International Journal of GEOMATE, May 2023, Vol. 24, Issue 105, pp.26-32
ISSN: 2186-2982 (P), 2186-2990 (O), Japan, DOI: https://doi.org/10.21660/2023.105.g12105
Geotechnique, Construction Materials and Environment

FINITE-ELEMENT ANALYSIS OF BEAM RESTING ON FOOTING

* Radhi Alzubaidi[1], Husain M Husain[2] and Samir Shukur[3]

[1]College of Engineering, University of Sharjah, UAE,
[2]College of Engineering, University of Baghdad, Iraq
[3]College of Engineering, University of Kufa, Iraq

*Corresponding Author, Received:14 Feb. 2022, Revised: 20 Feb. 2023, Accepted: 12 March 2023

ABSTRACT: The use of unreinforced concrete members in foundation for construction projects is limited and needs further investigations. In this study, a nonlinear three-dimensional finite-element technique is used to analyse the load deflection of an unreinforced concrete beam resting on strip footings using the computer program ANSYS 5.4. The nonlinear equation was solved by the incremental and iterative running load procedure. A case study showed an excellent agreement between the theories of finite elements and practical case. We explored a new direction of using the computer program to evaluate the effects of the width, depth, and length of the foundations. In the analysis of the effect of the depth, six values of depth were used while the length and width were constant. With the increase in the depth, the deflection decreased. The effect of the width of the foundation was also investigated. With the increase in the width of the foundation, the deflection increased due to the increase in the concrete mass. With the increase in the length of the foundation, the deflection increased due to the decrease in the stiffness of the foundation. The present finite-element and available experiential results are in a good agreement. The differences do not exceed 6% in the ultimate load prediction and 15.6% in the deflection in case of using the interface element and critical state model. With the use of the developed computer program in this study, a distinctive agreement was obtained between the experimental and theoretical load–deflection curves and those obtained by four applications using the finite-element technique.

Keywords: Strip footing, Finite element, Deflection, Depth, Length

1. INTRODUCTION

The design of different types of foundations has led to the concept of transmission of load to soil through development of a specific member system. The foundation types vary according to different requirements. Concrete or steel members such as beams or plates resting on layered soils attract the attention of both structural and geotechnical engineers.

They merge the effects of superstructure and substructure element. The safety of the implemented Arab design approach has been investigated [1]. The authors used a finite-element method. Prior to the application of the finite-element software, a numerical model has been validated by comparing finite-element results to experimental results, obtained from previous studies. The results of the finite-element method validated the results of the analytical analysis obtained from previous studies. High compressive and tensile stresses were obtained for the traditional method in Arab countries, where the increase ratios reached 96% on the top surfaces of the reinforced concrete footing. An interaction analysis has been carried out for design of an open plane frame resting on soil [2], where the soil has been modelled as a four-node isoparametric element, while a plane strain approach has been utilised to represent the soil. The more realistic half-space continuum and plane-strain approaches have been compared to analyse the approximation involved in the later type of representation of the soil. The sagging moments in superstructure beams obtained using the plane-strain model were always larger than those obtained from the elastic half. A numerical analysis of deepest excavation has been carried out, for which most designers have not focused on structural modelling [3]. They modified a three-dimensional (3D) finite-element method to model their solutions.

The case study was on a building project in Bangkok. The structure models included a diaphragm wall, diagonal braces, and bored piles. The types of elements implemented to model the structures distinctively affected the finite-element analysis.

A finite-element modelling of a beam resting on a two-parameter layered soil has been also carried out [4]. In the model analysis, the strain energy, shear strain of the beam element, and soil foundation have been considered. The shear deformations had crucial influences on the beam, structure, and interface behaviours. In addition, development of a methodology for identification of optimal design parameters for a system of beams resting on a stone column–improved soft soil has been investigated [5].

