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CALIBRATION OF NON-LINEAR TO LINEAR SOIL MODULUS ON PILE FOUNDATION DUE TO LATERAL LOADING

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ABSTRACT: Pile foundation modeling is a common practice to evaluate soil deformation due to vertical and lateral loading. This approach is particularly crucial for geotechnical structures with significant lateral loading, including retaining walls, skyscrapers, transmission towers, and offshore structures. One prevalent assumption in pile foundation modeling involved linear modulus of elasticity of the soil with increased depth. This assumption has traditionally been deemed valid for the response of pile foundation to the natural conditions encountered in the field. However, the soil is not an elastic material, which means the assumption of using linear modulus of elasticity in foundation modeling must be evaluated further. The deflection of the pile foundation is analyzed using linear, layered, and non-linear modulus of elasticity. The Reese Matlock model was adopted to model linear elastic modulus, while the Winkler model was used for layered and non-linear elastic modulus. Non-linear modulus of elasticity was obtained from three different sites, namely Citarum, Dompak, and Batang. The results showed a difference in the average comparison of the average Reese Matlock and Winkler models at the Citarum Project by 2.54%, 4.35%, and 5.08%. At the Dompak Project, the differences are around 2.57%, 4.37%, and 6.63%, and at the Batang Project, the differences are roughly 2.28%, 3.07%, and 5.58%. The analysis showed that the surface deflection was determined by non-linear elastic modulus analysis with an excess of about 10% to 14% at the first 10% of the pile length.

Keywords: Pile, Lateral-Loading, Elastic-Modulus, Lateral-Deflection

1. INTRODUCTION

Pile foundation is primarily designed to support axial loads, but in some cases, it must withstand significant lateral loading, for example, retaining walls, high-rise buildings, transmission towers, and offshore structures. The behavior of laterally loaded pile depends on their strength and the properties of the surrounding soil, represented by the elastic modulus of k_s. The response of the structure due to lateral loading has been recorded from a series of field-based lateral loading tests [1] and mathematical model simulations [2-4].

A new method was used in this study to examine the deflection response of piles embedded in layered soil when subjected to lateral loading, as shown in (Fig. 1). This analysis relies on differential equations to provide a comprehensive understanding of lateral deformations, bending moments, and shear forces acting along the entire length of the pile. The soil layer had a definite impact on the pile response and was considered for proper analysis and design [5].

In geotechnical engineering analysis, a soil model for addressing lateral earth pressure was introduced by previous studies [6-8]. The general analysis is based on the finite element method or equivalent spring representing the soil media. Finite element analysis has become prevalent in Indonesia in foundation engineering. In Indonesia, finite element analysis is widely applied in foundation engineering to determine lateral loading distribution for seismic design [9].

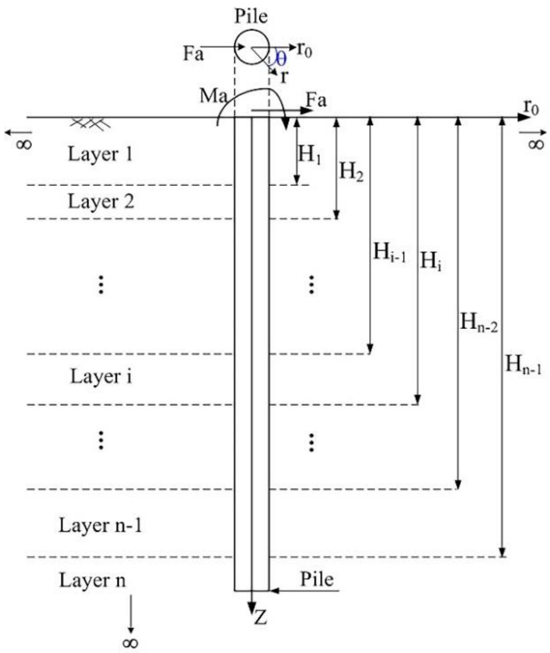

Fig. 1 Laterally loaded pile in a layered elastic medium.

