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A YIELD FUNCTION AND UNDRAINED BEHAVIOR OF K_0 CONSOLIDATED CLAYS

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ABSTRACT: The Cam Clay model, now sometimes called the classical model, cannot reproduce strain-softening behavior. The minimum principal effective stress σ_3 constant line for K_0 normally consolidated clay is a straight line with slope three in the space of mean effective stress p and axial differential stress q, which is located inside the yield surface in the Cam Clay model. In other words, there is a region in the Cam Clay model where the stress state of normal consolidation is judged to be over-consolidation. In this study, undrained and drained triaxial compression tests were conducted to clarify the problems of the Cam Clay model described above, and an anisotropic yield function was proposed. Results of the consolidated drained triaxial test showed that the plastic strains occurred when the applied minimum principal stress σ_3 exceeded the past maximum pressure. This result shows that the yield surface for normally consolidated clay is located near the constant line of the minimum principal effective stress. The void ratio of the clay decreased despite the unloading test into the yield surface of the Cam Clay model, which exhibits elastic behavior. This experimental result can emphasize that the stress change in the yield surface causes plastic strain in the clay. The strain softening behavior can be approximated by the proposed yield function if the degree of strain softening is small.

Keywords: Anisotropic yield function, Earth pressure at rest, Triaxial test, Clay, Cam Clay model

1. INTRODUCTION

In order to use the finite element method to predict ground displacement due to loading on soft clay, it is essential to have constitutive equations in multiple dimensions for the geomaterials [1]. The yield stress in a multidimensional consolidation analysis is defined by a yield function, meaning the limit of the stress space for the elastic behavior of the clay, and useful constitutive equations for clay are based on elastoplastic theory. Knowing the yield stress is important because the void ratio of a normally consolidated clay decreases significantly after the yield stress [2,3].

The Cam Clay model developed by Roscoe et al. [4] is one of the representative elastoplastic soil models based on the concept of the critical state, and its yield function is derived from assumptions about energy dissipation during plastic deformation. The Cam Clay model, now called the classical model, cannot reproduce strain-softening behavior [5,6]. The constant line of minimum principal effective stress σ_3 for K_0 normally consolidated clay is a straight line with slope 3 in the space of mean effective stress p and deviator stress q, which is located inside the yield surface in the Cam Clay model. In other words, the Cam Clay model determines the stress state of normal consolidation to be over-consolidation when it is actually normal consolidation.

Since the Cam Clay model was first published, many constitutive models have been developed and proposed. The representative model in Japan is the Sekiguchi-Ohta model [7], and the following constitutive models have been proposed since then to faithfully simulate the mechanical behavior of soil. The concept of rotational hardening, which assumes that the development of anisotropy is accompanied by the development of plastic deformation [8], the concept of sub-loading yield surface, which can describe the plastic deformation behavior in the over-consolidation region represented by cyclic stress [9] and the concept of super-loading yield surface, which can describe the soil structure [10,11]. Mizuno et al. also proposed the yield function of the original Cam Clay type of the Sekiguchi-Ohta model to a modified Cam Clay type with an elliptical shape [12]. These proposed models have been incorporated into finite element analysis programs to predict settlement and lateral displacement, and their analytical accuracy has been discussed. However, the above-mentioned strain softening [13,14] and the definition of yield stress by minimum principal stress σ_3 need to be examined based on laboratory test results.

In this study, triaxial tests were conducted on K_0 consolidated clays under loading and unloading, drained, and undrained conditions to experimentally investigate the yield function problem related to the above Cam Clay model [15]. An anisotropic yield function that can reproduce the strain-softening behavior of K_0 normally consolidated clays was proposed. The parameters required for this model can be easily determined from conventional tests except for one parameter, which is equivalent to the modified Cam Clay yield function by setting the parameters. The proposed yield function was used to simulate triaxial test results and to evaluate its capability.

2. RESEARCH SIGNIFICANCE

In recent years, the design of variable verification types for earth structures has been progressing rapidly. Finite element analysis is used for the design of consolidation settlement and horizontal displacement of clay ground, and it has been pointed out that the prediction accuracy of horizontal displacement is particularly inferior to that of vertical displacement. Although various anisotropic yield functions have been proposed to improve the analysis accuracy, in this paper, the yield stress of the minimum principal stress, which particularly affects horizontal displacement, was clarified by an anisotropic (K_0) consolidation test. Furthermore, we succeeded in reproducible calculations of the laboratory tests that were carried out by proposing a new anisotropic yield function.

