

INFLUENCE OF STRIP FOOTING SIZE ON THE ULTIMATE BEARING CAPACITY FORMULA FOR SANDY SOILS

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ABSTRACT: For the estimation of the ultimate bearing capacity (UBC) of footings, most contemporary formulas employ a linear yield function in the shear stress-normal stress space. However, it is well known that the general property in the failure envelopes of sandy soils manifests the non-linear effect of the stress level on the peak friction angle. The focus of this research study is the assessment of the UBC of surface strip footings ascribed to the effect of confining stress level and relative density (D_r) on the shear strength of sandy soils. The rigid plastic finite element method (RPFEM), using the confining stress dependence property of Toyoura sand, is utilized in non-linear finite element analyses. The results of the UBC analyses are ascertained to be consistent with those of the centrifuge experiments in the published references. The ground failure domains in the case of the non-linear shear strength model are gleaned smaller than those in the case of the linear shear strength one. The analysis results are compared with prevailing guidelines, for instance, the Architectural Institute of Japan (AIJ) and the Japan Road Association (JRA), which are developed for the mean shear strength property of sandy soils. The applicability of the UBC formula to effective stress analysis is also discussed, and the modified formula is developed.

Keywords: Ultimate bearing capacity, Strip footing, Size effect, Relative density, Sandy soils

1. INTRODUCTION

The design life span of a building substantially depends upon a meticulous assessment of the bearing capacity of its footing. Most of the formerly developed well-known ultimate bearing capacity formulas, for instance, Meyerhof [1], do not appraise the effect of footing size on the shear strength property of sandy soils. On the other hand, numerous experimental studies have now ratified that the typical yielding phenomenon of sandy soils is non-linear in shear stress-normal stress space [2–4]. Therefore, the influence of stress level on failure surface also needs to be investigated against classical bearing capacity theories [5]. Likewise, the effect of non-linearity in shear strength property of sandy soil on UBC needs to be well conceived against the Mohr-Coulomb failure criterion [6,7]. The UBC formulas being commonly used in Japan are those recommended by AIJ [8] and JRA [9], which consider the same modification coefficient to account for the effect of footing size on UBC in the absence of surcharge load. The simplified AIJ UBC formula is indicated in Eq. (1) in the case of surface strip footing under centric vertical load only.

$$q_u = \frac{1}{2} \gamma B \eta_\gamma N_\gamma \quad \text{where} \quad \eta_\gamma = \left(\frac{B}{B_o} \right)^{-\frac{1}{3}}, \quad B_o = 1 \text{ m} \quad (1)$$

Here, N_γ depicts the well-known bearing capacity factor [1], γ denotes the soil unit weight, B and B_o account for the footing size and reference value in it, respectively, and η_γ symbolizes the size effect modification coefficient for N_γ .

The size effect of footing on UBC is governed by the confining stress dependency of shear strength parameter ϕ [10–13]. In both AIJ and JRA formulas, the stress level effect is estimated in terms of footing size only, for the centric vertical load without surcharge. Nonetheless, prominent experimental studies have highlighted the significance of soil unit weight (γ) in determining the stress level effect on UBC in terms of product γB [14–16]. Furthermore, the size effect has also been attributed to the influence of relative density experimentally [17]. The size effect of footing on UBC has recently been investigated in some of the prominent research studies by using different analysis techniques [18–21]. However, the detailed literature review indicated that the effect of soil unit weight has not yet been widely investigated in estimating the influence of stress level on UBC. Consequently, a simplified UBC formula to well estimate the size effect in terms of contributing factors, i.e., γ and B , is still lacking. Therefore, this study is focused on the estimation of the size effect

of footing on UBC under centric vertical load in the case of total and effective stress analysis conditions. RPFEM, using non-linear shear strength characteristics of Toyoura sand, is utilized in finite element analysis. The numerical method is validated against the centrifuge experiments in the published references [22,23]. Finally, the new UBC formula is proposed in terms of the size effect modification coefficient concerning the influence of stress term γB , and its performance is gauged in comparison with the existing UBC formulas/guidelines. The proposed UBC formula is superior since it better estimates the size effect in terms of the stress term γB . This is primarily because the mean effective stress in the failure domain is governed by both γ and B through its dependency on the limit bearing pressure. The performance of the proposed UBC formula is well ascertained corresponding to the wide variation in the soil shear strength characteristics. It is inferred that the findings of the current research can be helpful in improving the existing UBC codes/guidelines. This study highlighted the variation in the size of the failure domain through the effect of stress level contributed by γ and B . Such findings will aid in the strengthening of engineering decisions in situations where the ground water table is subject to change, particularly in urban areas with buildings nearby. The proposed UBC formula is simple and can instantly be used by the practitioners to estimate the influence of stress level for an extensive range of subsoil conditions.

