clayey layer strength).

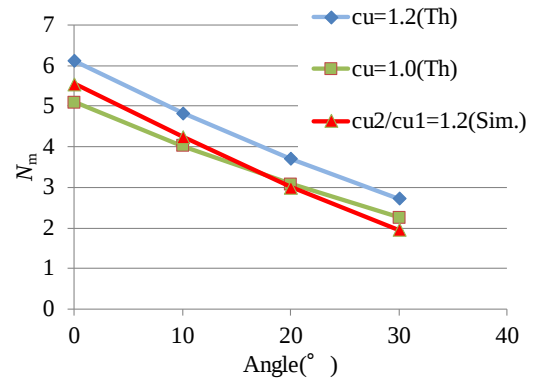
$$q_f = c \times i_c \times \alpha \times N_c \quad (13)$$

where q_f , i_c , α , and N_c are the ultimate bearing capacity (kN/m²), a correction factor of the inclined load ($i_c = (1 - \theta/90)^\alpha$), (θ : inclined angle (°)), shape coefficient ($= 1.0$), and coefficient of bearing capacity ($= 5.1$ at $\phi = 30^\circ$), respectively.

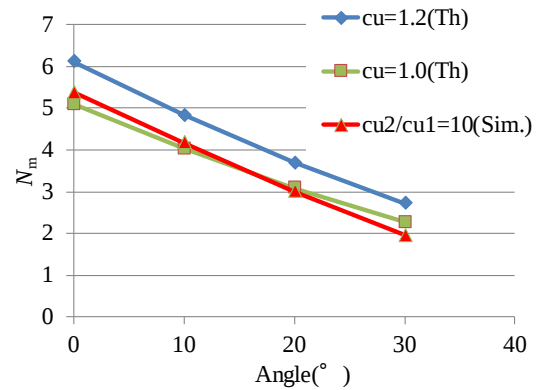
Figure 7 shows the shear strain contours at failure for $B/H = 2.0$, $c_{u2}/c_{u1} = 1.2$ and 10.0, and angle = 0 and 30°. At $c_{u2}/c_{u1} = 1.2$ and inclined angle = 0°, the simulation provides values for N_m between those for $c_u = 1.0$ (Th) and $c_u = 1.2$ (Th). As the inclined angle increases, the simulated value of N_m approaches the theoretical value of N_m at $c_u = 1.0$ (Th). At $c_{u2}/c_{u1} = 10.0$ and inclined angle = 0°, the effect of c_{u2}/c_{u1} is not evident as B/H is small. Therefore, shear failure of the upper clayey soil layer occurs (shown Fig. 7(a, c)). However, as the inclined angle increases, the failure mode changes from total wedge failure to slip failure for the upper clayey soil layer (shown Fig. 7(b, d)). Therefore, for larger inclined loads, the simulated bearing capacity approaches the bearing capacity calculated using the strength of the upper clayey layer of soil.

Figure 8 shows the relationship between the inclined angle and N_m at $B/H = 20.0$ and $c_{u2}/c_{u1} = 1.2$ and 10.0. Figure 9 shows the shear strain contours at failure for $B/H = 20.0$, $c_{u2}/c_{u1} = 1.2$ and 10.0, and inclined angle = 0 and 30°. At $c_{u2}/c_{u1} = 1.2$ and inclined angle = 0°, the simulated value of N_m lies between the N_m at $c_u = 1.2$ (Th). The failure mode in Fig. 9(a) indicates that the strength of the lower clayey layer plays a dominant role in bearing capacity.

As the inclined load increases, the simulated values of N_m approach the theoretical values of N_m at $c_u = 1.0$ (Th) because of the slip failure mode of the upper clayey soil layer (shown in Fig. 9(b)). At $c_{u2}/c_{u1} = 10.0$ and inclined angle = 0°, the simulated value of N_m is 16.9. From Eq. (2), the vertical bearing capacity during squeeze breakdown is 14.1, and thus, the numerical result is slightly larger than the theoretical value. As the inclined load increases, the simulated value of N_m approaches the theoretical value of N_m at $c_u = 1.0$ (Th). This is due to the slip failure of the upper clayey soil layer, in a manner similar to that described above (shown in Fig. 9 (d)).