The dependence of pile response on input parameters emphasizes the importance of making correct assumptions about soil properties. The assumption must be precise and reliable to mimic the actual pile behavior in the field. A common and unrealistic assumption is related to linear modulus elasticity in the depth of the pile foundation. Soil properties vary non-linearly with depth. Reese, L.C., and Matlock (1960) conducted a study investigating moments, horizontal forces, soil reactions, and deflections of the pile using an analogy with beams on elastic support, as shown in (Fig. 2). The results obtained were in line with the outcome of the actual lateral test.

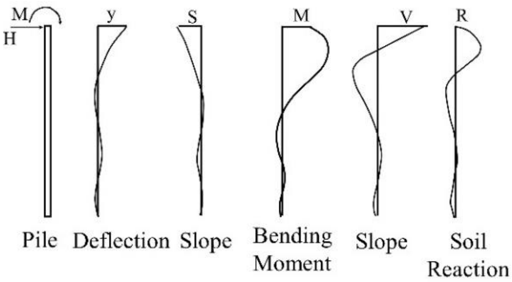

Fig. 2 Pile deformation due to horizontal load H and moment M [10].

Winkler [11] introduced another idealization using spring stiffness to capture non-linearity of the soil along the pile, as shown in (Fig. 3).

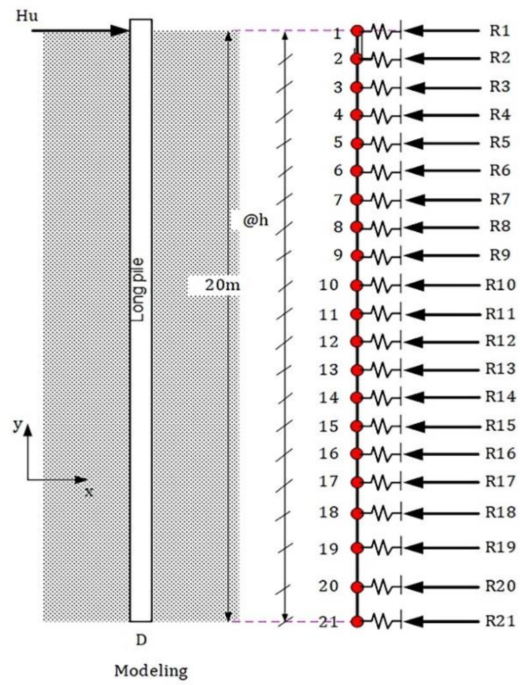

Fig. 3 Soil reaction along the pile [11]

Rahmadi [12] conducted a study which focused on evaluating the impact of lateral loading on pile foundations using the laboratory-scale model. The study aimed to determine the effect of the Winkler soil elasticity modulus model on pile-soil interaction with lateral loading. This was realized using a spring model to simulate soil behavior along the pile depth, including conducting laboratory experiments to calibrate the horizontal deflection model. The results showed that the difference in horizontal deflection between the Winkler model and the laboratory test was 7.73%. The difference in horizontal deflection between the Reese Matlock model and the laboratory test was 4.43%. It was also observed that the influence of the Winkler soil modulus of elasticity became prominent at approximately 40% of the pile length. A close agreement was found between the Winkler spring and the laboratory test models when analyzing the behaviour of a single pile subjected to lateral loading.

The assumptions used in pile foundation modeling considered the varying physical parameters of the soil to obtain a response model that reflected field conditions. Pile foundation modeling, based on the assumption of linear soil modulus of elasticity with increasing depth, is considered valid in estimating its response under conditions similar to those found in the field. The soil is not an elastic material. Therefore, there is a need to further investigate the assumption of using linear modulus of elasticity in foundation modeling.