2. RESEARCH SIGNIFICANCE

The size of the footing significantly affects the ultimate bearing capacity through confining stress dependency of shear strength parameters. This study thoroughly investigated the size effect phenomenon by examining the influence of various contributing factors, such as footing size, soil unit weight, and relative density, based on the mechanical properties of Toyoura sand. The effective stress analysis assisted in better understanding the size effect phenomenon. The simplified UBC formula proposed in this research can estimate the UBC of surface strip footing under centric vertical load in the case of various sandy soils with sufficient accuracy.

3. CONSTITUTIVE EQUATION FOR RIGID PLASTIC FINITE ELEMENT METHOD

The rigid plastic constitutive equation was initially proposed for frictional materials by Tamura [24]. Furthermore, RPFEM has been validated to effectively solve complex stability problems in geotechnical engineering [25–28]. In this research, the non-linear hyperbolic function is employed to

express the yield function of sandy soils (Eq. (2)).

$$f(\sigma) = aI_1 + (J_2)^n = b \quad (2)$$

Here, a and b express the material shear strength characteristics, i.e., internal friction and cohesion, respectively, while n denotes the non-linearity of the yield function in relation to the first stress invariant, I_1 , and the second invariant of deviator stress, J_2 . The non-linear rigid plastic constitutive equation (Eq. (3)) used for numerical computations in this study was previously proposed and published by the coauthors [29]. In this equation, the stress is uniquely established for the plastic strain rate.

$$\sigma = \frac{3a}{n} \left\{ \frac{1}{2n^2} \left[\left(3a \frac{\dot{\epsilon}}{\dot{\epsilon}_v} \right)^2 - 3a^2 \right] \right\}^{\frac{1-n}{2n-1}} \frac{\dot{\epsilon}}{\dot{\epsilon}_v} + \left(\frac{b}{3a} - \frac{1}{3a} \left[\frac{1}{2n^2} \left(3a \frac{\dot{\epsilon}}{\dot{\epsilon}_v} \right)^2 - 3a^2 \right]^{\frac{n}{2n-1}} \right) I \quad (3)$$

where $\dot{\epsilon}_v$ and $\dot{\epsilon}$ depict the volumetric strain rate and the norm of the strain rate, respectively.

4. NON-LINEAR SHEAR STRENGTH OF SANDY SOILS

In this study, Toyoura sand is used to set the confining stress dependent shear strength property of model sandy soil. The physical properties of Toyoura sand are enlisted in Table 1 after Tatsuoka [30]. Figure 1 manifests the influence of confining pressure on the shear strength of Toyoura sand based on the experimental study of Tatsuoka [31]. It demonstrates that the internal friction angle, ϕ , decreases with the increase in confining pressure at a given void ratio, e .

Table 1 Physical characteristics of Toyoura sand

Property	Value
Specific gravity of solids, G_s	2.64
Max. void ratio, e_{\max}	0.977
Min. void ratio, e_{\min}	0.605
Mean grain size, d_{50}	0.16 mm
Uniformity coefficient, C_u	1.46
Grain shape	angular to sub-angular
Minerology	quartz (90%), chert (4%)

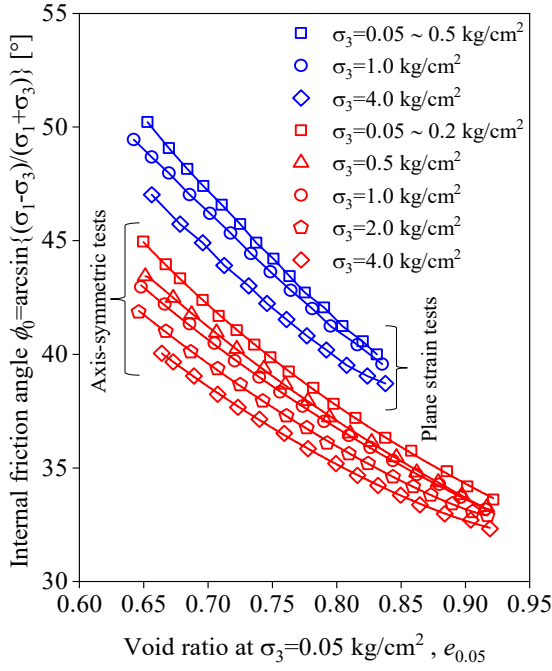


Fig. 1 Effect of confining pressure on the shear strength of air-pluviated Toyoura sand^[31]

In this study, triaxial compression test results are used to arrange the correlation between ϕ and I_1 (Fig. 2) with the aim of estimating the influence of pressure level on ϕ . Figure 2 specifies the reduction in internal friction angle with the increase in the first stress invariant. The negative values of I_1 express the compression stress. Moreover, the experimental data is extrapolated to estimate the y-intercept, i.e., internal friction angle, ϕ_0 , against each relative density, D_r , as indicated through dashed lines in Fig. 2, to normalize the internal friction angle corresponding to the various stress levels.

