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NUMERICAL AND ANALYTICAL STUDY FOR SOLVING HEAT EQUATION OF THE REFRIGERATION OF APPLE

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ABSTRACT: In this paper, we study the mathematical Heat and Mass transfer model, Apple shipments from Australia to England began to decline several decades ago. Due to a disorder known as "brown heart," which developed due to insufficient cooling. When placed on deck, the apples are normally warm and must be refrigerated to keep them fresh. Storing in the cold Breathing generates heat as well. This heat was thought to be the cause. The generation successfully counteracted the apple's cooling, resulting in dark fruit. "Brown Heart ".This was the issue that resulted in the Awberry. To investigate how heat is distributed within a room. The location where heat is produced. At first, Awberry assumed the apple was in the beginning.at a constant temperature In the suitable value, we can assume that this temperature is zero. Select a temperature scale from the drop-down menu. $t = 0$ is the current time. It must be concluded that the generation of heat inside the apple is not "Brown Heart "cause. We now know that the brown core is caused by an excessive concentration of carbon dioxide and an insufficient amount of oxygen in the stockpile. It affects the metabolic activities that occur in apples and leads to a decrease in temperature separation. We have solved the heat equation for cooling apples using the method of separating variables in addition to numerical methods and clarification of the results obtained, including comparing the exact solution with the numerical solution. In terms of discovering analytical and numerical solutions, the approach is quite effective and useful.

Keywords: Heat Equation, Separations of Variables, Refrigeration of Apple, Numerical Method, Matlab.

1. INTRODUCTION

Due to inadequate preservation facilities, over 30% of the perishable goods in India end up in garbage. Furthermore, due to a lack of equipment and infrastructure, only a small amount of fruits and vegetables are grown in developing nations for local markets or export. Various strategies must be used to reduce losses throughout the harvest, handling, storage, packaging, and processing of fresh fruits and vegetables into products that are suited for storage and have enhanced storage qualities. Fruits, vegetables, and other food products with flesh are best preserved using the cold preservation method since it preserves their flavor, aroma, and texture. Refrigeration regulates microbial, biochemical, and physical processes Food goods are processed, which stops deterioration and lengthens storage life. To enhance the economic performance along the entire cold chain, the producer must remove the field heat from the cultivated fruits and vegetables [1].This guarantees that the produce is handled properly and lowers losses. Once the fruit is in the hands of the final consumer who is purchasing for his consumption, the need for low temperature must be maintained throughout the remainder of the voyage. Produce must have a longer shelf life since it cannot be eaten immediately after it is produced; consumption takes time. Preservation works to extend the shelf life of some fruits from two days to ten days. Therefore, it's crucial to extend the produce's shelf life. Since the quality of perishable goods declines over time, it is now required to preserve those features [2]. Precooling after harvest to eliminate field heat is essential for maintaining quality. Precooling aids in slowing down enzymatic and respirational activity. Additionally, it slows down microbial expansion, which prevents food from deteriorating [3]. Precooling reduces moisture loss and permits a decrease in ethylene production, which postpones ripening and ultimately improves quality and shelf life. There are essentially two pre-cooling methods. They are water cooling and forced air precooling. Each technique has benefits and drawbacks of its own. Precooling is the cooling process utilized in forced air systems. A specific speed of cold air is flown over the product in this situation. An external refrigeration cycle that uses a refrigerant to cool the air maintains the air's temperature. In the forced air cooling method, a centrifugal fan is used to push air onto the fruit, whereas water is used in place of air in the water cooling method. Since there is a surplus of air and a shortage of water, air cooling is favored to water cooling. Additionally, air cooling has lower chilling losses than water cooling. Therefore, forced air pre cooling is most frequently used [4].

To reduce post-harvest deterioration, agricultural goods should be chilled from ambient temperatures to their optimal storage temperatures [5]. After harvest, there is a significant loss of fresh fruits due to rot and shriveling as a result of inappropriate storage and handling conditions [6].