A finite-difference-based simulation model and evolutionary multi-objective optimisation model have been employed. The authors aimed to minimise the settlement at the centre of the beam and maximum shear force. The evaluation of the system showed that the stiffness of the stone columns or modular ratio and flexural rigidity of the beam are most important parameters for an optimal design. A method for a nonlinear analysis of Euler–Bernoulli beams resting on a heterogeneous multilayer soil has been developed [6]. The authors obtained the governing differential equations for beam and soil displacements using the virtual work. The equations have been solved using one-dimensional finite-element and finite-difference methods. Beam responses with an accuracy comparable to those obtained from an equivalent two-dimensional (2D) finite-element analysis were obtained within seconds. A boundary element method has been developed [7] to analyse elastic foundation finite beams on 2D plane-strain and 3D multilayer isotropic soils. The authors analysed the solution of multilayer elastic soils, which was a kernel function of a BEM analysis. With the displacement and stress of coordination between the beam and soil, the solution was obtained for beams resting on a multilayer soil.

2. RESEARCH SIGNIFICANCE

A nonlinear 3D finite-element analysis was carried out to predict the load–deflection behaviour of unreinforced concrete beams resting on soil using the computing program ANSYS5.4. The use of an unreinforced concrete beam leads to a cheaper concrete member, which can be used as a foundation for different types of soil. This is a new trend for finite-element evaluation through a computer program. This study is of significance considering the findings in the case study with the developed theories.

3. FOUNDATION ANALYSIS METHOD

A computer program capable of analysis of a combined footing and irregular structure with openings or notches and non-prismatic sections, resting on a layered medium, has been developed using the finite-element method [8], where the thin-plate bending theory has been employed. Combined footings and mat foundations have been analysed [9] using the finite-grid method. In addition, nomograms have been prepared to aid the designer for a rapid prediction of displacements and bending moments.

The results have been compared to those obtained by finite elements and Hereby solution. In addition, the properties of each soil and foundation, variations in thickness and configuration of foundation, effect of adding stone columns, and probable existence of soil voids have been analysed. In addition, a nonlocal viscoelastic foundation model has been proposed [10] to analyse the dynamics of beams with different boundary conditions using the finite-element method. A case study predicted that the finite-element technique is efficient for a dynamic analysis of beams with nonlocal viscoelastic foundations.

Further, the behaviour of a shallow foundation resting on multilayer and homogeneous soils under dynamic loading has been investigated [11]. The authors used the 2D finite-element software Cyclic TP to model the soil foundation system. They compared the results to understand the effect of the layered soil on the dynamic load response of the soil foundation system. In addition, the bearing capacity and settlements of a strip footing resting on a soil have been calculated using finite elements and analytical models [12]. The geogrid improved the bearing ability of the footing and reduced the settlement.

4. INTERFACE MODEL

The use of finite elements for the analysis of soil–structure interaction (SSI) problems has been limited. Owing to difficulties in representing the interface between the soil and structure, most of the analysis has been performed using one of two limiting assumptions regarding the characteristics of the soil–structure interface.

1 - The interface is perfectly rough, without possibility for slip between the soil and structure.

2 - The interface is perfectly smooth, without possibility for shear stress that would retard the relative movement between the soil and structure [13]. Numerous types of finite elements have been developed to represent the soil–structure interface in a finite-element analysis more realistically. The seismic characteristics of SSI in rigid rock have been analysed by the finite-element method; there are many methods modified to input earthquake on the lateral boundary of the finite-element model [14]. The formulations and ABAQUS implementations of these boundaries were used. The accuracy properties of these boundaries were then compared by numerical examples including free-field and SSI problems. The comparison studies indicated that the

free and VS boundaries failed to reproduce the free-field and SSI responses when a relatively small SSI model was employed. The particle finite-element method (PFEM) has also been investigated [15] for an analysis of various complex coupled problems in mechanics involving fluid–soil–structure interactions (FSSIs). The authors showed examples of application of the PFEM to solve FSSI problems including motion of rocks by water streams, erosion of a river bed near a bridge foundation, stability of breakwaters and construction of sea waves, and landslides.
Nonlinear analyses of SSI problems have been reviewed [16]. The authors discussed coupled finite-element modelling of an SSI system with soil nonlinearity and interface element modelling. The focus has been on advantages and disadvantages of the methods discussed according to their applicability, accuracy, and quality to idealise the superstructure and soil.

