IMPROVEMENT ON NUMERICAL SOLUTION OF 1-D FLOW WITH HYPER-CONCENTRATED SEDIMENT

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ABSTRACT: The flow phenomena in a mountainous area are often dominated by the situation where the streams have steep slopes and potential to carry the high sediment concentration. After the initiation of the flow with certain characteristic including sediment concentration within the flow, the flow would then erode and deposit the sediment in the next reach of the stream. Such phenomena would take place until the equilibrium state is met. The gradient of flow velocity, depth of flow, and sediment concentration are studied considering the equation of mass and momentum conservation, followed by the establishment of a linear equation. The improvement of the numerical solution persists in the form of the utilization of MS-Excel program as a technique of solving a linear three system of equations which was found to be practical. Such a technique was found to be effective to investigate the sensitivity analysis of the results due to some changes on various dependent parameters including the slope parameter. The three systems of equation and the proposed numerical solution technique offer further works associated with the hyper-concentrated flow through both numerical and physical model.

Keywords: Sediment concentration, velocity gradient, depth of flows, conservation of mass and momentum

1. INTRODUCTION

There are various technical problems related to alluvial rivers, including sediment transport, aggradation, and degradation. These problems depend on the physical properties of the sediment particles and their interaction with water as its media of transportation. From geological viewpoints, these sediment particles were formed by a relatively long process of rock weathering. The physical properties of the sediment particles in one river might be different from another river. For rivers originate from an active volcano, the sediment ejected from the volcanic activity and the corresponding processes will also affect the properties. Generally, rivers originate from active volcanos are having steep slopes and potential to produce critical flow. Therefore, the entrainment of the sediment into the flow will then also be very potential to form the flow with a certain degree or intensity of sediment concentration.

The sediment particle properties that may affect the sediment transport mechanism include the particle shape, concentration, mass density, and grain size distribution. The sediment transport mechanism in the volcanic rivers often particular where the sediment concentration within the flow is very high, in such that the solid material is much higher than the liquid materials. Figure 1 shows an illustration of flow in the Gendol River, Mt. Merapi area, Indonesia. Such flow is often called lahar flow or debris flow. A slightly different from lahar flow, hyper-concentrated flow is more dominated by the sediment with a more extensive portion of fine materials. The behavior of rainfall is a dominant factor affecting the initiation of the debris flow, not only the series of rainfall (working rainfall) within a 24 hours occurrence but also the rainfall those take place several days before, further called as antecedent rainfall [1,2,3]. The antecedent rainfall has been used to establish the warning criteria for the volcanic rivers in Mt. Merapi Area, Indonesia.



Fig. 1 Lahar flow at Gendol River, Mt. Merapi

Debris flow that originates from the volcanic rivers bring damage to the hydraulic structures at downstream rivers and frequently caused reduce in the hydraulic capacity of the structures [4]. This also may cause upstream flooding during highintensity rainfall events. Significant damage to buildings, critical infrastructures, and human life, as caused by lahar has been reported by Stuart [5]. The degree of the damage is controlled by properties of the lahar, location of elements at risk, and susceptibility of these elements to the lahar.

In such condition, the sediment transport takes place neither through the bed load nor the suspended load mechanism. Two approaches of further analytical study have been utilized so far, that are the Bagnold dispersive flow and the Bingham plastic fluid [6]. The Bagnold approach assumes that the sediment particles move in the flow as a granular and non-cohesive material. Further development of the Bagnold approach is carried out based on the method of grain to grain basis. In the other hand, Bingham approach assumes that the flow behaves like a fluid (well-mixed sediment and water) with a specific viscosity [7].

2. NUMERICAL MODELING OF HYPER-CONCENTRATED FLOW

There are many descriptions to define the term hyper-concentrated flow from the hydraulics viewpoint. There are two types of debris flow, namely the inertial debris flow and viscous debris flow [8]. The inertial debris flow has very different characters with the viscous debris flow. Hyperconcentrated flow is a debris flow contains mainly fine particles, could be the inertial or viscous debris flow type, with small Reynold number like mudflow in the Orcher Plateau China or large Reynold number like flood flow in the Yellow River. In 2009, Nemec proposed the definition of the hyper-concentrated flow that he considered reasonable. Hyper-concentrated flow is a turbulent subaerial flow that is excessively dense and hence deposits sediment mainly or entirely in a nontractional manner [9]. The flow may be channelized or not, and its deposit will be normal-graded and non-stratified, except possibly at the top. Such flows in terrestrial settings are often called turbulent debris flows but are referred to as high-density turbidity currents in subaqueous settings, because a subaqueous turbulent sediment-gravity flow is a turbidity current. Other definition said that the hyper-concentrated flow is a flow containing the fine particles [10]. The above description was provided with the systematic classification on types of hyper-concentrated flows as presented in Table 1. The hyper-concentrated flow under this definition is classified as mudflow, mud flood and debris flow, each of them has its own features or characteristics.