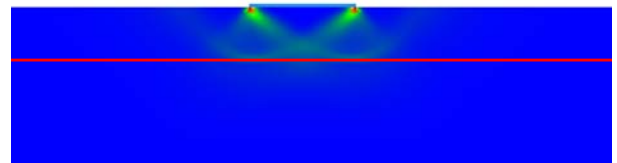


(a) $B/H = 2.0, c_{u2}/c_{u1} = 1.2$

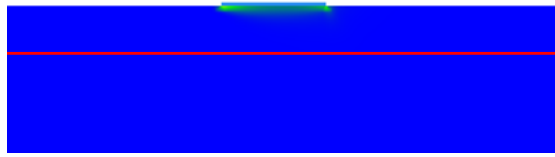


(b) $B/H = 2.0, c_{u2}/c_{u1} = 10.0$

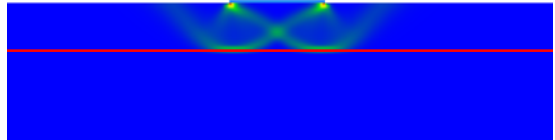
Fig. 6 Relationship between inclined angle and N_m



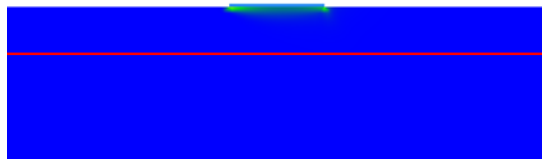
(a) $B/H = 2.0, c_{u2}/c_{u1} = 1.2, \text{angle} = 0$



(b) $B/H = 2.0$, $c_{u2}/c_{u1} = 1.2$, angle = 30

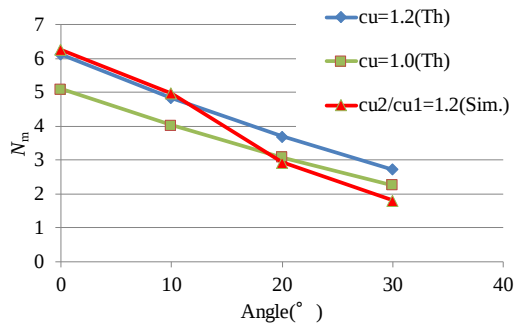


(c) $B/H = 2.0$, $c_{u2}/c_{u1} = 10.0$, angle = 0

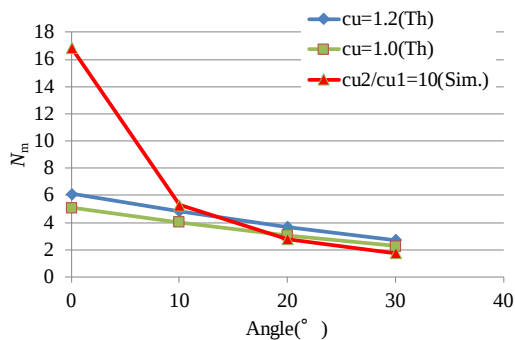


(d) $B/H = 2.0$, $c_{u2}/c_{u1} = 10.0$, angle = 30

Fig 7. Shear strain contours

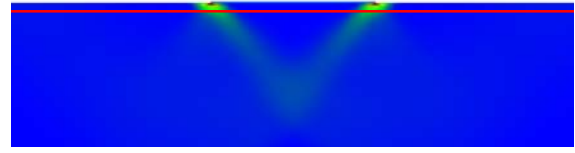


(a) $B/H = 20.0$, $c_{u2}/c_{u1} = 1.2$



(b) $B/H = 20.0$, $c_{u2}/c_{u1} = 10.0$

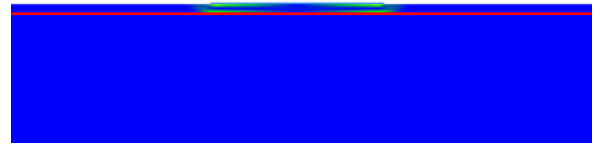
Fig. 8 Relationship between inclined angle and N_m