5. MODELLING OF A MATERIAL

The most powerful representation that incorporates concrete and soil behaviour is the elastoplastic model. The model exhibits nonlinearities, failure, and residual strain upon loading of the initial stress conditions. Several locations of yield points are introduced due to various material types. However, the yield is the starting point of plastic behaviour and endpoint of elastic behaviour for the elastoplastic material. The criterion used to decide which combination of stress causes yielding is referred to as yield criterion. In this study, the concrete is modelled by a Willam–Warnke yield surface, while the soil is modelled by a Drucker–Prager yield surface.

5.1 Modelling of a Concrete Material

There are many constitutive models developed to predict the response of plain and reinforced concrete structures under various stress states. The main constitutive models are the elasticity-based and plasticity-based models. The elasticity-based models describe the concrete as a linear elastic or nonlinear elastic material.

The elasticity models have been used to study the nonlinear responses of plain and reinforced concrete beams, panels, and shells in which the main nonlinearity is introduced by cracking and widening of cracks in the concrete [17]. The elastic model for concrete in compression can be significantly improved by assuming a nonlinear elastic stress–strain relationship. Nonlinear elastic stress–strain relationships have been proposed [18] for concrete materials, which can be generally classified as follows.

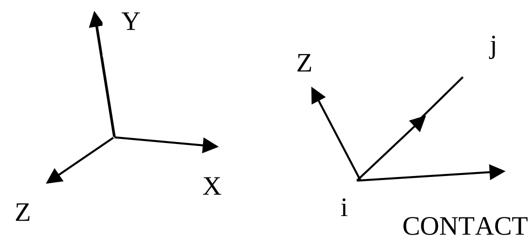
1- Total stress–strain in the form of a secant stiffness relation.
2- Incremental stress–strain in the form of a tangential stiffness formulation.

5.2 Finite-Element Representation of the Interface

We present a technique for solving 2D and 3D interface problems. The elements (contact 52) shown in Fig. 1 are used. The interface element includes normal and sliding forces [19, 20].

This element satisfies one of the two conditions if the elastic COULOMB friction is used.
1- Stuck element (no sliding)

$\mu|f_n| > |f_s|$,



52 Fig. 1 3D point to point (ANSYS Manual 1997)

Where

μ = coefficient of friction,
f_n = normal force,
$f_n = K_n (u_{n,j} - u_{n,i} + d)$,
K_n = normal stiffness,
$u_{n,j}$ = displacement of node j in the normal direction,
$u_{n,i}$ = displacement of node i in the normal direction,
d = distance between nodes.
$f_s = K_s (u_{s,j} - u_{s,i} - u_o)$,
f_s = sliding force,
K_s = sticking stiffness,
$u_{s,j}$ = displacement of node j in the sliding direction,
$u_{s,i}$ = displacement of node i in the sliding direction,
u_o = sliding distance of nodes i and j with respect to each other.

$$\{F\} = [K]\{\Delta\}, \quad (1)$$

$$\{F\} = \begin{bmatrix} fn \\ fsy \\ fsz \\ fn \\ fsy \\ fsz \end{bmatrix} \quad (2)$$

where

fn = normal force,
fs = stick force (in the y and z directions).

$$\{\Delta\} = \begin{bmatrix} u_i \\ v_i \\ w_i \\ u_j \\ v_j \\ w_j \end{bmatrix} \tag{3}$$

$$[K] = \begin{bmatrix} kn & 0 & 0 & -kn & 0 & 0 \\ 0 & ks & 0 & 0 & -ks & 0 \\ 0 & 0 & ks & 0 & 0 & -ks \\ -kn & 0 & 0 & kn & 0 & 0 \\ 0 & -ks & 0 & 0 & ks & 0 \\ 0 & 0 & -ks & 0 & 0 & ks \end{bmatrix} \tag{4}$$

1. Sliding element

If $\mu|f_n| = |f_s|$,

sliding occurs in both directions. The stiffness matrix (in the element coordinates) is

$$[K] = \begin{bmatrix} kn & 0 & 0 & -kn & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -kn & 0 & 0 & kn & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

5.3 Computer Program ANSYS

The computer program ANSYS 5.4 was used to analyse plain concrete beams resting on soil [21]. The program can solve linear and nonlinear problems including the effect of cracking, crushing, yielding of reinforcement (if existing), creep, bond slip, and temperature change, with 165 different elements.