2. STUDY SIGNIFICANCE

This study analyzed the response of the soil around a foundation when subjected to lateral loading. It used two theories for this purpose, namely Winkler 1867, which modeled the soil as a spring system with non-linear stiffness ($\boldsymbol{k_s}$) and the Reese, L.C. and Matlock 1960 model, assumed linear stiffness along the pile depth. The analysis was conducted using the Mathlab R2013a program. By comparing the results obtained from these two approaches, the validity of assuming linear soil elastic modulus along the pile depth in simplifying the modeling process could be assessed.

3. METHODOLOGY

The average value of soil elastic modulus is frequently adopted to represent its variation along the pile depth. However, further analyses are needed to justify the reliability of this assumption. This study aims to validate these assumptions by considering several conditions of soil modulus elasticity values. These include average modulus elasticity along the pile by Reese Matlock (1960) y_{RM} , layered modulus elasticity $\boldsymbol{y_L}$ **,** average modulus elasticity along the pile derived from Matlab $\boldsymbol{y_R}$ **,** and non-linear modulus elasticity $\boldsymbol{y_{Wk}}$ by Winkler (1867). For the validation process, the input values are sourced from field data on three projects, namely the Citarum, Dompak, and Batang projects in Jakarta, Tanjung Pinang Riau, and Central Java, respectively, as shown in (Table 1). The respective soil properties from the three locations are shown in (Table 2).

Table 1. The elasticity of the soil modulus value

Depth (m)	Citarum L = 20 m			Dompak L = 18 m			Batang L = 17 m		
	Modulus Young's (*10^4) (kN/m^2)								
	y_L	y_R & y_{RM}	y_{Wk}	y_L	y_R & y_{RM}	y_{Wk}	y_L	y_R & y_{RM}	y_{Wk}
1.0	3.0	53.7	2.8	3.7	135.8	2.9	4.0	161.6	2.7
2.0	3.0	53.7	4.2	3.7	135.8	4.2	4.0	161.6	3.9
3.0	3.0	53.7	5.1	3.7	135.8	5.2	4.0	161.6	4.8
4.0	3.0	53.7	6.0	3.7	135.8	6.0	4.0	161.6	5.5
5.0	3,0	53.7	6.5	3.7	135.8	6.7	4.0	161.6	6.2
6.0	3.0	53.7	7.1	3.7	135.8	7.4	4.0	161.6	6.8
7.0	3.0	53.7	7.7	3.7	135.8	7.8	4.0	161.6	7.4
8.0	3.0	53.7	8.2	3.7	135.8	8.3	4.0	161.6	7.9
9.0	3.0	53.7	8.7	3.7	135.8	8.8	4.0	161.6	8.4
10.0	3.0	53.7	9.2	3.7	135.8	9.3	4.0	161.6	8.8
11.0	3.0	53.7	9.7	3.7	135.8	9.8	4.0	161.6	9.3
12.0	3.0	53.7	10.1	3.7	135.8	10.2	4.0	161.6	9.7
13.0	3.0	53.7	10.5	400.0	135.8	10.5	540.0	161.6	10.1
14.0	3.0	53.7	10.9	400.0	135.8	10.9	540.0	161.6	10.5
15.0	172.0	53.7	10.7	400.0	135.8	11.3	540.0	161.6	10.8
16.0	172.0	53.7	11.1	400.0	135.8	11.6	540.0	161.6	11.2
17.0	172.0	53.7	11.4	400.0	135.8	12.0	540.0	161.6	11.5
18.0	172.0	53.7	11.7	400.0	135.8	12.3	-	-	-
19.0	172.0	53.7	12.1	-	-	-	-	-	-
20.0	172.0	53.7	12.4	-	-	-	-	-	-