In practice, storage facility managers frequently define and apply storage conditions based on past experience, allowing for certain product losses due to non-optimal storage conditions. Local heat and mass transfer intensities are usually overlooked while assuming global heat and mass transfer rates through product layers. As a result, the complex interactions between products in the same layer as well as between layers are ignored. Computer modeling and computational fluid dynamics have been used to investigate challenges in the agriculture and food industries [5-10] .

For decades, numerical and experimental methods have been used to study conjugate heat transfer and fluid flow in a channel containing heated components [10-13]. Despite the fact that such issues were primarily focused on the cooling of electronic components, several post-harvest processing processes make use of the same problem configuration. The conjugate heat transfer problem for laminar flow over a three-heated obstacle array.

[14] used a control volume model to tackle the problem. A similar correlation was utilized to explore the same topic in reference [15], but this time with experimental data rather than numerical simulations. Young and Vafai [16] looked into the cooling of heated blocks, focusing on the heat transfer process's conjugate behavior. Fruit cooling is a conjugate heat transfer phenomenon, and a thorough understanding of the problem requires modeling and simulation, in which the local air velocity fields, as well as the mutual heat transfer between each fruit and the surrounding cooling agent, are analyzed. Fuji apples are often stored and chilled in rectangular crates with lateral holes to allow cool air to enter. The temperature is normally controlled at or below freezing.

Two consecutive apples will be separated by around one diameter. As a result, the heat emitted by the leading apple will almost surely delay the cooling of the following lining apples during the chilling process. The influence of the leading apple on the following apple as a function of the coolant air flow can be used to investigate the overall cooling performance of two apples.

2. RESEARCH SIGNIFICANCE

Perishable goods, particularly fruits and vegetables, benefit greatly from cold storage. By controlling the marketing window and supplies, it stabilizes prices and aids in the scientific preservation of perishables. Additionally, it protects the primary producer from distress sales and motivates farmers to increase their output. It has become necessary to build cold storage facilities in the producing and consuming centers to handle the current and projected production of fruits and vegetables due to the drop in prices of fruits and vegetables right after harvest and to prevent spoilage of fruits and vegetables worth crores of rupees. India is the world's second-biggest producer of veggies and the world's greatest producer of fruits. Despite this, there aren't many fruits and vegetables available per person because post-harvest losses account for between 25% and 30% of production.

A significant amount of product also loses quality before it is consumed, which is another factor. This is primarily due to the perishable nature of the produce, which needs a cold chain structure to preserve the quality and prolong the shelf-life if consumption is not intended right away after harvest. Farmers are forced to sell their produce right away after harvest due to a lack of cold storage and related cold chain infrastructure, which leads to surplus situations and low price realization. Farmers occasionally do not even receive their harvesting and shipping charges, let alone the cost of produce or profit. As a result, our production is not stabilizing, and the farmers switch to a different crop the following year after burning their fingers on one crop, perpetuating the vicious cycle.

Despite taking the risk of growing high-value fruits and vegetables year after year, our farmers continue to live in poverty. Accessible cold storage will go a long way toward lowering the possibility of a distressed sale and ensuring higher returns. This article makes an effort to teach readers about a variety of broad technical and financial features of a cold storage unit in order to assist lenders and project developers. Due to the uneven distribution of airflow in industrial cooling rooms, it is difficult to maintain uniform cooling and cold storage of fresh produce. Computational fluid dynamics (CFD) is a simulation tool that uses a powerful computer and applied mathematics to model fluid flow situations for the prediction of heat, mass, and momentum transfer as well as for the best design in industrial processes. CFD has recently been used in the food production sector. the use of CFD in the food processing sectors, including mixing, drying, and sterilizing. These localities have experienced significant development in recent years.