6. NUMERICAL APPLICATIONS

Numerical cases are considered to compare the results obtained by the present method of finite elements to those obtained from experimental or analytical solutions.

Numerous numerical examples have been analysed by ANSYS 5.4. The examples are also used to check the validity of the material models and applicability and capability of the analysis method used in this study, where different types of elements are employed. The theoretical study is approximate in nature mainly due to the following factors.

1- Approximation in the material modelling of concrete and soil.
2- Approximation inherent in the discretisation in the finite-element technique.
3- Approximation in the integrations used in the numerical analysis.
4- Approximation due to the type of procedure used in solving the nonlinear system of equations.

The main results obtained by the solution is this study are the load–deflection response, crack propagation, and stress distribution.

7. TEST RESULTS AND DISCUSSION

A long strip footing with a uniformly distributed load of 450 MPa lying on a surface of a homogeneous isotropic linear elastic half-space was numerically analysed [21]. Table 1 shows details of the problem. According to the symmetry, only one half of the case is considered. A mesh of 1338 elements was used to model the concrete, soil, and interface. Details of the geometry and finite-element mesh are shown in Fig. 2. Fig. 3 shows load–deflection results obtained by this study, compared to results predicted in [22] using finite elements and infinite elements for the soil.

The results were in a good agreement. A recycled tire-derived aggregate mixed with a kaolin soil has been used to modify the ultimate bearing capacity of soil for a strip footing resting on the soil [23]. The bearing capacity of the soil increased as the percentage of the mixer increased. In addition, a finite-element analysis of a plain concrete pier under two sinusoidal wave inputs with different frequencies was performed, and the failure mechanism was investigated [24]. A tensile failure was caused by a friction force at the joint. The direction of the friction force changed depending on the input frequency. The crack patterns at the bottom surface of the beam at the initial cracking and at failure are shown in Fig. 4.

The cracking appeared at a load of 300 kPa. The initial cracks appeared at the bottom region of the beam. As the load increased, the stress in the tension zone increased and cracks extended towards the top and sides of the distribution stress.

Table 1 Material properties of the strip footing

Concrete	Young's modulus	E_c (/mm^2) = 23000
	Compressive strength	f_c (N/mm^2) = 25
	Tensile strength	f_t (N/mm^2) = 2.5
	Poisson' ratio	$v_c = 0.2$
Soil	Young's modulus	E_{so} (N/mm^2) = 12
	Poisson' ratio	$V_{so} = 0.3$
	Cohesion	C (N/mm^2) = 0
	Angle of internal friction	38
Interface	Factor friction	0.3

7.1 Parametric Study

Parameters that affect the behaviour of the plain concrete strip foundation on the soil under a static load include the beam's dimensions, the length, width, and depth.

7.2 Effect of the Beam Depth

The effect of the beam depth on the deflection has been investigated [21], where six values of depth have been used, while the length and width have been constant, 6 and 0.5 m, respectively. Fig. 5 shows a distinctive decrease in deflection with the increase in the depth of the beam under the same load. This result seems reasonable considering the increase in the flexural rigidity of the strip footing.