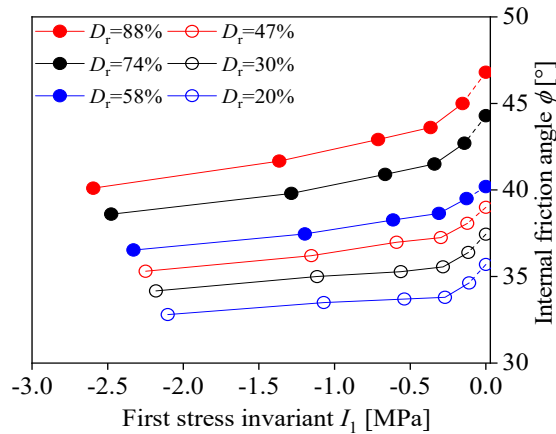


Fig. 2 Relation between ϕ and I_1 for Toyoura sand (after Tatsuoka^[31])

By using the least squares method, normalized relationships are established between ϕ/ϕ_0 and I_1

corresponding to each relative density as well as the mean trendline as can be observed in Fig. 3. Due to the similarity in soil response having different D_r under various stress levels, it is meaningful to establish a simplified relation based on the mean property of Toyoura sand. Likewise, based on the experimental study, Hettler [32] also ascertained the similar effect of confining pressure on ϕ in the case of different sands and relative densities. Their experimental study concluded that the parameter determining the effect of stress level remained almost unchanged in the case of three different sands i.e., Darmstadt, Degebo and Eastern Scheldt with different densities.

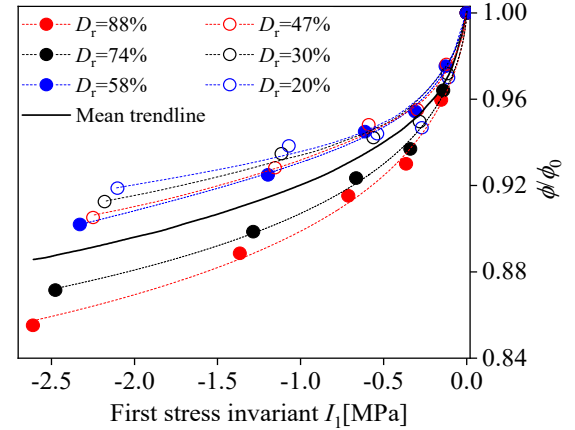


Fig. 3 Normalized relationship between ϕ/ϕ_0 and I_1 for Toyoura sand

The non-linear parameters can be easily set based on the relationships plotted in Fig. 3 corresponding to each relative density. Established on the mean property of Toyoura sand, the non-linear shear strength parameters can be set for the model sandy soil against various internal friction angles as can be observed in Table 2.

Table 2 Parameters for the model sandy soil

ϕ_0 (°)	a	b	n
30	0.175		
35	0.216	1	0.526
40	0.257		

5. VALIDATION OF THE NUMERICAL METHOD

In this study, UBC analysis is carried out by using the RPFEM code developed by the coauthors and previously published in the research article [29]. In this section, the estimated ultimate bearing capacity based on the RPFEM analysis is compared with the centrifuge model loading tests on surface

strip footings in Fig. 4 for four different relative densities of 58%, 74%, 85%, and 88% [22,23]. In Fig. 4, RPFEM analysis results indicate fairly good agreement with the experimental findings and a similar effect of footing size can be witnessed. It substantiates that the RPFEM using the non-linear shear strength property of sandy soil can well estimate the size effect of footing on UBC.