Item	Mudflow	Mud flood	Debris flow
The look Features	 High viscosity and yield stress High concentrati on (C) of silts and clays 45% <c< 55%</c< Low velocity 	 Turbulent Non-cohesive particles C as high as 40% High velocity 	 Dispersive, low viscosity Large clastic particles Non- cohesive High velocity
Effective solution	 Storage basins Deflection walls 	 Straight channels Lined canals, berm and levee channels Drop structures, energy dissipators 	 Concrete Sabo dams Steel frames and debris rakes

Table 1 Classification of hyper-concentrated flow [10].

Debris and hyper-concentrated flows are among the most destructive of all water-related disasters [11]. Debris flows would be more potential to occur when the heavy rainfall takes place; this is due to the condition that the slope materials become saturated with rainwater resulting in slope failures and mass movement. The reported incidents of catastrophic landslides and debris flow in Southern Thailand Peninsula have increased significantly in recent years owing to increased human settlement and land-use changes in hilly areas [12]. There were several experimental works on hyper-concentrated flow those have been carried out to further familiarize the behavior of the hyper-concentrated flow [13,14].

In the present work, the hyper-concentrated flow equation is developed based on the threeequation model, and further consideration on the determination of sediment entrainment coefficient will be treated to meet the sensibility of the system of the equation to cope with the physical processes of erosion and deposition. Consider a flow in a channel reach shown in Fig. 2; to give a better classification of the equation, the 'water deposition' term is introduced.



Fig. 2 Erosion and deposition in a channel reach

The mass flow of water-sediment mixture in a channel reach can be expressed in the form of conservation equation as follows.

$$\frac{d}{dx}(\rho_m Uh) = \rho_s \frac{d}{dx}(UCh) + \rho_w \frac{d}{dx}((1-C)Uh) \quad (1)$$

where:

 $\rho_m = \text{mass density of the fluid mixture}$ C = average sediment concentration in the flow U = average velocity ever the cross section h = average depth of flow dx = distance of the channel reach

The standard equation of momentum conservation is then utilized to establish the three systems of equation (see the sketch in Fig. 3).



Fig. 3 Sketch of conservation of momentum equation

The related basic equations are described as follow:

$$W = \rho_m ghdx \tag{2}$$

$$f = \rho_m U_*^2 dx \tag{3}$$

$$P_{1} = \frac{1}{2} \rho_{m} g h_{1}^{2}$$
(4)

$$P_2 = \frac{1}{2} \rho_m g h_2^2 \tag{5}$$

$$U = \frac{1}{2} \left(U_1 + U_2 \right) \tag{6}$$

$$h = \frac{1}{2} \left(h_1 + h_2 \right) \tag{7}$$

$$\rho_m \left(U_2^2 h_2 - U_1^2 h_1 \right) = P_1 - P_2 + W \sin \theta - f \tag{8}$$

The derivation of the equation system of momentum conservation of water-sediment mixture into dx as shown in Eq. (8) can be written as follows:

$$\frac{d}{dx}(\rho_m U^2 h) = -\frac{1}{2}g\frac{d}{dx}(\rho_m h^2) + \rho_m gh\sin\theta dx - \rho_m U_*^2 dx \qquad (9)$$

where:

g = acceleration due to gravity θ = channel bed slope U_* = shear velocity near the bed

Eq. (1) can actually be split into two equations

of conservation, i.e., sediment flow conservation and water flow conservation as shown in Eqs. (10) and (11), respectively.

$$\rho_s \frac{d}{dx} (UCh) = \rho_s C_* V_s E_s - \rho_s C_b V_s$$

or

$$\frac{d}{dx}(UCh) = V_s(C_*E_s - C_b) \tag{10}$$

and

$$\rho_{w} \frac{d}{dx} ((1-C)Uh) = \rho_{w} \left(\frac{1-C_{*}}{C_{*}}\right) C_{*} V_{s} E_{s} - \rho_{w} \left(\frac{1-C_{*}}{C_{*}}\right) V_{s} C_{b}$$

or

$$\frac{d}{dx}((1-C)Uh) = V_s\left(\frac{1-C_*}{C_*}\right)(C_*E_s - C_b) \quad (11)$$

where:

 ρ_w = mass density of water

 ρ_s = mass density of sediment

 C_* = sediment conservation in the bed

 V_s = fall velocity

 E_s = coefficient of sediment entrainment from the bed

- C_b = sediment concentration of debris flow near the bed,
- α = is a coefficient of maximum conservation of sediment in the flow near the bed, ≈ 2 .