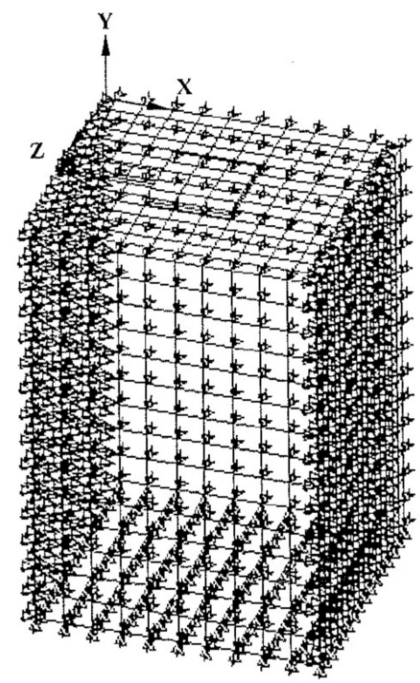
7.3 Effect of the Beam Width

The effect of the beam width on the deflection was investigated using eight values of the beam width while maintaining the length and depth constant. Fig. 6 shows a considerable increase in the deflection of the beam with the increase in the width, which may be attributed to the increase in the concrete mass.

7.4 Effect of the Beam Length

The effect of the length of the beam was studied by considering seven values of the length while maintaining the width and depth constant. The results are shown in Fig. 7, where the deflection increased as the length increased. This is also attributed to the increase in the mass of the concrete. That is generally true for most types of beams, including unreinforced beams. The longer will bend more due to the longer the beam, the more it will bend under a given load. The amount of deflection also depends on the material properties of the beam.

Fig. 2 Finite-element mesh of the strip footing



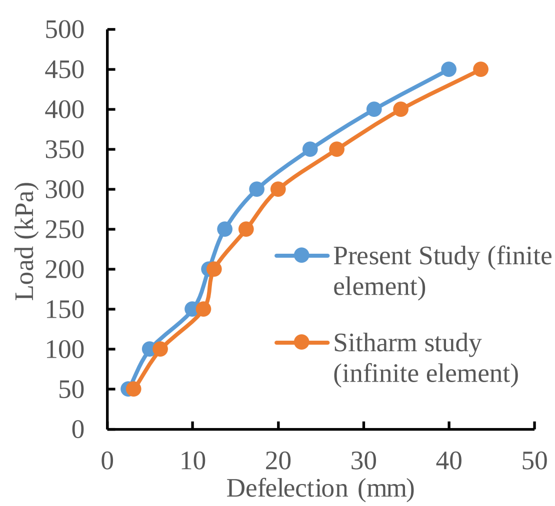


Fig. 3 Load–deflection curve of the strip footing

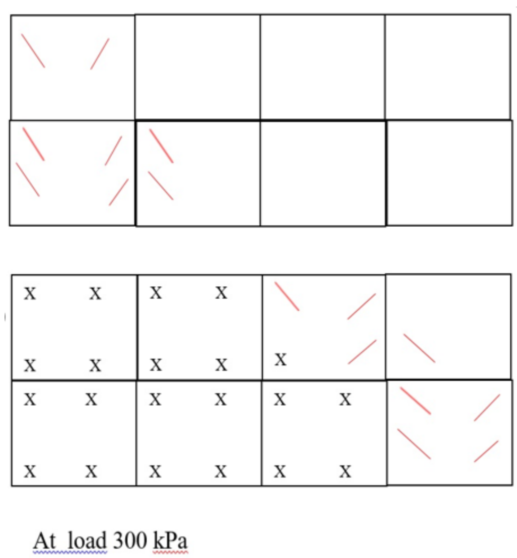


Fig. 4 Crack patterns of the strip footing

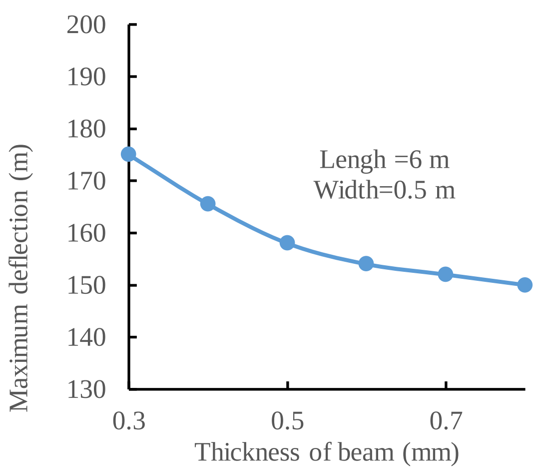


Fig. 5 Effect of the thickness on the load–deflection curve for the strip footing

8. CONCLUSIONS

A numerical analysis of a strip beam foundation resting on a soil under a static load was presented. An eight-node brick was used to model the concrete foundation. The soil was modelled by an eight-node brick element, while the interaction between the soil and concrete foundation was simulated by the contact interface element. Based on the present finite-element program analysis, the following conclusions can be summarised.