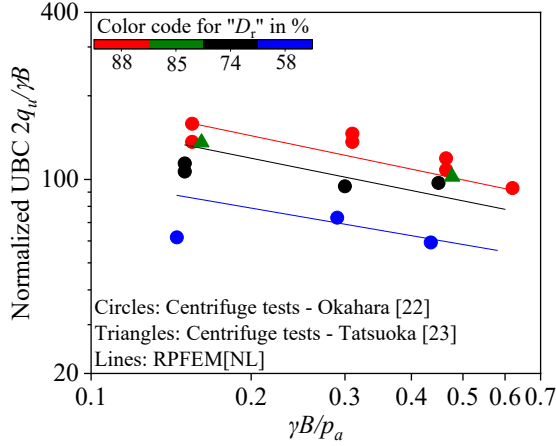


Fig. 4 Validation of the numerical method in the case of Toyoura sand

6. UBC ANALYSIS FOR MODEL SANDY SOIL

6.1 Drucker-Prager Criterion and Conventional UBC Estimation

This section analyzes the UBC of surface strip footing under plane strain condition by employing the linear shear strength model i.e., RPFEM Drucker-Prager (DP) using the shear strength parameters identical to those gleaned from conventional triaxial compression tests. The UBC is estimated for a wide range of footing sizes (1 m to 30 m) and soil shear strength parameters to properly investigate the correctness of the applied method. The footing is considered as a perfectly rigid mass, while the boundary conditions are defined to be wide enough so as not to have any influence of rigidity on the collapse mechanism. The analysis results are plotted in comparison with the AIJ, JRA, and one of the conventional UBC formulas, i.e., Meyerhof [1], in the case of centric vertical load only (Fig. 5). As the Drucker-Prager criterion is a simplified Mohr-Coulomb criterion; thereby, the RPFEM(DP) results are in good agreement with the Meyerhof [1]; however, there is a marked difference with those from the AIJ and JRA formulas. This distinction is primarily because both AIJ and JRA formulas consider the size effect of footing in UBC estimation. The good agreement in UBC results amongst RPFEM(DP) and Meyerhof indicates that the finite element mesh, boundary

conditions and loading arrangements set in this study can well simulate the footing-soil system. The AIJ and JRA formulas were developed semi-experimentally. Therefore, the correctness of the size effect modification coefficients in these formulas need to be well investigated based on the shear strength property of real sandy soils, such as Toyoura sand, thereby endorsing the necessity of this research study. Furthermore, the size effect of footing on UBC can be widely investigated with the advantage of numerical method, which otherwise becomes difficult in the case of model tests.

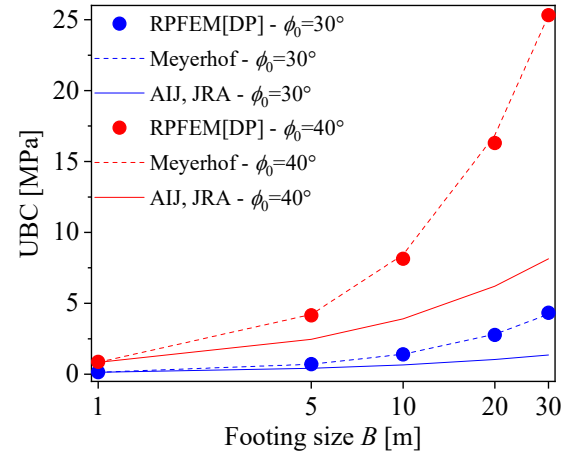


Fig. 5 Comparison of the UBC results in the case of $\gamma=18 \text{ kN/m}^3$

6.2 UBC Analysis for Model Sandy Soil Using Non-Linear Shear Strength Property

In this section, UBC analysis is conducted for the model sandy soil using the non-linear shear strength parameters set in Table 2. The UBC results through the non-linear (NL) RPFEM are arranged in Fig. 6 for the given range of footing sizes and shear strength parameters in the case of $\gamma=18 \text{ kN/m}^3$.

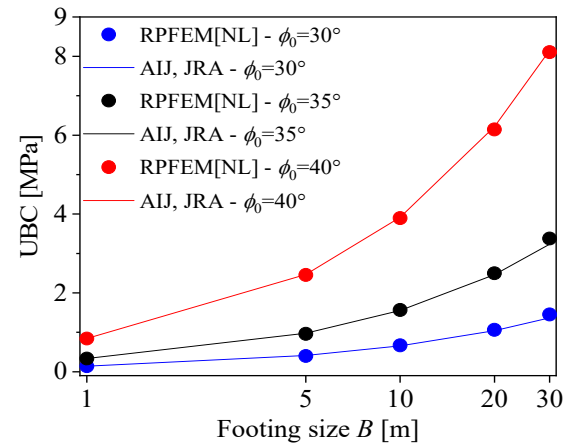


Fig. 6 Comparison of the UBC results for model sandy soil in the case of $\gamma=18 \text{ kN/m}^3$

Figure 6 illustrates the good agreement of UBC results retrieved through RPFEM(NL) and AIJ and JRA UBC formulas in the case of $\gamma=18 \text{ kN/m}^3$. It is inferred that RPFEM(NL) using the rigid plastic constitutive equation and shear strength property of Toyoura sand can well ascertain the influence of footing size on UBC.