3. YIELD FUNCTION AND UNDRAINED PATH

3.1 Yield Function

In order to define the yield function of the Cam Clay model, it has been postulated that the external work acting on a clay element is equal to the dissipation energy due to the plastic deformation and that Eq. (1) is expressed as follows [4].

$$p * d\upsilon_p + q * d\varepsilon_p = M * p * d\varepsilon_p \tag{1}$$

where p is the effective mean stress, q is the deviator stress, M is the slope of Critical State Line (CSL), $d\upsilon_p$ and $d\varepsilon_p$ are the increments of the plastic volumetric and the deviator strain. By assuming that normality rule exists and substituting Eq. (2) in Eq. (1), it is possible to drive the yield function F. Yielding of the clay occurs whenever the stress satisfies the yield function, i.e., Eq. (3).

$$\frac{dq}{dp} * \frac{d\varepsilon_p}{d\upsilon_p} = -1 \tag{2}$$

$$F = \eta - M * \log\left(\frac{p_c}{p}\right) = 0 \tag{3}$$

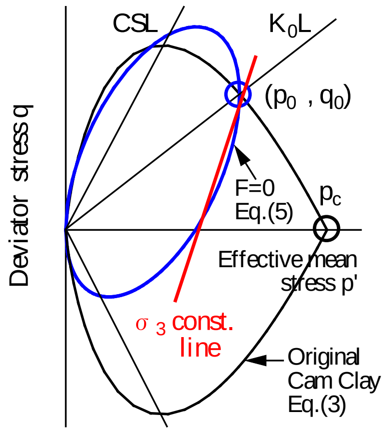


Fig.1 The yield surfaces of normally isotropic and K_0 consolidated clay

where $\eta(=q/p)$ is the stress ratio and p_c is a hardening parameter and analogous to a pre-consolidation pressure.

Dafalias et. al. [16] postulated the following plastic work dissipation equation to derive the yield function for normally K_0 consolidated clays.

$$p * \mathrm{d}\upsilon_p + q * d\varepsilon_p \\ = p\sqrt{(\mathrm{d}\upsilon_p)^2 + (N * d\varepsilon_p)^2 + 2 * \beta * d\upsilon_p * d\varepsilon_p} \tag{4}$$

$$F = q^2 - 2 * \beta * p * q + \beta^2 * p * q \\ + N^2 * (p^2 - p * p_0) = 0 \tag{5}$$

Where N and β are material constants related to the shape of the yield surface, p_0 is the effective mean stress in the particular case of K_0 consolidated clays.

The yield surfaces of normally isotropic and K_0 consolidated clay are shown in Fig.1 [4]. The yield surface of Original Cam Clay by Eq. (3) is well-known and widely used and its shape looks like a leaf. If N=M and β=η_0=0, Eq. (5) corresponds to that of Modified Cam Clay. An anisotropic surface with a blue line is the transformed and rotated ellipse of Modified Cam Clay. A red line shown in Fig.1 has an incline of 3 in 1 in which the minimum principal effective stress of a normally consolidated clay is held constant. The red line is located inside of the yield surface given by Eq. (3), although Eq. (5) with a blue line close enough to a red line. Even if the stress state for the over-consolidated clay moves to the right side of the red line, only elastic strains will be predicted by means of the yield function, i.e., Eq. (3). Plastic strain never occurs even if the minimum principal effective stress exceeds the past maximum value. As a logical consequence, the definition of the yield function is