1-Ability to use unreinforced concrete beams in the foundation with a good performance even after

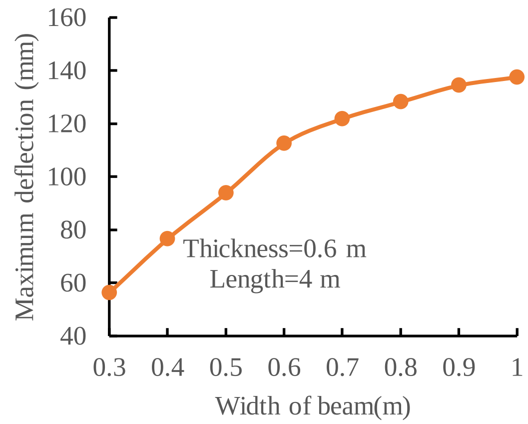


Fig. 6 Effect of the width on the load–deflection curve for the strip footing

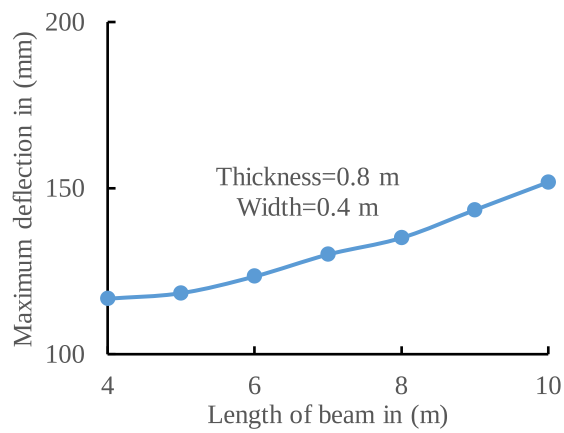


Fig. 7 Effect of the length on the load–deflection curve

occurrence of cracks.

1- The results obtained by the present finite-element method showed that the computational model used in this study is suitable for the prediction of the load–deflection behaviour of unreinforced concrete foundations under a static load. The comparison between the numerical and available experimental results showed a good agreement.
2- The increase in the depth of the strip footing led to a decreased deflection under the same static load. This seems reasonable due to the increase in the flexural rigidity.
3- With the increase in the depth of the strip footing, the inflection distinctively increased due to the increase in the concrete mass. With the increase in the length of the strip footing, the deflection of the beam increased due to the decrease in the stiffness of the strip footing beam.

9. REFERENCES

[1] Magoob H. and Elbbasy A., Finite Element Verification of the Unreliability of Using Structural Plain Concrete Footing under Reinforced Concrete Footing, Case Studies In Construction Materials, Elsevier, Vol. 15, 2021, pp.1-15.

[2] Rao P., Rambabu K. and Allam M., Representation of Soil Support in Analysis of Open Plan Frames, Computers and Structures, Vol. 56, Issue 6, 1995, pp.917-925.

[3] Likitlersuang S. Chheng and Keawsawasvong S., Structural Modelling in Finite Element of Deep Excavation, Journal of Geoengineering, Vol. 14, Issue 3, 2019, pp.121-128.

[4] Boudaa S., Khalfallh S. and Bilotta E., Static Interaction Analysis Between Beam and Layered Soil Using a Two-Parameter Elastic Foundation, International Journal of Advanced Structural Engineering, Vol. 11, 2019, pp.21–30.

[5] Deb K. and Dhar A., Parameter Estimation for a System of Beam Resting on Stone Column-Reinforced Soft Soil, International Journal of Geomechanics, Vol. 13, 2013, Issue 3, pp.222-233.