However, the effect of soil unit weight γ needs to be investigated in terms of the size effect of footing. This is mainly because the mean effective stress in the failure domain is governed by both footing size B and soil unit weight γ as a function of the limit bearing pressure. Moreover, often practical situations involving frequent variations in the groundwater table come across, ultimately affecting the soil strength characteristics. It is therefore meaningful to thoroughly examine the effect of soil unit weight γ while determining the size effect modification coefficient. The typical ground failure domain in the case of non-linear RPFEM analysis is smaller than that of the Drucker-Prager criterion as can be observed in Fig. 7. This phenomenon can be understood through the effect of stress level on ϕ . In the case of non-linear RPFEM analysis, reduction in ϕ is considered with incrementing stress levels, thereby resulting in the smaller failure domain. Moreover, the failure domain at $\gamma=8 \text{ kN/m}^3$ is obtained larger than at $\gamma=18 \text{ kN/m}^3$ in the case of non-linear shear strength, mainly because of the difference in the mean effective stress in both cases. In the case of $\gamma=8 \text{ kN/m}^3$, stress level is reduced thereby rendering higher shear strength and increased size of soil bearing zone as compared to $\gamma=18 \text{ kN/m}^3$.

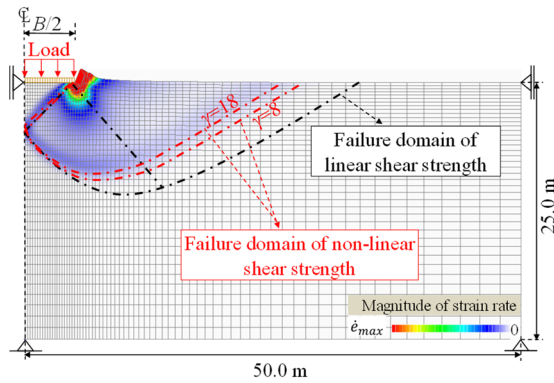


Fig. 7 Ground failure domains in the case of $\phi_0=35^\circ$, $B=10 \text{ m}$ and variation in $\gamma (\text{kN/m}^3)$

7. UBC FORMULA FOR SANDY SOIL

Based on the UBC analysis results the influence of soil unit weight γ and footing size B is explicitly determined in terms of the size effect modification coefficient η_γ . In this study, the size effect modification coefficient η_γ is defined as Eq. (4):

$$\eta_\gamma = \frac{2q_u}{\gamma B N_\gamma} \quad (4)$$

The numerical case studies have manifested that the term γB governs the UBC through the confining stress dependency of shear strength parameters. For instance, in the case of $\phi_0=35^\circ$, the UBC of the footing size of 5 m at $\gamma=18 \text{ kN/m}^3$ is 948 kPa, while that of the footing size of 10 m at $\gamma=9 \text{ kN/m}^3$ is 949 kPa. The analysis results are arranged in Fig. 8 in terms of normalization variable $\gamma B/p_a$ against $2q_u/\gamma B N_\gamma$ in the case of $\phi_0=35^\circ$ for variation in γ . It helps in understanding the effect of γ on normalized UBC. Here, p_a symbolizes the normalization factor i.e., standard atmospheric pressure, $p_a=101.325 \text{ kPa}$.

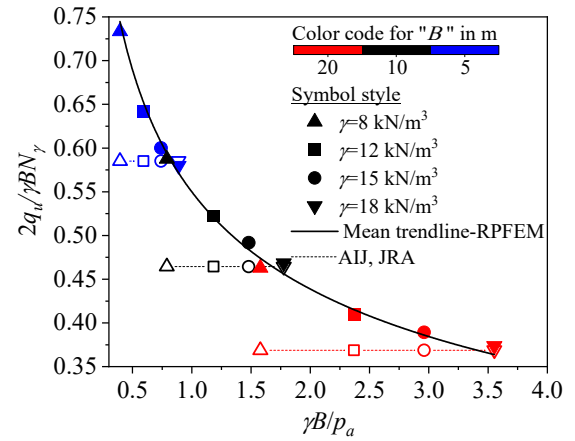


Fig. 8 Effect of γ on $2q_u/\gamma B N_\gamma$ in the case of $\phi_0=35^\circ$

Figure 8 manifests the decrease in normalized UBC with the incrementing stress levels by increasing γ in the case of model sandy soil, while the AIJ and JRA formulas do not explicitly consider such an effect of soil unit weight. Likewise, previous research studies have also corroborated that the bearing capacity factor N_γ decreases with the increase in soil unit weight γ [33].