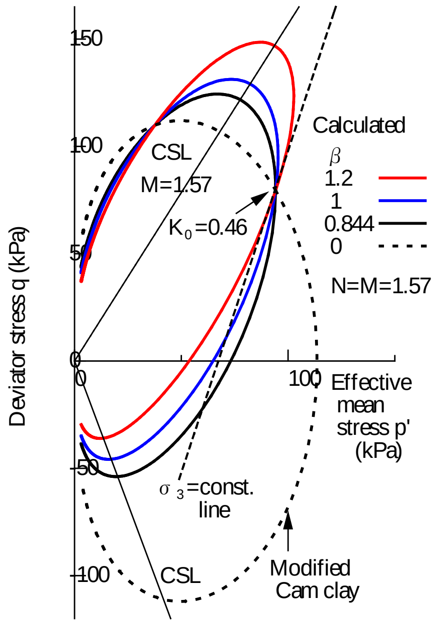


Fig.2 Effect of β value on undrained paths

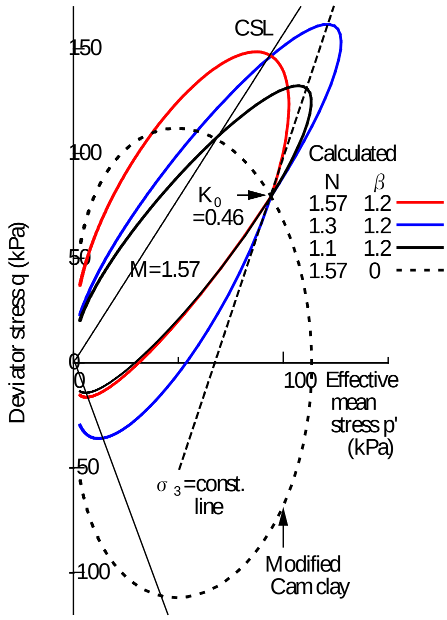


Fig.3 Effect of N value on undrained paths

ambiguous.

Although the yield criterion should be noted in determining the stress level at which plastic deformation begins, it is not clear whether the shape of the yield surface depends on the consolidated stress state. It is difficult to obtain a yield function that can strictly define the stress space in which the clay behaves elastically. However, the validity of these yield functions should be examined by experimental evidence.

3.2 Undrained Path

The volumetric strain increment dv of a soil element is expressed by Eq. (6) as the sum of elastic and plastic strain components both with superscripts e and p, respectively. Using yield function F and plastic potential Q, the hardening parameter H is given by Eq. (7).

$$dv = dv^p + dv^e = \left(\frac{1}{H}\frac{\partial F}{\partial p}\frac{\partial Q}{\partial p} + \frac{1}{K}\right)dp + \frac{1}{H}\frac{\partial F}{\partial q}\frac{\partial Q}{\partial p}dq \quad (6)$$

$$\mathrm{H} = -\frac{\partial F}{\partial v^p}\frac{\partial Q}{\partial p} \quad (7)$$

Where K is the elastic bulk modulus, calculating the undrained path in the consolidated undrained triaxial test is possible by making the volumetric strain increment in Eq. (6) equal to zero.

With the above condition dv=0, Eq. (8) and Eq. (9) yield the p-q relation derived from Eq. (3) and Eq. (5), respectively. It should be emphasized that the undrained path is not affected by the plastic potential Q, as $\partial Q/\partial p$ in the volumetric strain is eliminated in this derivation.

$$\eta = \frac{M}{\Lambda} * \ln\left(\frac{p_c}{p}\right) \quad (8)$$

$$p = p_0\left[\frac{\eta_0^2 - 2*\beta*\eta_0 + N^2}{\eta^2 - 2*\beta*\eta + N^2}\right]^{\Lambda} \quad (9)$$

where $\Lambda = 1 - \kappa/\lambda$, κ and λ are the swelling and compression index, N is the stress ratio at the maximum deviator stress in the undrained triaxial test, β is equal to the stress ratio η_0 (= q_0/p_0) at K_0 consolidation.

The proposed model expressed by Eq. (9) can explain the strain softening behavior if N is less than M. The yield surface, shown by the blue line in Fig.1, is expressed by using M=1.57 and N=1.35.

An example of the undrained path affected by β value with N=M is shown in Fig.2. Fig.3 shows the effect of the N value on the undrained path. When η <M, the deviator stress q shows the maximum value, and then q decreases until η =M. Reduction of deviation stress, i.e., the peak drop, can be calculated.

4. EXPERIMENTAL PROCEDURE AND CLAYS TESTED

Undisturbed saturated, normally consolidated alluvial clays were taken by the thin wall sampling for the residential development in the suburbs of Tokyo. The physical and mechanical properties of the clay sample and soil constants used in the calculation are shown in Table 1.