3. THE HEAT EQUATION'S DERIVATION

Consider a heat-conducting homogeneous rod that extends down the $x-$axis from $x = 0$ to $x = L$ in order to derive the heat equation (see Figure 1). The rod is insulated laterally such that heat only travels in the $x-$direction, has a uniform cross section A, and a constant density. Let c stand for the rod's specific heat, and let $u(x,t)$ represent the temperature of the cross section at the position x at any instant in time t. (the amount of heat required to raise the temperature of a unit mass of the rod by a degree). The amount of heat is in the portion of the rod that lies between the cross sections at x and $x + \Delta x$.

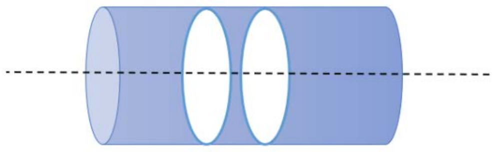

Fig.1 Heat conduction in a thin bar.

$$Q(t) = \int_x^{x+\Delta x} c\rho\, Au(s,t)ds. \quad (1)$$

On the other hand, according to Fourier's rule of heat conduction, the rate at which heat moves into the segment across the cross section at x is proportional to the cross section and the gradient of the temperature at the cross section:

$$-kA\frac{\partial u(x,t)}{\partial x}, \quad (2)$$

where stands for the rod's thermal conductivity. Heat moves in the direction of lowering temperature, as seen by the sign in Equation 2 Similar to this, the rate of heat flow through the segment's cross section at $x + \Delta x$ equals

$$-kA\frac{\partial u(x+\Delta x,t)}{\partial x}, \quad (3)$$

The change in the heat content of the segment $x \le s \le x + \Delta x$ must match the difference between the amount of heat that flows in through the cross section at x and the amount of heat that flows out through the cross section at $x + \Delta x$. Therefore, by removing Equation (3) by deriving the result from Equation (2) and converting it to the time derivative of Equation (1). It is thin enough to maintain a consistent temperature across the entire cross section.

$$\frac{\partial Q}{\partial t} = \int_x^{x+\Delta x} c\rho\, A\frac{\partial u(s,t)}{\partial t}ds$$
$$= kA\left[\frac{\partial u(x+\Delta x,t)}{\partial x} - \frac{\partial u(x,t)}{\partial x}\right] \quad (4)$$

According to the mean value theorem for integrals, if the integrand in Equation 4 is a continuous function of s,

$$\int_x^{x+\Delta x} c\rho\, A\frac{\partial u(s,t)}{\partial t}ds = \frac{\partial u(\xi,t)}{\partial t}\Delta x,$$
$$x < \xi < x + \Delta x, \quad (5)$$

so that Equation (4) becomes

$$c\rho\Delta x\frac{\partial u(\xi,t)}{\partial t} = k\left[\frac{\partial u(x+\Delta x,t)}{\partial x} - \frac{\partial u(x,t)}{\partial x}\right] \quad (6)$$

Equation 6 two sides are divided by cx, and the limit is assumed to be $\Delta x \rightarrow$.

$$\frac{\partial u(x,t)}{\partial x} = \alpha^2\frac{\partial^2 u(x,t)}{\partial x^2} \quad (7)$$

using $\alpha^2 = \frac{k}{c\rho}$. The one-dimensional heat equation is also known as equation(7). The diffusivity inside the solid is measured α^2.We must include the phrase $\int_x^{x+\Delta x} f(s,t)\,ds$ if the rod receives heat from an outside source at a rate of $f(x,t)$ per unit volume per unit time. Equation 4 time derivative term is denoted by $f(s,t)$.As a result, in the $\lim_{\Delta x \to 0}$,

$$\frac{\partial u(x,t)}{\partial t} - \alpha^2\frac{\partial^2 u(x,t)}{\partial x^2} = F(x,t) \quad (8)$$

where $F(x,t) = f(x,t)/(c\rho)$ is the source density. This equation is called the nonhomogeneous heat equation.