Considering the effect of soil unit weight and the findings of experimental studies, it is pertinent to figure out the modification coefficient for N_γ . The RPFEM analysis results are plotted in Fig. 9 for an extensive range of shear strength parameters, i.e., $\phi_0=30^\circ$, 35° and 40° , footing sizes of $B=1 \text{ m}$ to 30 m , and soil unit weights of $\gamma=8 \text{ kN/m}^3$ to 18 kN/m^3 in the case of model sandy soil to perfectly grasp the influence of stress level and to propose a relationship for the modification coefficient η_γ . In Fig. 9 the mean trendline is set by using the least squares method. Although the range in $\gamma B/p_a$ encountered in practice lies between 0.1 and 0.6 [33], however, in this study, analysis results are investigated for a comparatively wider range to

broadly fathom out the effect of stress level on UBC.

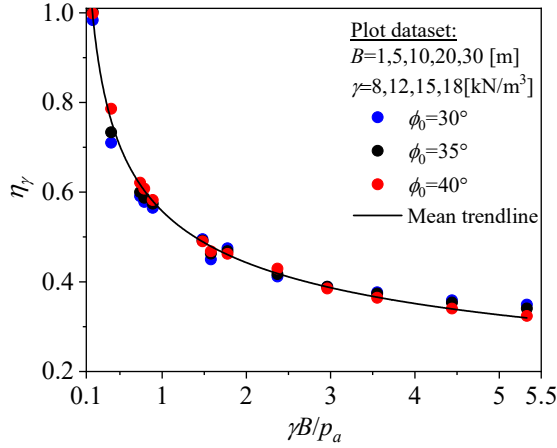


Fig. 9 Relationship between η_γ and $\gamma B/p_a$ in the case of model sandy soil

Based on the analysis results arranged in Fig. 9, the size effect modification coefficient is defined as Eq. (5) [34]. In this study, the modified UBC formula for strip footing on sandy soil under centric vertical load is proposed as Eq. (6) in terms of modification coefficient η_γ and bearing capacity factor N_γ [1].

$$\eta_\gamma = 0.55 \left(\frac{\gamma B}{p_a} \right)^{-\frac{1}{3}} \quad \text{where } 0 \leq \eta_\gamma \leq 1 \quad (5)$$

$$q_u = \frac{1}{2} \gamma B \eta_\gamma N_\gamma \quad (6)$$

Established on the experimental study of ϕ and c - ϕ soils in purview of bearing capacity factors N_c , N_γ , and N_q , Ohsaki [35] recommended cutting off the bearing capacity factors at and beyond $\phi=40^\circ$. This is because such soils having angle of internal friction larger than 40° are not often encountered in practice and a little error in laboratory measurement of the angle of internal friction may result in undue overestimation of ultimate bearing capacity. Similarly, the AIJ guidelines also recommend cutting off the bearing capacity factors at and beyond $\phi=40^\circ$. Therefore, the current research study also recommends cutting off the bearing capacity factors at and beyond $\phi=40^\circ$ for the sake of avoiding any overestimation in UBC due to the reasons outlined above.

8. EFFECT OF RELATIVE DENSITY ON UBC

In this section the UBC analysis is conducted for the Toyoura sand corresponding to each relative

density. The non-linear relationships developed in Fig. 3 are employed in UBC analysis for the given relative density. The normalized UBC is arranged against the normalized stress term $\gamma B/p_a$ in Fig. 10 for the extensive variation in relative density of Toyoura sand i.e., $D_r=20$ -88%. Figure 10 illustrates that the influence of stress level becomes less marked with the reduction in relative density of Toyoura sand and vice versa. This is primarily because in the case of dense sandy soils dilatancy is greatly suppressed due to the close packing of grains at higher stress. This observation is consistent with the centrifuge experimental study [17]. The performance of proposed UBC formula in the case of various relative densities of Toyoura sand and other sandy soils i.e., Degebo and Eastern Scheldt is investigated against the RPFEM(NL) analysis results in Figs. 11 and 12, respectively. In Figs. 11 and 12, the upper limit on bearing capacity factor N_γ is not applied in the UBC formula corresponding to $\phi=40^\circ$.