Note: yL = Layering; y_R & y_{RM} =Average; y_{Wk}=Non-Linear

Table 2. Soil properties data in the field

Project	Depth (m)	Type of soil	Gs	w (%)	γ_b (kN/m^3)	γ_d (kN/m^3)	E (kN/m^2)	Soil parameters φ(°)	Soil parameters c(kN/m^2)
Citarum L_{pile} = 20m	0 to 4	Silty clay	2.5	20.0	15.8	13.17	3.0E+04	17.0	3.00
	4 to 14	Sand	2.7	20.0	19.7	16.42	3.00E+04	35.0	0.02
	14 to 60	Silty sand	2.6	20.0	17.6	14.,67	1.72E+06	35.0	0.02
Dompak L_{pile} = 18m	1 to 6	Silty sand	2.5	20.0	15.5	12.92	3.70E+04	15.0	2.20
	6 to 12	Sandy silt	2.5	20.0	15.8	13.17	3.70E+04	20.0	2.10
	12 to 22	Sand	2.7	20.0	16.6	13.83	4.00E+06	25.0	0.05
	22 to 34	Sand Gravel	2.7	20.0	19.5	16.25	4.00E+06	35.0	0.02
Batang L_{pile} = 17m	1 to 8	Silty and	2.6	40.0	16.6	11.86	4.00E+04	25.0	2.05
	8 to 12	Silty sand	2.6	35.0	16.5	12.22	4.00E+04	25.0	2.30
	12 to 20	Sand	2.7	40.0	18.0	12.86	5.40E+06	30.0	0.05
	20 to 30	Sand Gravel	2.7	40.0	19.0	13.57	5.40E+06	35.0	0.02

Note: G=specific gravity; w=water content; γ_b= unit weight; γ_d=dry unit weight; E= Elasticity modulus

Lateral load tests in the field have also been carried out and the results are shown in (Fig. 4) which will be used to compare with the results of analytical calculations in this study.

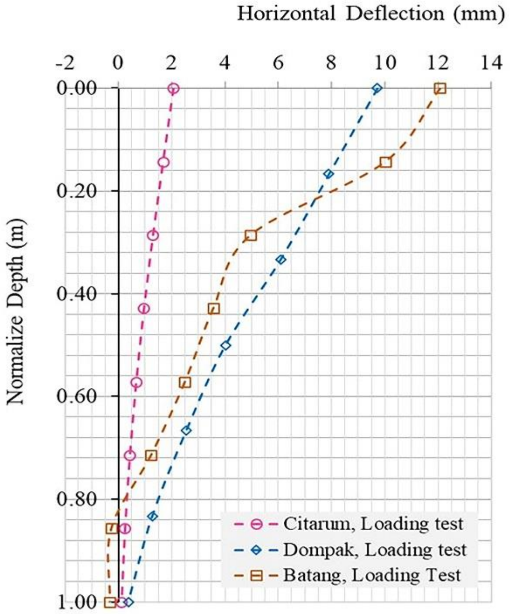

Fig. 4 Lateral loading test in the field

3.1 The sequential analysis is as follows:

3.1.1 Step 1

This step was carried out to retrieve lateral loading and original soil data from the field for the Citarum (L_{pile} = 20.0 m; D_{pile} = 1.2 m), Dompak (L_{pile} = 18.0 m; D_{pile} = 1.0 m), and Batang projects (L_{pile} = 17.0 m; D_{pile} = 0.6 m). These data would be used to conduct a comparative deflection analysis using the Winkler concept assisted by Matlab.

3.1.2 Step 2

Each project data was applied to the basic Winkler concept to re-analyze the horizontal deflection, as explained in the following equation:

$$R_1 = \frac{1}{2}.ks.B.h.yi$$

$$R_2\ s/d\ R_{120} = ks.B.h.yi$$

$$R_{21} = \frac{1}{2}.ks.B.h.yi \quad (1)$$

Parameter $\boldsymbol{R_i}$ refers to soil reaction at each node, while $\boldsymbol{k_s}$, B, h, and $\boldsymbol{y_i}$, in Eq. (1), refer to the subgrade modulus, the pile foundation width, the distance, and the displacement of each node, respectively. The constant displacement matrix $\boldsymbol{y_i}$ is symbolized by $[\boldsymbol{A}]$, while $[A]^{-1}$ is the inverse of the constant displacement matrix $\boldsymbol{y_i}$. The displacement vector $\boldsymbol{y_i}$ at each node $\{\boldsymbol{y_i}\}$ and the constant vector of the equation symbolized by {B} are solved using the matrix method in Eq. (2):

$$[A]x\{y_i\} = \{B\} \text{ atau } \{y_i\} = [A]^{-1}x\{B\} \quad (2)$$

By applying the second-order differential formulation as stated in Eq. (3), 21 equations and unknown variables ($\boldsymbol{y_i}$) were derived as follows:

$$\text{EI}\ \frac{d^2y}{dx^2} = -\text{M} \rightarrow \Sigma\text{M} = 0 \text{ and } \Sigma\text{H} = 0$$

$$Node\ 2, \Sigma M_2 = 0,$$

$$C(y_{n-1} - 2y_n + y_{n+1}) + R_1.h - P.h = 0$$

$$Node\ 3, \Sigma M_3 = 0,$$

$$C(y_{n-1} - 2y_n + y_{n+1}) + R_1.2h + R_2.h - P.2h = 0$$

$$\text{Node } 21, \Sigma M_{21} = 0,$$

$$R_1.4h + R_2.3h + R_3.2h.... + R_{20}.h - P.20h = 0$$

$$\Sigma H = 0,$$

$$R_1 + R_2 + R_3 + R_4..... + R_{20} - P = 0 \quad (3)$$

By converting Eqs. (2) and (3) into matrix form, final solutions can be obtained using the *Matlab* R2013a program.

3.1.3 Step 3

Each project also re-analyzed the horizontal deflection through the method developed by Reese et al., (1960), to calculate moments and horizontal loads, soil reactions, and deflections along piles based on the analogy of beams on elastic supports. Mathematically, it can be represented by Eq. (4), as follows:

$$y(x) = F\,(x, T, Ls, k, EI, H, M) \quad (4)$$

$y(x)$ is the deflection, L_s is the length of the pile, EI is the soil stiffness, and x is the depth measured from the ground. H, M, T, and k refers to the shear stress, moment, subgrade modulus, and stiffness factor, respectively. Therefore, deformation due to H and M can be separated, as shown in (Fig. 5).

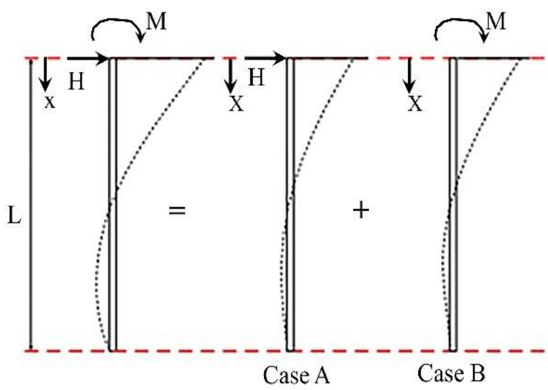

Fig. 5 Superposition due to H and due to M

In each case, there are six terms with two dimensions, namely cases A and B, which are represented by Eq. (5):

$$\text{Case A}: \left(\frac{y_A.EI}{H.T^3}\right);\left(\frac{x}{T}\right);\left(\frac{L_s}{T}\right);\left(\frac{k.T^4}{EI}\right) \quad (5)$$
$$\text{Case B}: \left(\frac{y_B.EI}{M.T^2}\right);\left(\frac{x}{T}\right);\left(\frac{L_s}{T}\right);\left(\frac{k.T^4}{EI}\right)$$

The superposition of deflection for cases A and B governs Eq. (6). y_x is the total displacement due to the moment and lateral force.