4. HEAT EQUATION OF SEPERATION OF VARIABLES

we show how the axisymmetric heat equation may be solved in an endlessly long cylinder by using the separation of variables.
The heat equation is in circular coordinates and is

$$\boldsymbol{\frac{\partial u}{\partial t} = \alpha^2\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \quad ,0 \le r < b, \;\; 0 < t,} \quad (9)$$

where $\boldsymbol{\alpha^2}$represents the thermal diffusivity and $\boldsymbol{r}$ represents the radial distance. Assume that we heated this cylinder's temperature to uniform T0 and then allowed it to cool.
To cool by maintaining the surface's temperature at zero beginning at time t = 0. Assuming the answer is of the form $\boldsymbol{u(r,t) = R(r)T(t)}$, we can start by assuming that

$$\frac{1}{R}\left(\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr}\right) = \frac{1}{\alpha^2 T}\frac{dT}{dt} = -\frac{k^2}{b^2} \quad (10)$$

Only negative values of the separation constant produce nontrivial solutions.

$R(r) = J_0(\frac{kr}{b})$, where J_0 is the Bessel function of the first kind and zeroth order, is the nontrivial solution. When the separation constant is zero, $R(r) = ln(r)$, which is infinite, is produced. At the start. The modified Bessel function $I_0(\frac{kr}{b})$ is produced by positive separation constants. The boundary requirement that $u(b,t) = R(b)T(t) = 0 \; or \; R(b) = 0$ cannot be satisfied by this function, despite the fact that it is finite at the origin. $J_0(k)$ must equal 0 in order to satisfy the boundary condition $R(b) = 0$. There are k_n constants, which are infinite, according to this transcendental equation. The temporal portion of the solution solves the differential equation for each k_n.

$$\frac{dT_n}{dt} + \frac{k_n^2 a^2}{b^2} T_n = 0, \tag{11}$$

which has the answer

$$u_n(r,t) = A_n \, J_0(k_n \frac{r}{b}) \mathrm{e}^{\frac{-k_n^2 a^2}{b^2} t} \tag{13}$$

All of the specific answers are superimposed linearly to produce the overall solution.

$$u(r,t) = 2T_0 \sum_{n=1}^{\infty} \frac{1}{k_n J_1(k_n)} \, J_0(k_n \frac{r}{b}) \mathrm{e}^{\frac{-k_n^2 a^2}{b^2} t} \tag{14}$$

Finding An is the last thing we need to do. Considering the initial condition of $u(r,0) = T_0$

$$u(r,0) = T_0 \sum_{n=1}^{\infty} A_n J_0\left(\frac{k_n r}{b}\right) \tag{15}$$

$$A_k = \frac{1}{c_k} \int_0^L x \, f(x) J_n \, (\mu x) dx, \tag{16}$$

$$c_k = \frac{1}{2} L^2 J_{n+1}^2 \, (\mu x) \tag{17}$$

$$T_n(t) = A_n \mathrm{e}^{\frac{-k_n^2 a^2}{b^2} t} \tag{12}$$

As a result, the product solutions

Equations 8 and 9 provide

$$A_n = \frac{2T_0}{J_1^2(k_n) b^2} \int_0^b r J_0\left(k_n \frac{r}{b}\right) dr$$
$$= \frac{2T_0}{k_n^2 J_1^2(k_n) b^2} \left(\frac{k_n r}{b}\right) J_1\left(\frac{k_n r}{b}\right)\Bigg|_0^b$$
$$= \frac{2T_0}{k_n J_1(k_n)} \tag{18}$$

from the equation

$$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x) \tag{19}$$

Consequently, the whole response is

$$u(r,t) = 2T_0 \sum_{n=1}^{\infty} \frac{1}{k_n J_1(k_n)} \, J_0(k_n \frac{r}{b}) \mathrm{e}^{\frac{-k_n^2 a^2}{b^2} t} \tag{20}$$