The first and second of the right-hand terms of Eq. (10) are the erosion and deposition of sediment, respectively (Portion 1 and 2 of Fig. 2), whereas the first and second of the right-hand terms of Eq. (11) are the erosion and deposition of water, respectively (Portion 3 and 4 of Fig. 2). As it is found by Garcia and Parker [15,16], the coefficient of sediment entrainment Es must empirically be given, and in the case of three-equation model of turbidity currents, Garcia and Parker gave the empirical relation for E_s as shown in Eq. (12).

$$E_s = \frac{U_*}{V_s} \left(\left(\sqrt{(\rho_s - \rho_w)gd} \right) \frac{d}{v} \right)^{0.75}$$
(12)

where:

d = representative diameter of the grain

v = kinematic viscosity

The shear velocity U_* can be expressed as follows:

$$\frac{\rho_w U_*^2}{\rho_s d^2 D^2} = a_1 \left(C_* - C_b \right)^{-2}$$
(13)

where:

$$D =$$
 gradient velocity or $\frac{dU_*}{dz_*}$

 a_1 = coefficient of 'dispersive stress', ranges from 0.1 - 0.2.

Substitute the value of U_* from Eq. (13) to Eq. (12) will yield the value of E_s to

$$E_{s} = \frac{dD}{V_{s}\left(C_{*} - C_{b}\right)} \sqrt{\frac{a_{1}\rho_{s}}{\rho_{m}}} \left(\sqrt{\left(\rho_{s} - \rho_{w}\right)gd} \frac{d}{\gamma}\right)^{0.75}$$
(14)

The value of E_s in Eq. (14) is yet to be established in the purpose of debris-flow modeling. For the initial attempt, this paper will assume the value of E_s as a function of constant K, as shown in Eq. (15).

$$E_s = \frac{K}{C_* - C_b} \tag{15}$$

where:

$$K = \frac{dD}{V_s} \sqrt{\frac{a_1 \rho_s}{\rho_w}} \left(\sqrt{\left(\rho_s - \frac{a_1 \rho_s}{\rho_w}\right)} g d \frac{d}{v} \right)^T$$
(16)

v

(Y is to be determined empirically).

Based on the assumption that the debris flow is a good mixture of water-sediment, the mass density of the debris flow can then be written as;

$$\rho_m = \rho_s C + (1 - C) \rho_w \tag{17}$$

Eqs. (10) and (11) can be written as;

$$UC\frac{dh}{dx} + Uh\frac{dC}{dx} + Ch\frac{dU}{dx} = V_s\left(C_*E_s - C_b\right)$$
(18)

and

$$U(1-C)\frac{dh}{dx} - Uh\frac{dC}{dx} + h(1-C)\frac{dU}{dx} = V_s \left(\frac{1-C_*}{C_*}\right) (C_*E_s - C_b)$$
(19)

It is apparent in Eq. (17) that the mass density of the debris flow changes over the process or the mechanism of the flow. In differential term, this phenomenon can be written as;

$$\frac{d}{dx}(\rho_m) = (\rho_s - \rho_w)\frac{dC}{dx}$$
(20)

Further differential manipulation of Eq. (9) by making use of Eq. (20) will form the following equation;

$$\rho_m \left(U^2 + gh \right) \frac{dh}{dx} + \left(U^2 h + \frac{1}{2} gh^2 \right) \left(\rho_s - \rho_w \right) \frac{dC}{dx} + 2\rho_m Uh \frac{dU}{dx} = \rho_m \left(gh \sin \theta - U_*^2 \right)$$
(21)

System of Eqs. (18), (19) and (21) is a threeequation model for debris flow, which in matrix form can be written as;

$$\begin{bmatrix} UC & Uh & Ch \\ U(1-C) & -Uh & h(1-C) \\ \rho_m(U^2 + gh) & (U^2h + \frac{1}{2}gh^2)(\rho_s - \rho_w) & 2\rho_mUh \end{bmatrix}$$
$$\begin{bmatrix} \frac{dh}{dx} \\ \frac{dC}{dx} \\ \frac{dU}{dx} \end{bmatrix} = \begin{bmatrix} V_s(C_*E_s - C_b) \\ V_s\left(\frac{1-C_*}{C_*}\right)(C_*E_s - C_b) \\ \rho_m(gh\sin\theta - U_*^2) \end{bmatrix}$$
(22)

The solution of Eq. (22) requires an initial value of h, C, and U.