Table 1 Physical and mechanical properties of clay samples

Site	G_s	w_L (%)	I_p (%)	λ	κ	M	K_0
Kashiwa	2.643	112	50	0.200	0.030	1.48	0.40
Soka	2.672	83	39	0.182	0.041	1.54	0.43
Hitachi	2.657	121	69	0.460	0.120	1.57	0.46

4.1 Consolidated Undrained (CU) Triaxial Test

In order to examine the effects of secondary compression and the stress state in the anisotropic consolidation on the yield surface, consolidated undrained triaxial compression and extension tests are performed by the strain control with the strain rate of 0.1 %/min.

The specimens with a diameter of 5 cm and a height of 12 cm wrapped with the drained paper are consolidated under K_0 conditions before the undrained shear test and the pore water pressure during undrained shearing is measured at the bottom of the specimen. The influence of the secondary consolidation period on the undrained path, that is, the yield function, was investigated as remarkable secondary compression was observed during K_0 consolidation.

The lateral pressure of some specimens was increased after K_0 consolidation and aniso-tropically consolidated under constant axial stress for one day. This stress change occurred inside the initial yield surfaces of Cam Clay. The effect of the increase of the lateral pressure is investigated by the undrained path. And the constants M and N were determined using the maximum stress ratio η_{max} and the peak deviator stress q_p, respectively in CU tests.

4.2 Consolidated Drained (CD) Triaxial Test

The void ratio change for the over-consolidated clay was measured in order to examine the effects of dilatancy on the initial yield surface. Over-consolidated clays ought to exhibit elastic behavior. No volumetric strain is observed without the change of the effective mean stress, as is clear from Eq. (6).

For CD tests, the saturated clay specimens are first subjected to K_0 normally consolidated conditions. Each stress increment duration of one day is used for complete drainage so that the excess pore water pressure becomes equal to zero. The conditions for the axial and radial stress increment with the experimental results are shown in the latter discussion.

Back pressure is applied of 98.1 kPa in both triaxial tests during consolidation and shearing.

5. TEST RESULTS AND DISCUSSIONS

5.1 p_c in the Isotropic Consolidation Test

To investigate the relationship between the hardening parameter p_c of the Cam Clay model and the consolidation yield stress σ_y (p_y), a one-dimensional and isotropic consolidation test was performed on Hitachi clay. The results are shown in Figs. 4 and 5. In the one-dimensional consolidation test, two sets with similar sampling depths, a total of four specimens, were pre-consolidated at an overburden pressure of σ_v=80 kPa. Each one of the clays was then consolidated by incremental loading (white symbols in Fig. 4), and the other clays were unloaded to 10 kPa and then reloaded by incremental loading (black symbols in Fig. 4).

In the isotropic consolidation test, initial consolidation pressures of 10 kPa and 20 kPa were applied to two cylindrical specimens, each 5 cm in diameter and 10 cm high, and then 20 kPa load increments were applied in stages.

Fig. 4 shows the e-log p relation in the one-dimensional consolidation test. From the e-log p curve, as shown in black symbols obtained by reloading after the release of consolidation pressure, the void ratio and compression index decrease, but the vertical yield stress hardly change. Therefore, the settlement prediction by the compression index obtained from the laboratory consolidation test may lead to underestimation. So, it is difficult to infer the actual behavior in the field by using the relationship measured from the laboratory sample that released the overburden pressure. The effective mean stress as the consolidation yield stress obtained from the one-dimensional consolidation test is equal to 50 kPa since K_0 =0.46.

According to the results of the isotropic consolidation test in Fig.5, it is possible to confirm the value of the hardening parameter, p_c because the deviator stress is kept to zero. The value of p_c as Original Cam Clay becomes 86 kPa. As shown in Fig.5, it is clearly understood that the yield stress is approximately equal to the effective mean stress obtained from the one-dimensional consolidation test and is much smaller than the hardening parameter p_c in the case of Original Cam Clay. Therefore, plastic strains have occurred of the stress change inside of the yield surface. This leads to doubt about the assumption of the yield function.