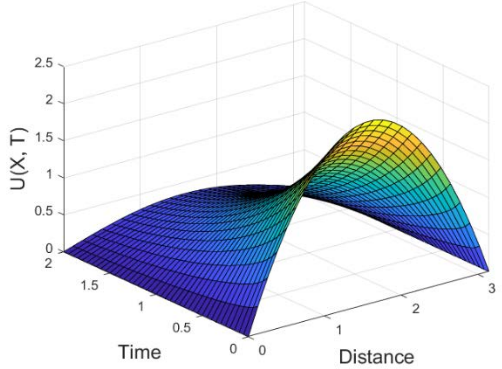

Fig.2 The temperature $u(x,t)$ within a thin bar as a function of position x and time $\alpha^2 t$ when we maintain both ends at zero and the initial temperature equals $x(\pi - x)$.

5. HEAT EQUATION OF REFRIGERATION OF APPLE

The nonhomogeneous heat equation becomes spherical because of the spherical geometry.

$$\frac{1}{\alpha^2}\frac{\partial u}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial u}{\partial t}\right) + \frac{F}{\kappa}, 0 \le r < b, \;\; 0 < t, \tag{21}$$

where α^2 represents the thermal diffusivity, b represents the apple's radius, κ is the thermal conductivity, and F is the heating rate (per unit time per unit volume).When we try to employ variable separation on Equation (21), we find that the $\frac{F}{\kappa}$ It prevents us from doing so. To get around this problem, we ask the more straightforward issue of what occurs after a very long time.

We expect a balance to be reached eventually, in which conduction takes the heat generated within the apple to the surface, where the environment absorbs it. We hope for a steady-state solution $w(r)$ where heat conduction removes the heat generated within the apples. The differential equation in its most basic form

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{\partial w}{\partial t}\right) = -\frac{F}{\kappa}, \tag{22}$$

Provides the steady-state value. In addition, just as we added a transient solution to allow our solution to satisfy the starting condition, we must have one here as well, and the governing Equation

$$\frac{\partial w}{\partial t} = \frac{a^2}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial v}{\partial r}\right). \tag{23}$$

First, solve Equation (23).

$$w(r) = C + \frac{D}{r} - \frac{Fr^2}{6\kappa} \tag{24}$$

Because the answer must be finite at $r = 0$, the constant D equals zero.
Therefore, the steady-state solution must satisfy the boundary condition w(b) = θ.

$$C = \theta + \frac{Fb^2}{6\kappa} \quad (25)$$

We introduce a new dependent variable $y(r,t) = RV$ in the transient issue (r,t). Equation 3 can be replaced with Equation 22 thanks to the new dependent variable.

$$\frac{\partial y}{\partial t} = a^2 \frac{\partial^2 y}{\partial r^2}, \quad (26)$$

We can resolve The $R(r)$ equation to become $R(r)T(t)$ if we assume $y(r,t) = R(r)T(t)$ and only have a negative separation constant.

$$\frac{d^2R}{dr^2} + k^2R = 0, \quad (27)$$

which has the answer

$$R(r) = Acos(kr) + Bsin(kr) \quad (28)$$

Because the solution, Equation (28), must disappear at r = 0 in order for v(0, t) to stay finite, the constant A equals zero. However, because for every time = $w(b) + v(b,t)$ and $v(b,t) = R(b)T(t)/b = 0, R(b) = 0$. As a result, $k_n = n/b$, and

$$u_n(r,t) = \frac{B_n}{r} \sin(\frac{n\pi r}{b}) \exp\left(-\frac{n^2\pi^2a^2t}{b^2}\right) \quad (29)$$

The entire solution is obtained via superposition, which equals

$$u(r,t) = \theta + \frac{F}{6\kappa}(b^2 - r^2) + \sum_{n=1}^{\infty} \frac{B_n}{r} \sin(\frac{n\pi r}{b}) \exp\left(-\frac{n^2\pi^2a^2t}{b^2}\right) \quad (30)$$