Normalization of flow depth (h), flow velocity (U), reach (x), and settling velocity (V_s) in the system of Eq. (22) over their corresponding initial value would give the normalized values as follows;

$$h_{n} = \frac{h}{h_{initial}}; \text{and } dh = h_{initial} dh_{n}$$

$$x_{n} = \frac{x}{h_{initial}}; \text{and } dx = h_{initial} dx_{n}$$

$$U_{n} = \frac{U}{U_{initial}}; \text{and } dU = U_{initial} dU_{n}$$

$$V_{s_{n}} = \frac{V_{s}}{U_{initial}}; \text{and } dV_{s} = V_{s_{initial}} dV_{s_{n}}$$

$$(23)$$

Normalization of Eq. (17) towards ρ_{w} , would make the mass density of the mixture (ρ_{mn}) become:

$$\rho_{mn} = \frac{\rho_m}{\rho_w} = 1 + (S - 1)C$$

and

$$\frac{d\rho_{mn}}{dx_n} = (S-1)\frac{dC}{dx_n} \tag{24}$$

where S is a specific mass density of sediment.

Normalization of U^{*2} would give:

$$U_{*n}^{2} = \frac{a_{1} dV_{s_{n}}^{2} \frac{u_{n}^{2}}{h_{n}^{2} dU_{*}^{2}} S}{\left(C_{*} - C_{b}\right)^{2}}$$
(25)

where $\left(\frac{dU_*}{dz_*}\right)^2$ is the square of the velocity gradient.

Base on the above normalization, the three-equation model of hyper-concentrated flow can be written as;

$$\begin{bmatrix} U_{n}C & U_{n}h_{n} & Ch_{n} \\ U_{n}(1-C) & -U_{n}h & h_{n}(1-C) \\ \rho_{mn}(U_{n}^{2} + \frac{h_{n}}{F^{2}}) & (U_{n}^{2}h_{n} + \frac{1}{2}\frac{h_{n}^{2}}{F^{2}})(S-1) & 2\rho_{mn}U_{n}h_{n} \\ \end{bmatrix} \begin{bmatrix} \frac{dh_{n}}{dx_{n}} \\ \frac{dC}{dx_{n}} \\ \frac{dU_{n}}{dx_{n}} \end{bmatrix} = \begin{bmatrix} V_{s_{n}}(C_{*}E_{s} - C_{b}) \\ V_{s_{n}}\left(\frac{1-C_{*}}{C_{*}}\right)(C_{*}E_{s} - C_{b}) \\ \rho_{mn}\left(\frac{h_{n}}{F^{2}}\sin\theta - U_{*n}^{2}\right) \end{bmatrix}$$
(26)

where:

$$F = \frac{U_{initial}}{g h_{initial}}$$
(27)

The solution of Eq. (26) can be carried out numerically either by utilization of the Euler method or Runge-Kutta method. Besides, the elimination technique by Microsoft Excel, need to be introduced to obtain the value of flow depth h, concentration C, and velocity U and their variation with the change of distance x_n as shown on Eqs. (28), (29) and (30), respectively.

$$h_{n(i+1)} = h_{ni} + \frac{x_n}{6} \left(k_1 + 2k_2 + 2k_3 + k_4 \right)$$
(28)

$$C_{(i+1)} = C_i + \frac{x_n}{6} \left(k_1 + 2k_2 + 2k_3 + k_4 \right)$$
(29)

$$U_{n(i+1)} = U_{ni} + \frac{x_n}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$
(30)

where:

$$k_1 = f\left(h_{ni}, C_i, U_{ni}\right) \tag{31}$$

$$k_{2} = f\left(h_{ni} + \frac{\Delta x_{n}}{2}k_{1}, C_{i} + \frac{\Delta x_{n}}{2}k_{1}, U_{ni} + \frac{\Delta x_{n}}{2}k_{1}\right)$$
(32)

$$k_{3} = f\left(x_{ni} + \frac{\Delta x_{n}}{2}, h_{ni} + \frac{\Delta x_{n}}{2}k_{2}, C_{i} + \frac{\Delta x_{n}}{2}k_{2}, U_{ni} + \frac{\Delta x_{n}}{2}k_{2}\right)$$

$$U_{ni} + \frac{\Delta x_{n}}{2}k_{2}\left(33\right)$$

$$k_{4} = f(h_{ni} + \Delta x_{n}k_{3}, C_{i} + \Delta x_{n}k_{3}, U_{ni} + \Delta x_{n}k_{3})$$
(34)

Remarks:

 $k_1 = f(h_{ni}, C_i, U_{ni})$ means that the solution of a system of three-equation utilizes the corresponding value of h_{ni}, C_i, U_{ni} , hence also the k_2, k_3 , and k_4 . The Euler method is basically the first order of the solution of the ordinary differential equation, whereas the Runge-Kutta is the second order of the solution. The Runge-Kutta method is supposed to perform more accurate results. However, some logical or physical behavior may justify the results.