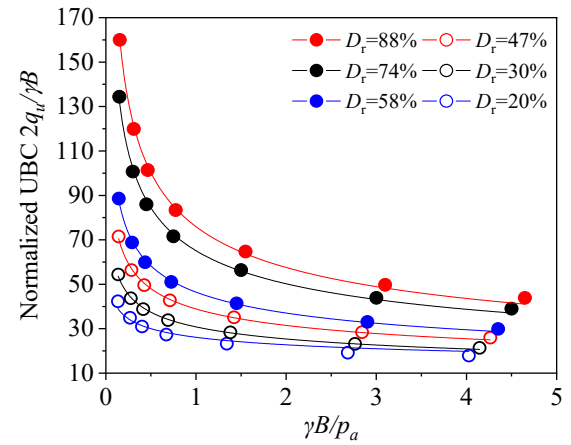


Fig. 10 Analysis results in the case of Toyoura sand

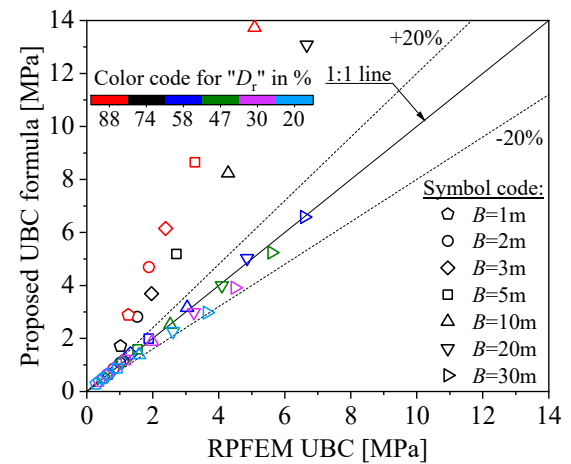


Fig. 11 Performance of the proposed UBC formula for variation in relative density in the case of Toyoura sand

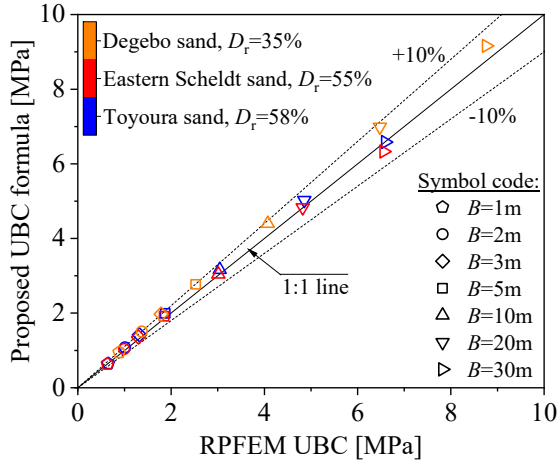


Fig. 12 Performance of the proposed UBC formula for variation in relative density in the case of different sandy soils

Figures 11 and 12 indicate that the proposed UBC formula can well estimate the UBC for loose to medium density of Toyoura sand and other sandy soils. However, in the case of dense Toyoura sand i.e., $D_r=74\%$ and 88% , UBC is overestimated if the upper limit in N_f is not applied (Fig. 11). In the case of Toyoura sand $\phi_0 \approx 40^\circ$ corresponds to $D_r=58\%$. Therefore, the proposed UBC formula can reasonably well estimate the UBC for all soils and relative densities by cutting off the bearing capacity factors at and beyond $\phi_0=40^\circ$.

9. PERFORMANCE OF THE PROPOSED UBC FORMULA

The performance of the proposed UBC formula is evaluated through its comparison with the other prevailing UBC formulas in the literature. The UBC results are compared in Fig. 13 by extensively varying the footing sizes and soil conditions. The proposed UBC formula well agrees with the AIJ and JRA formulas in the case of $\gamma=18 \text{ kN/m}^3$. However, in the case of $\gamma=8 \text{ kN/m}^3$, the AIJ and JRA formulas underestimate the UBC by about 20% primarily due to the influence of γ on stress level. The proposed UBC formula is also compared with some other commonly used international UBC guidelines such as from the American Association of State Highway and Transportation Officials (AASHTO) [36], U.S. Federal Highway Administration (FHWA) [37], Eurocode [38] and U.S. Army Corps of Engineers (USACE) [39]. The UBC formulas proposed in these guidelines significantly overestimate the results due to the lack of due consideration to the size effect phenomenon [36–39]. The performance of proposed UBC formula is also assessed against the RPFEM(NL) analysis results by widely varying the footing sizes and soil conditions as can be observed in Fig. 14.