Finally, we use the initial condition u(r, 0) = 0 to calculate the coefficients B_n.Therefore,

$$B_n = \frac{-2}{b}\int_0^b r\left[\theta + \frac{F}{6\kappa}(b^2 - r^2)\right]\sin(\frac{n\pi r}{b})\,dr = \frac{2\theta b}{n\pi}(-1)^n + \frac{F}{\kappa}\left(\frac{b}{n\pi}\right)^3 (-1)^n \quad (31)$$

The entire package is

$$u(r,t) = \theta + \frac{2\theta b}{r\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(\frac{n\pi r}{b}\right)\exp\left(-\frac{n^2\pi^2a^2t}{b^2}\right) + \frac{F}{6\kappa}(b^2 - r^2) + \frac{2Fb^3}{rk\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}\sin(\frac{n\pi r}{b})\exp\left(-\frac{n^2\pi^2a^2t}{b^2}\right) \quad (32)$$

The temperature distribution due to the imposition of the temperature on the apple's surface is given by the first line of Equation (24). In contrast, the second line raises the temperature due to interior heating.

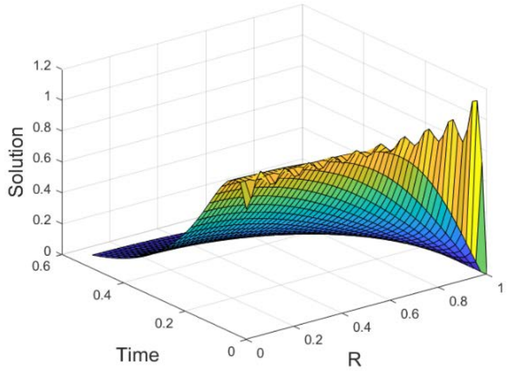

Fig.3 The temperature $u(r,t)/T_0$ within an infinitely long cylinder at various points r/b and times a_2t/b_2 that we heated to the beginning temperature T_0 temperature uniformity After that, the surface was forced to cool.

Returning to our original question of whether the inside heating is powerful enough to offset the cooling caused by refrigeration, we apply the second line of Equation (32) to determine how much the temperature deviates from what we predict. Because of this, the maximum temperature is found in the center of each apple, and it is the only one of its kind. This problem has piqued your curiosity. Assuming that the radius of the apple is b = 4 cm, a1 G/ = 1.33105C/s, as well as a 2.55 103 = 2.55 103 = 2.55 103 = 2.55 103 = 2.55 103 =The temperature effect of heat generation is particularly high at cm2/s.
When the temperatures within the apples reach 0.0232 C after around 2 hours, the temperature inside the apples is small, only 0.0232 C.
Equilibrium. As a result, we must conclude that heat generation within the apples is not the cause of the problem. brown heart's cause .Brown heart is now known to be caused by an excess of carbon dioxide in the storage hold and an insufficient amount of oxygen. This environment, it's assumed, impacts the apple's metabolic activities, leading to low-temperature disintegration.

6. TRANSIENT CONDUCTION WITH A HEAT SOURCE

The following equation governs one-dimensional transient conduction (21).By introducing the following nondimensional quantities, we may reduce this equation to a dimensionless form

$$\xi = \frac{x}{L} \qquad \tau = \frac{\alpha t}{L^2} \qquad Bi = \frac{hL}{k}$$

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} \qquad \sum = \frac{L^2 F}{k(T_i - T_\infty)}$$

$$\chi = \frac{-F''L}{k(T_i - T_\infty)}$$

Where$\theta = \theta(\xi, \tau)$, L, is the domain length,T_i is an arbitrary temperature that usually represents the

beginning temperature,F'' is the heat flux, and for T_∞ a convective boundary is the fluid temperature. When a convective boundary condition is not employed, is an arbitrary temperature that must deviate from. The governing equation changes when these variables are included