$$y_x = y_A + y_B = A_y\left(\frac{H.T^3}{EI}\right) + B_y\left(\frac{MT^2}{EI}\right) \quad (6)$$

Eqs. (7) to (10) were obtained from the derivative of Eq. (6):

$$\text{Slope}, S_x = S_A + S_B = A_s\left(\frac{H.T^2}{EI}\right) + B_s\left(\frac{M.T}{EI}\right) \quad (7)$$

$$\text{Moment}, M_x = M_A + M_B = A_m(H.T) + B_m(M) \quad (8)$$

$$\text{Shear}, V_x = V_A + V_B = A_v(H) + B_v\left(\frac{M}{T}\right) \quad (9)$$

$$\text{Soil reaction}, p_x = p_A + p_B = A_p\left(\frac{H}{T}\right) + B_p\left(\frac{M}{T^2}\right) \quad (10)$$

Coefficients A and B are resolved numerically using the table provided by Reese, L.C. and Matlock, (1960).

4. RESULTS AND DISCUSSION

This study considered data from three projects conducted on sandy silt soil, each with varying densities, pile dimensions, and locations. The horizontal forces were applied and analyzed with the concepts, of y_{Wk}, y_{RM} and, y_L developed by Winkler (1867), Reese Matlock (1960) and assisted by Matlab. The outcomes were compared to the results of linear deflection, y_R as shown in Figure 5.

More details, in (Fig. 6), y_{Wk} and y_L show higher non-linear lateral deflection, compared to y_{RM} and y_R. This difference ranged from approximately 10% to 14% within the initial 10% of the pile length. The comparison of the average horizontal deflection for Citarum Dompak and Batang Project results is shown in Table 3.

According to Table 3, $\frac{y_R}{y_{RM}}$ shows negligible value as both methods rely on average modulus elasticity. However, the difference is more pronounced as non-linear soil properties are considered in the analyses $\frac{y_R}{y_L}$ and $\frac{y_R}{y_{WK}}$.

Comparison presented in Table 3 shows that the difference in horizontal deflection between the Reese Matlock model and the linear model reached a maximum value at 2.57% and a minimum at 2.28%. Meanwhile, the difference value found between Linear model and Layered model reached its maximum at 4.37% and its minimum at 3.07%. Consequently, the difference between Winkler and linear models reached 6.63% at maximum and 5.08% at minimum.