[6] Haldar S. and Basu D., Analysis of Beams on Heterogeneous and Nonlinear Soil, International Journal of Geomechanics, Vol. 16, 2016, Issue 4, pp.4016004-1- 04016004-10.

[7] AI Z., Li Z. and Cheng Y., BEM Analysis of Elastic Foundation Beams on Multi-layered Isotropic Soils, Soils and Foundations, Vol. 54, 2014, Issue 4, pp.667-674.

[8] Munther J., Mats and combined Footings Analysis by Finite Element Method, ACI Journal, Vol. 68, 1971, Issue 12, pp.945-949.

[9] Noori F., Effect of Foundation Soil Rigidity on Concrete Pressure Distribution Beneath Rafts Foundations, MSc Thesis, 1990, University of Technology, Bagdad, Iraq.

[10] Friswell M., Adhikari S. and Lei Y., Vibration Analysis of Beams with Non-Local Foundations Using the Finite Element Method, International Journal for Numerical Methods in Engineering, Vol. 71, 2007, pp.1365-1386.

[11] Verma A. and Mohanty S., Finite Element Analysis of Foundation on Layered and Homogeneous Soil Deposit under Dynamic Loading, 2018, Indian Geotechnical Conference, pp.1-9, Bengaluru, India.

[12] AlJezanwi D. and AlAzawi A., The Behavior of Strip Footing Resting on Soil Strengthened with Geogrid, Civil and Environmental Engineering, Vol. 17, 2021, Issue 2, 2021, pp.597-609.

[13] Clough G. and Duncan J., Finite Element Analysis of Retaining Wall Behavior, Journal of Soil Mechanics and Foundation Division , ASCE, Vol. 97, 1971, Issue SM 12, PP.1657-1673.

[14] Li Y. Zhao, M. Xu C., Du X. and Li Z., Earthquake Input for Finite Element Analysis of Soil-Structure Interaction on Rigid Bedrock, Tunneling and Underground Space Technology, Vol. 79, 2018, pp.250-262.

[15] Oñate E., Celigueta M., Idelsohn S., Salazar F. and Suárez B., Possibilities of the Particle Finite Element Method for Fluid–Soil–Structure Interaction Problems, Computational Mechanics, Vol. 48, 2011, pp.307-318.

[16] Dhadse G., Ramtekkar G. and Bhatt G., Finite Element Modelling of Soil Structure Interaction System with Interface, A Review Archives of Computational Methods in Engineering, Vol. 28, 2021, pp.3415–3432.

[17] Chen W. and Saleeb A., Constitutive Equations for Engineering Material, John Willey and Sons, 1982, New York.

[18] Yasuhiro C. and Chen W., Hypoelastic – Perfectly Plastic Model for Concrete Material, Journal of Engineering Mechanics, ASCE, Vol. 113, 1987, Issue 12, pp.840- 1858.

[19] Mazurkiewics M. and Ostachowicz W., Theory at Failure Element Method for Elastic Contract Problems of Solid Bodies, Computers and Structures, Vol. 17, 1983, Issue 1, pp.51-59.

[20] ANSYS Manual Version 5.4, Swanson Analysis System Inc., 1997, Houston, Pennsylvania.

[21] Shukur S., Analysis of Unreinforced Concrete Beams on Soil by Finite Elements, MSc thesis, 2002, University of Kufa, Iraq.

[22] Sitharm T. and Kamar I., Non-Linear Analysis of Geomechanical Problems Using Coupled Finite and Finite Element, Geotechnical and Geological Engineering, Vol.16, 1998, pp.129-149.

[23] Arefnia A., Dehghanbanadaki D. and Kassim K., Ultimate Bearing Capacity of Strip Footing Resting on Clay Soil Mixed with Tire-derived Aggregates, Frontiers of Structural and Civil Engineering, Vol. 15, 2021, pp.1016–1024.

[24] Furukawal A., Kawamatsu1 Y. and Kiyono1 J., Study on Failure Mechanism Plain Concrete Pier with Cold Joint During Earthquakes, International Journal of GEOMATE, Vol. 21, Issue 84, 2021, pp.1-8.