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \xi^2} + \sum \qquad (33)$$

Fixed Temperature

$$\theta = \theta_w$$

Specified Flux

$$\frac{\partial \theta}{\partial \xi} = \chi_w$$

Convective

$$\frac{\partial \theta}{\partial \xi} = -Bio\theta_w$$

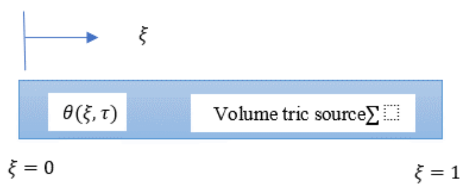

Fig.4 Source geometry for a one-dimensional transient heat transfer.

The boundary at is represented by the negative sign in the convective boundary condition, while the values at the wall are represented by the subscript. Equation (33) is a parabolic partial differential equation with one spatial dimension. Separation of variables is one approach of solution that is applicable to a limited number of boundary conditions. This is an example of a solution of this type.

due to the equal spacing of the curves at The temperature at which convection occurs is represented by the upper curve in Figure 5. The temperature drops quickly at tiny levels of due to convection and conduction. As the steady-state temperature profile determined by the energy source approaches, the temperature of the surface begins to climb. The profile at the moment of lowest temperature is displayed as a dotted line in Figure 6. Running pdepe for a long time yielded the steady-state solution. The steady-state curve depicted in Figure 6 was generated using bvp4c to corroborate that finding.

Table 1 Input Values

Parameter	Value
BC	$\sum = 1$
$\xi = 0$	$Bi = 0.1$
$\xi = 1$	$\theta(1, \tau) = \theta_w = 0.55$
IC	$\theta(1, \tau) = 1 - 0.55\xi$
$0 < \xi < 1$	41
$0 < \tau < 1$	101

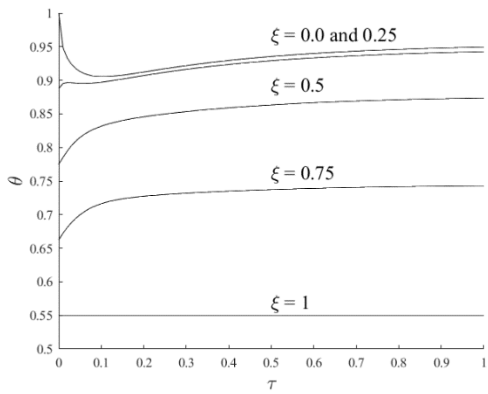

Fig.5 For the same data plotted against the spatial coordinate with time as a parameter, one-dimensional heat conduction using the data in Table 1.

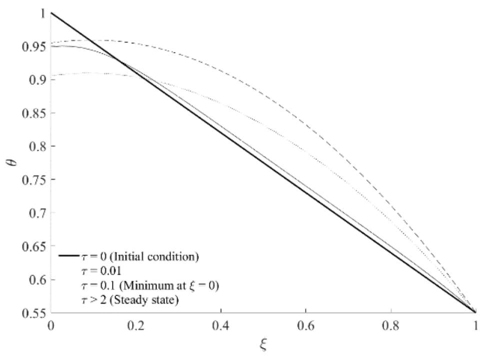

Fig.6 Heat conduction in one dimension using data from Table 1. Figure 3 shows the same data displayed versus time with location as a parameter.

7. CONCLUSION

The development and performance of a computer algorithm that calculates latent and sensible heat loads, as well as moisture loss and temperature distribution in the environment, are presented in this work. Apples should be refrigerated in quantity. This algorithm was created as a helper for both the is a refrigeration facility designer and operator who can simulate a wide range of commodities. In addition, the thermophysical properties of commodities and flow field parameters were discussed in this paper.which regulates heat and mass transmission in fresh apple.

8. ACKNOWLEDGMENTS

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