3. RESULTS AND DISCUSSION

Like those have been carried out by previous workers [17,18], at a given condition and channel geometry, several flow parameters are investigated. These include the variation of depth, concentration, and velocity at a certain distance would meet equilibrium state. The model is tested by applying the initial value of $h_{initial} = 0.60$ m; $U_{initial} = 0.4$ m/sec; $C_{initial} = 0.0$; with the related constants of C_* = 0.75; d = 1.0 mm; $\rho_s = 2.65 \text{ t/m}^3$; $\rho_w = 1.00 \text{ t/m}^3$; $\theta = 20^{\circ}$; g = 9.81 m/sec²; and $\alpha = 2$. The nature of Eqs. (15) and (18) require condition that K should be less than $C_*/4$. Applying the value of K = 0.08dan $dU_*/dz_* = 2.5$ into Eq. (26) and solution of differential equation of depth, concentration, and velocity by Euler and Runge-Kutta methods as shown in Fig. 4 and Fig. 5 for the slope $\theta = 20^{\circ}$ and 10° respectively.



Fig. 4 Normalized depth, concentration, and velocity at slope $\theta = 20^{\circ}$.



Fig. 5 Normalized depth, concentration, and velocity at slope $\theta = 10^{\circ}$.

It can be seen from Fig. 4 and 5 that the analysis results by the Runge-Kutta method are always

showing the higher increase of the depth, concentration, and velocity rather than that of the Euler method. At the steeper slope ($\theta = 20^{\circ}$), the depth of flow increased by approximately 70 times higher than the initial value, whereas at the lower slope ($\theta = 10^{\circ}$), the depth of flow increased by approximately 11 times higher than the initial value. Fig. 4 and 5 also indicate that the flow is analyzed until at the normalized distance of 100 times than the reach of 0.1 m throughout the simulation. At this condition, the concentration decreased until approximately 0.29 and 0.23 times lower than the initial value for the slope $\theta = 20^{\circ}$ and $\theta = 10^{\circ}$ respectively. In the other hand, the velocity decreased until 0.11 and 0.19 times smaller than the initial value for the slope $\theta = 20^{\circ}$ and $\theta = 10^{\circ}$ respectively. Further discussion is made in erm of the mass balance over the entire reach under studied. The magnitude of the mass balance is basically the multiplication of the mass density of a fluid mixture ρ_m , flow depth *h*, and flow velocity *U* (see Fig. 6). The asymptotic trends of the mass balance with two different slopes indicate the the Euler method performed more logical trend rather than the Runge-Kuta method.



Fig. 6 Normalized mass balance over the entire reach at slope $\theta = 20^{\circ}$ and $\theta = 10^{\circ}$.

It is seen from Fig. 6 that at a particular flow condition, the normalized mass balance, i.e. approximately 0.59 thru 0.60 for slope $\theta = 20^0$ and $\theta = 10^0$ respectively, take place at a normalized distance of approximately 40. For further model development, the application of the Euler method is more advisable.

4. CONCLUSIONS

The concluding remarks are as follow;

a) The improvement of the solution technique of Eq. (26) applying the MS-Excel is found to be effective rather than that of applying the ordinary matrix operation. Such effective term means that the solution performs the more stable, less time running, and more practical to examine the sensitivity of the result against various parameters. Besides, the Euler method tends to perform more logical results rather than the Runge-Kutta method.

- b) The value of K in Eq. (16) is obviously a function of grain diameter d and the value of Y is equal to 0.75. The value of dU_*/dz_* is basically the velocity gradient of the flow, it shows whether the flow is in the acceleration or deceleration condition. Unless relevant experimental work is conducted, the most appropriate value of such velocity gradient would not be able to be determined.
- c) Further studies should consider the utilization of different channel geometry and sediment properties (size, mass density, non-uniformity, etc.), as well as various constants (sediment entrainment function E_s , coefficient of sediment concentration near the bed α , and the gradient velocity dU*/dz*). Verification of this model through experimental works, either laboratory or field investigations, are still the subject of interest.

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