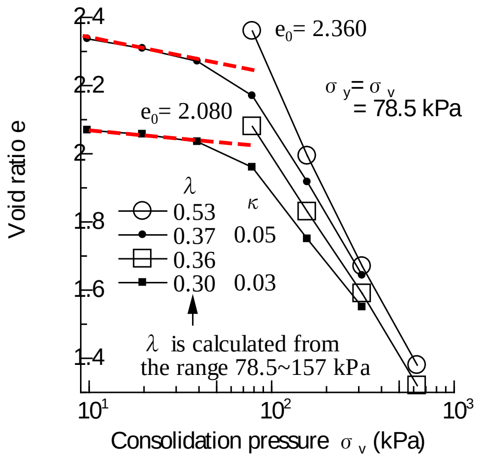


Fig.4 *e*-log*p* curves of one-dimensional consolidation test with Hitachi clay

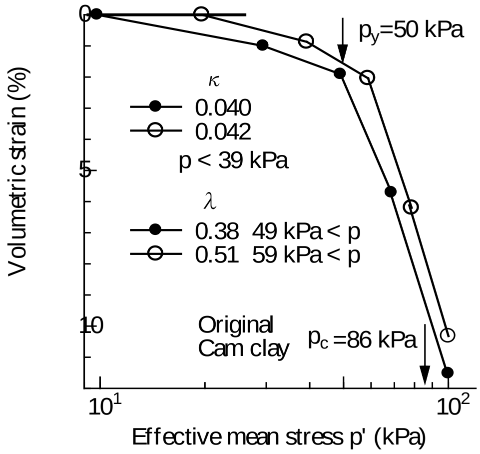


Fig.5 *e*-log*p* curves of isotropically consolidation test with Hitachi clay

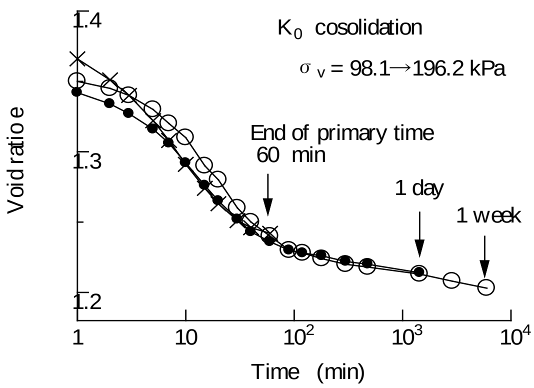


Fig.6 Consolidation time curves

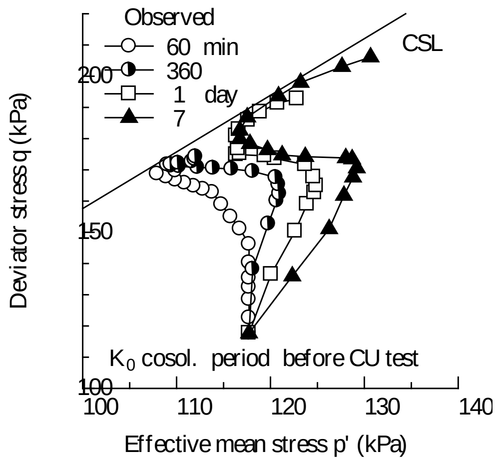


Fig.7 Observed undrained paths

5.2 Effects of Quasi-Over Consolidation

Before the undrained compression test, the consolidation settlement seems to continue almost infinitely under the sustained load, as shown in Fig.6. No method has been proposed to determine the appropriate time for the transition to the undrained shear test, though the undrained strength increases due to secondary compression.

Figs. 7 and 8 show that the undrained path is affected by the duration of quasi-over consolidation. The calculated undrained paths are approachable to the experimental results according to the value of constants β in Eq. (9). Therefore, it can be considered that the yield surface is also influenced by secondary compression during K_0 consolidation.

The effective stress path traverses along the path A⇒B⇒C⇒D as shown in Fig.9, if the lateral stress during K_0 consolidation increases with secondary compression. This stress change is equivalent to a kind of quasi-over-consolidation because the vertical stress is kept constant. And after K_0 consolidation, the effective stress is located inside the yield surface of Cam Clay.

It is clear that the undrained path is influenced by such stress history. It can be considered that there is a possibility of enlargement/deformation of the yield surface due to secondary compression.

5.3 Undrained Path in the Extension Side

Fig. 10 show the calculated and observed undrained paths in the triaxial compression and extension tests. The subscripts C and E in the figure for N and β mean compression and extension, respectively. As seen from these figures, the observed undrained path lies inside of the yield surface of Cam clay. The undrained path in the extension side should be located in a little bit outside of the yield surface as it can be known from the comparison between Eq. (3) and Eq. (8).