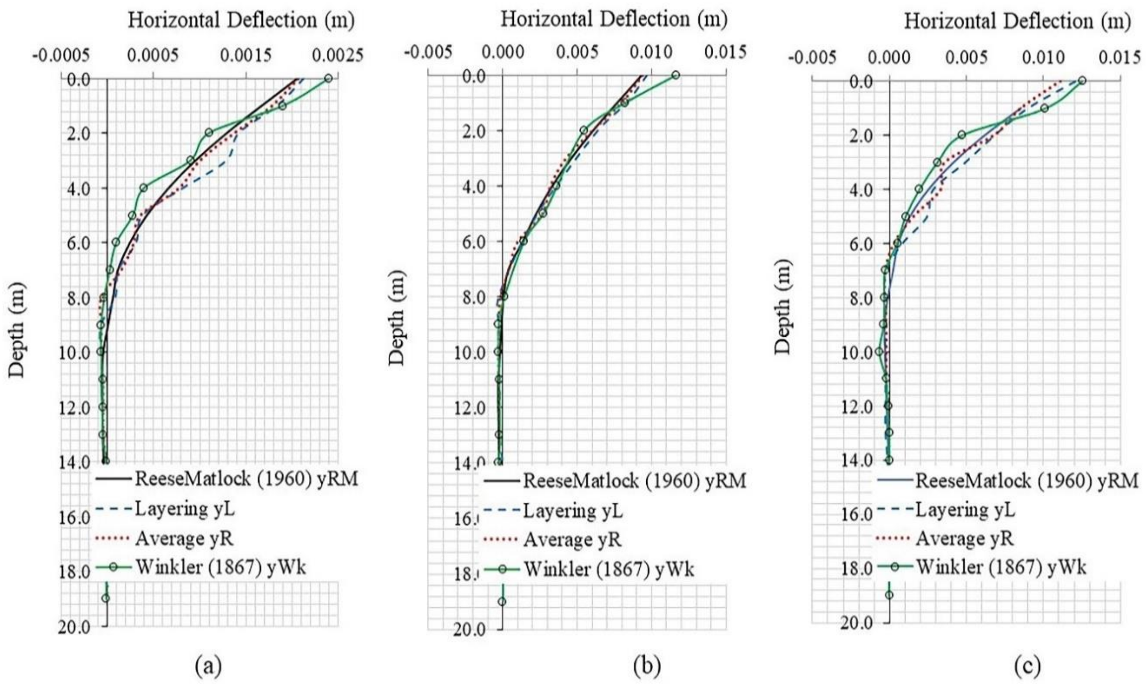

Fig. 6 Comparison of the horizontal deflection, (a) Citarum project, (b) Dompak Project, (c) Batang Project

Table 3. Comparison of horizontal deflection results

Field Data	$\frac{y_R}{y_{RM}}$	$\frac{y_R}{y_L}$	$\frac{y_R}{y_{Wk}}$
Citarum Project	2.54%	4.35%	5.08%
Dompak Project	2.57%	4.37%	6.63%
Batang Project	2.28%	3.07%	5.85%

Note: yL *= Layering;* y_R *&* y_{RM} *=Average;* y_{Wk}*=Non-Linear*

These results are consistent with the laboratory study conducted by Agus P. Rahmad and Sumiyati (2023). They stated that the difference in horizontal deflection between the Reese Matlock model and the laboratory test averaged 4.43%, while the difference in horizontal deflection between the Winkler model and the laboratory test averaged 7.73%. Their results were derived using silty sand soil samples and two physical models: a steel pole with diameter (D) of 0.02 m and length (L) of 0.6m and another steel pole with D = 0.015m and L = 0.5m**.**

Refer to (Fig. 7), it can be found that the results obtained from the current data (Citarum, Dompak, and Batang Project) are in good agreement with the measured data from Zhang (2015). In his publication, semianalytical solutions using the power series method were proposed to assess the behavior of a vertical pile with varying cross sections embedded in a multi-layered soil system. The lateral deflection ended at normalized depth equals 1, or it is approximately 40% of the pile length, calculated from the head of the pile.

Consistently, the loading test result from the field measurements show a similar trend with the current results. The lateral deflection is diminished at normalized depth equals to 1 or approximately 40% of the first meters of a pile length.

(Fig. 7) shows the smallest lateral deflection is found at Citarum project while the highest is found at Batang project. It is inversely proportional to the value of average elastic modulus at Citarum project, which has the highest number of Er = 57000 kN/m^2, followed by Dompak project at Er = 54000 kN/m^2 and Batang project at Er = 53000 kN/m^2. Similar result is also supported by Cao (2017). His work presented a higher elastic modulus at higher deflection resistance and smaller horizontal deflection.

The horizontal deflection analysis for each method used in conducting the projects are shown in (Fig. 8). The results show that horizontal deflection occurs in approximately 40% of the pile length, with detailed values provided in Table 4. The horizontal deflection is approximately zero at the deeper length (40% of the pile length). This trend is also consistent with the previous results in the laboratory study presented by Agus P. Rahmad & Sumiyati G. (2023) and Zhang (2015).

It is worth noticing that the average value of horizontal deflection along the pile is insignificant. However, the difference in horizontal deflection would be more obvious if looked at the first 10% of the pile length, as presented in (Fig. 8). The lateral deflections calculated using non-linear elastic modulus are higher than the ones calculated using the linear elastic modulus.