Figure 14 illustrates that the proposed UBC formula can estimate the UBC of strip footings against the RPFEM(NL) analysis results within $\pm 5\%$, thereby substantiating its wide applicability.

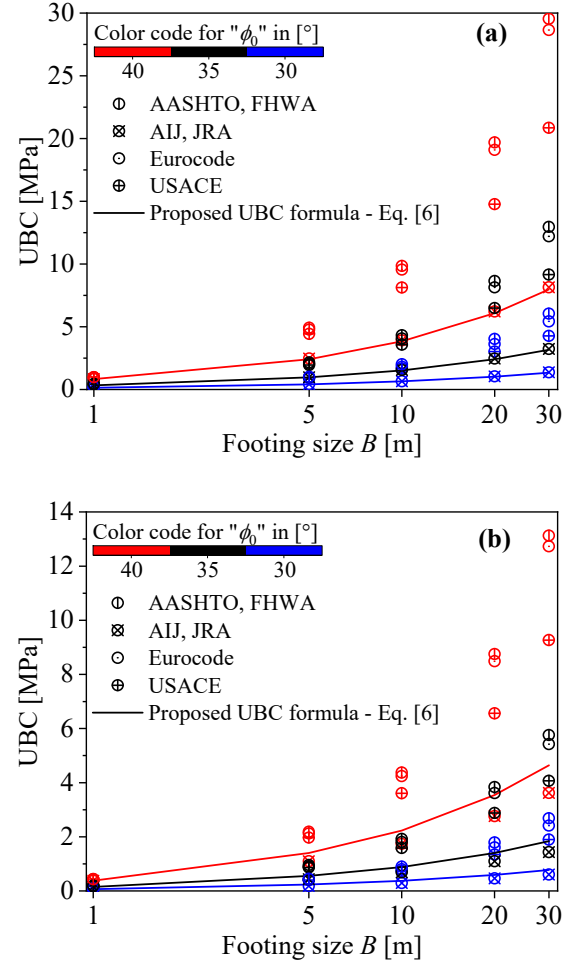


Fig. 13 Comparison of the various UBC formulas (a) $\gamma=18 \text{ kN/m}^3$ (b) $\gamma=8 \text{ kN/m}^3$

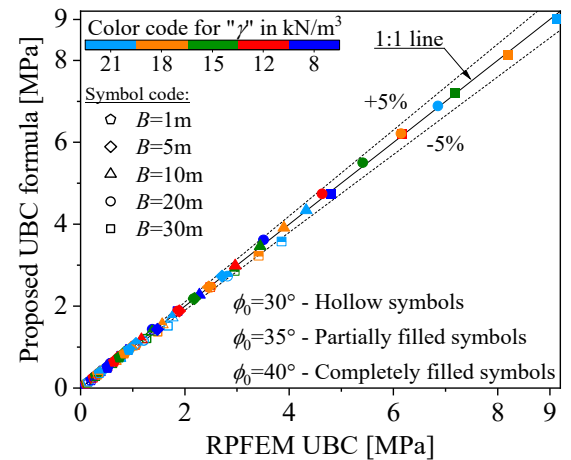


Fig. 14 Performance of the proposed UBC formula in the case of model sandy soil

10. CONCLUSION

A UBC formula was examined in detail and modified based on the mechanical property of Toyoura sand. The applicability of the analysis technique for UBC estimations was corroborated through the centrifuge experiments in the published references. Based on the numerical survey, the effect of soil unit weight was clarified to be well expressed in the past by normalized variable $\gamma B/p_a$ regarding size effect, therefore, the correction factor for N_γ was newly proposed. Furthermore, this study also investigated the effect of relative density on UBC in the case of Toyoura sand and other sandy soils i.e., Degebo and Eastern Scheldt. The proposed ultimate bearing capacity formula can well estimate the UBC of loose to medium dense sandy soils. The bearing capacity factors are proposed to cut off at and beyond $\phi_0=40^\circ$ for wider application of the proposed formula and conservative estimation of the UBC. The AIJ and JRA UBC formulas do not explicitly consider the effect of soil unit weight in size effect modification coefficient. Moreover, the applicability of international guidelines such as AASHTO, FHWA, Eurocode and USACE is found to be limited, ascribed to the size effect of footing on UBC. The proposed UBC formula was found to estimate the UBC within $\pm 5\%$ from the analysis results through extensive variation in the footing sizes and soil conditions.

11. ACKNOWLEDGMENTS

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