As the undrained path based on Eq. (9) depends

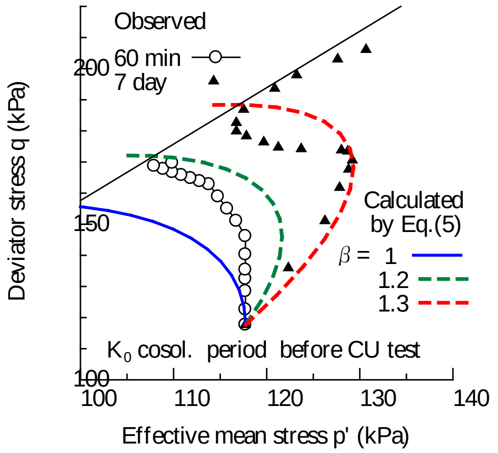


Fig.8 Undrained paths calculated with different β values

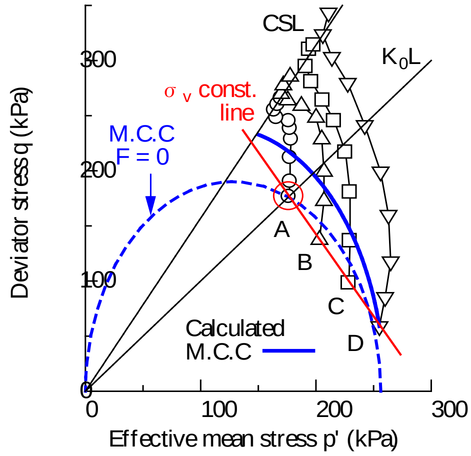
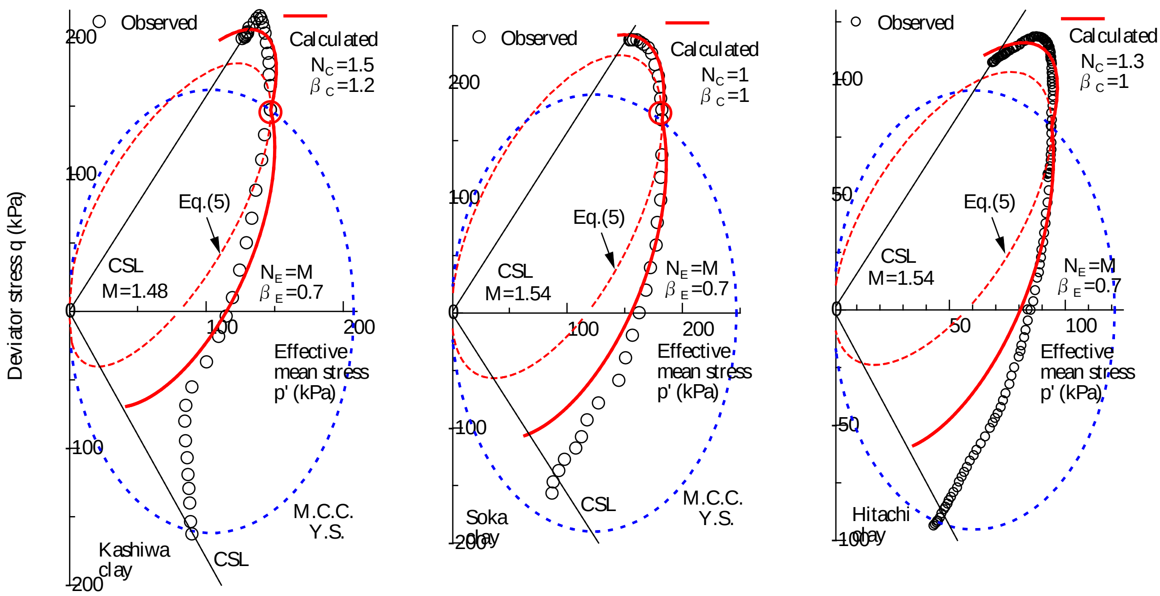


Fig.9 Undrained paths and quasi-overconsolidation



(a) Kashiwa clay (b) Soka clay (c) Hitachi clay

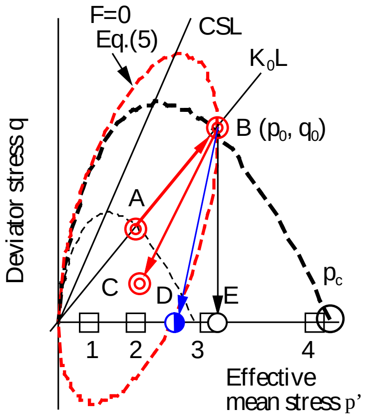
Fig.10 Yield function and undrained paths

on the constant β described in the figures. The constant β was varied so that the result of the calculation was close to the measured one. The undrained path on the compression side can be calculated close to the observed one though the calculation is greatly affected by the set value of the constant β.