(a) $B/H = 20.0$, $c_{u2}/c_{u1} = 1.2$, angle = 0



(b) $B/H = 20.0$, $c_{u2}/c_{u1} = 1.2$, angle = 30



(c) $B/H = 20.0$, $c_{u2}/c_{u1} = 10.0$, angle = 0



(d) $B/H = 20.0$, $c_{u2}/c_{u1} = 1.0$, angle = 30

Fig 9. Shear strain contours

5. CONCLUSION

Numerical simulations of the vertical bearing capacity of a footing in a two-layered clayey soil system were conducted using the RPFEM. When c_{u1} (strength of the lower clayey layer) $< c_{u2}$ (strength of the upper clayey layer) $< \infty$, failure occurs across both soil layers. The greater the values of B/H (B : foundation width, H : height of the upper clayey soil) and c_{u2}/c_{u1} , the greater the bearing capacity. When squeeze breakdown failure occurs, the bearing capacity increases linearly with respect to B/H . On the other hand, when c_{u2}/c_{u1} is small, the increase in bearing capacity levels off, reaching a constant value despite the increase in B/H . The result shows that this tendency is more prominent as c_{u2}/c_{u1} becomes smaller, and the bearing capacity becomes constant with a smaller B/H . In the case of $c_{u2}/c_{u1} > 3.0$, the whole failure mode with upper and lower clayey soil layer occurred from the squeeze breakdown failure mode. Thus, the findings proved that the bearing capacity calculation method proposed by Vesic [12] is inadequate and can result in an unsafe design. In the case of inclined loads, when the inclined load was larger, the simulated bearing capacity approached the

value of the bearing capacity calculated for the upper layer of clayey soil; for large inclined angles, the slip failure mode occurs in the upper layer of the clayey soil. Thus, the applicability of the RPFEM for the assessment of the ultimate bearing capacity of a two-layered clayey soil system is successfully demonstrated.

6. REFERENCES

- [1] Terzaghi, K. and Peck, R.B.: Soil Mechanics in Engineering Practice, John Wiley & Sons, pp. 472-569. 1967.
- [2] Architectural Institute of Japan, AIJ (1988, 2001), Recommendations for the design of building a foundation, pp.105-111, 2001. (in Japanese)
- [3] Kaneda, K., Aoki, M., Tanikawa, T., and Ohtsuka, S.: Ultimate vertical bearing capacity of clayey footing under squeeze breakdown using rigid-plastic finite element method, Seventh International Conference on Geotechnique, Construction Materials and Environment, Mie, Japan, Nov. 21-24, 2017, ISBN: 978-4-9905958-8-3 C3051.
- [5] Kaneda, K., Tanikawa, T., Hamada, J., Ohtsuka, S., and Aoki, M.: "Analysis of ultimate bearing capacity of footing of two-layered clayey soil system by rigid plastic finite element method", AIJ Journal of Technology and Design, Vol.19, No.43, pp. 881-885, 2013. (in Japanese)
- [6] Kaneda, K., Aoki, M., and Ohtsuka, S.: Ultimate vertical bearing capacity of spread foundation on sandy soils using rigid plastic finite element method, Numerical Analysis in Geotechnics – NAG2018, Ho Chi Minh City, Vietnam, 22nd March 2018.
- [7] Tamura, T., Kobayashi, S., and Sumi, T.: "Limit analysis of soil structure by rigid plastic finite element method", Soils and Foundations, Vol.24, No.1, pp.34-42. 1984.
- [8] Asaoka, A. and Ohtsuka, S.: "The analysis of failure of a normally consolidated clay foundation under embankment loading", Soils and Foundations, Vol.26, No.2, pp.47-59. 1986.
- [9] Hoshina, T., Ohtsuka, S. and Isobe, K., Ultimate bearing capacity of ground by Rigid plastic finite element method taking account of stress dependent non-linear strength property, Journal of Applied Mechanics, 6, pp.191-200, 2011.
- [10] Hoshina, T., Ohtsuka, S. and Isobe, K., Ultimate bearing capacity analysis of ground by rigid plastic finite element method taking account of stress dependent non-linear strength property, Journal of JSCE, Ser.A2 (Applied Mechanics), Vol.68, No.2 I_327-I_336, 2012. (in Japanese)
- [11] Nguyen Du L., Ohtsuka S, and Kaneda K.: Ultimate bearing capacity of footing on sandy soil against combined load of vertical horizontal and moment loads. *GEOMATE2015*, 2015.
- [12] Nguyen, Du L., Ohtsuka, S., Hoshina, T. and Isobe, K., Discussion on size effect of footing in ultimate bearing capacity of sandy soil using rigid plastic finite element method. Soils and Foundations. 56 (1), 93-103, 2016.
- [13] Vesic, A.S.: Bearing Capacity of Shallow Foundation, Found. Engineering ed. by Winterkorn & Fung, van Nostrand Reinhold, 1975.
- [14] Yamaguchi, H.: Theory and applications of bearing capacity and deformation of shallow foundation, Soil Japanese Society of Soil Mechanics and Foundation Engineering, No.2, pp.85-91, 1982. (in Japanese)
- [15] Meyerhof, G.G. and Chaplin, T.K.: "The compression and bearing capacity of cohesive layers", British Journal of Applied Physics, Vol.4, No.1, 1953.

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