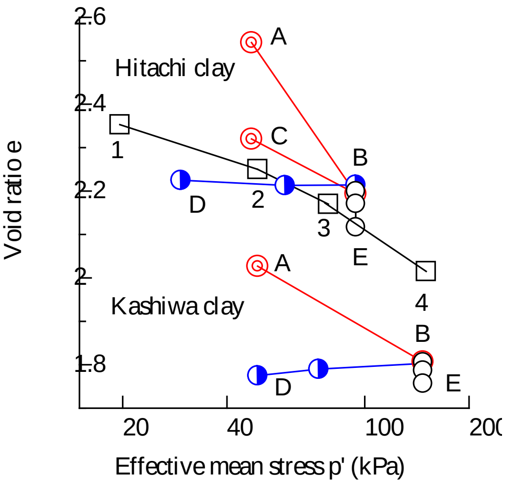
5.4 Void Ratio Change Inside of the Yield Surface

Fig.11 shows the yield surface and the change of the void ratio caused by the loading and unloading along those stress paths (A⇒B(⇒C), B⇒D, B⇒E and 1⇒2⇒3⇒4). If the stress at point A in Fig.11(a) lies on the yield surface and then is loaded beyond A to B, the clay will yield and cause the elastic and plastic strains corresponding to the normal consolidation line. The void ratio changes along the path A⇒B⇒C of Hitachi clay in Fig.11(b) represent e-log$p$ relation under K_0 compression and swelling.

On the other hand, when a clay sample brought to B is subjected to unloading paths such as B⇒D and B⇒E, their paths have to cause purely elastic strain. The change of the void ratio due to the loading path B⇒E must be zero. As the effective mean stress increment is equal to zero (dp=0) inside of the yield surface, the elastic strain increment must be zero. However, the experimental results show a reduction in the void ratio. If these



(a) stress paths



(b) *e*-log*p* relation

Fig.11 Stress paths and *e*-log*p* relation under K_0 compression and swelling

reductions are coursed by plastic strains, the loading path B⇒E must lie outside of the initial yield surface. For the loading path B⇒D, the change in the void ratio is very small. As the loading path B ⇒D is located near the constant line of the past minimum principal effective stress, it can be considered to be equivalent to the yield surface.

The stress path, such as 1⇒2⇒3⇒4 of Hitachi clay, corresponds to the isotropic compression. The void ratio decreases rapidly when the mean stress exceeds the past maximum mean stress (p_0). This means that the yield surface may cross the p-axis on the left side of p_0.

These experimental results signify that the assumption for the yield function of Cam Clay is doubtful and does not strictly define the limit of the elastic behavior. It can be considered that there is a limit to the ability of elastoplastic soil models.

6. CONCLUSIONS

This paper presents a yield function exhibiting strain-softening behavior for K_0 normally consolidated clays and evaluates the simulative capability using the undrained and drained triaxial test results. The proposed yield function is equivalent to that of Modified Cam Clay if the constant N=M and the initial stress ratio η_0 ($=q_0/p_0$) =0 in the isotropic consolidated clay. The following conclusion can be drawn from the comparison of the calculated and observed triaxial test results.

1) The calculated undrained paths using an anisotropic yield function agree well with observed ones of K_0 normally consolidated clays. The strain softening behavior can be approximated by the proposed yield function if the degree of strain softening is small.
2) Results of the consolidated drained triaxial test show that the plastic strains occurred when the applied minimum principal stress exceeded the past maximum pressure. It has been shown by the experimental results that the yield surface for K_0 consolidated clays is located near the constant line of the minimum principal effective stress.
3) The void ratio and volume of the clay decreased despite the unloading test into the yield surface of Cam Clay, which exhibits elastic behavior. This experimental result can emphasize that the stress change in the yield surface causes plastic strain in the clay, indicating that the existence of a yield plane is questionable.

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