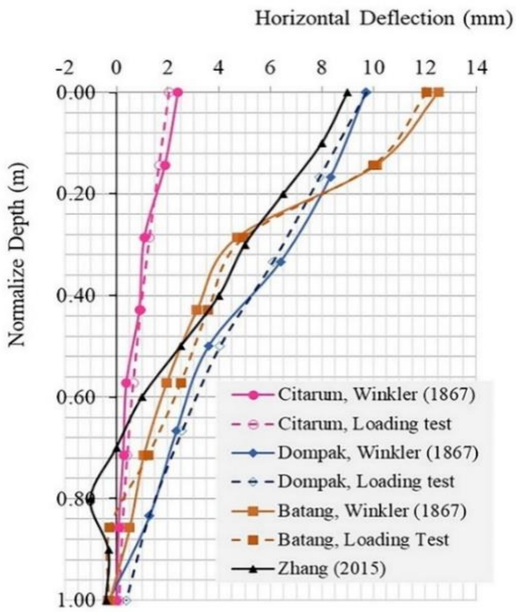

Fig. 7 Comparison of horizontal deflections the non-linear solution with field measurement

Table 4. The horizontal deflection effect occurs in about 40% of the pile length.

	Horizontal Deflection (m)		
Project / Method	Citarum L = 20m 40%L 8m	Dompak L = 18m 40%L 7,2m	Batang L = 17m 40%L 6,8m
Matlock (1960), y_{RM}	3.0 E-04	3.0 E-04	3.0 E-04
Layering, y_L	3.0 E-05	3.0 E-04	-5.0 E-06
Average, y_R	4.0 E-05	3.0 E-04	-2.0 E-04
Winkler (1867), y_{Wk}	-2.0 E-05	4.0 E-04	-1.0 E-04

5. CONCLUSION

1. In conclusion, several conditions for the soil modulus elasticity value were considered, such as average along the pile based on Reese Matlock (1960), $\boldsymbol{y_{RM}}$, layering, $\boldsymbol{y_L}$, average along the pile, $\boldsymbol{y_R}$, and layered by Winkler (1867), $\boldsymbol{y_{Wk}}$. The results showed that the average comparison of $\boldsymbol{y_R} / \boldsymbol{y_{RM}}$, $\boldsymbol{y_R} / \boldsymbol{y_L}$ and $\boldsymbol{y_R} / \boldsymbol{y_{Wk}}$ at the Citarum project was 2.54%, 4.35% and 5.08%. For the Dompak and Batang Projects, these comparisons were 2,57%, 4.37% and 6.63%, as well as 2,28%, 3.07% and 5.58%, respectively. These comparisons helped evaluate the assumption of using average soil elastic modulus along the pile in modeling to simplify the analysis, considering factors discussed in the subsequent points.
2. Horizontal deflection at the first meters of the pile length is inversely proportional to the average elastic modulus which depending on the pile dimensions and the applied lateral force.
3. The results obtained indicated that the horizontal deflection effect occurred in approximately 40% of the pile length and remained consistent in other areas with different soil properties and lateral forces.
4. As discussed in the third conclusion, it was recommended to consider the lateral bearing strength of the pile foundation, particularly in the first 40% of the pile length.
5. The evaluation proved that non-linear elastic modulus analysis determined the surface deflection, resulting in an excess of approximately 10% to 14% horizontal deflection at the first 10% of the pile length. These results should be considered in subsequent analysis of linear elastic modulus. This consideration was essential as it could potentially impact the recommended lateral bearing capacity of the pile foundation.
6. As discussed in the fourth conclusion, it was extremely essential to reconsider the use of the assumption of linear elastic modulus in the analysis, particularly in regions prone to earthquakes or strong winds. This was specifically relevant in the initial 10% of the pile length.
7. The use of average elastic modulus at the first 10% of pile length need to be considered in the lateral displacement analyses, as an alternative to the adoption of a safety factor.

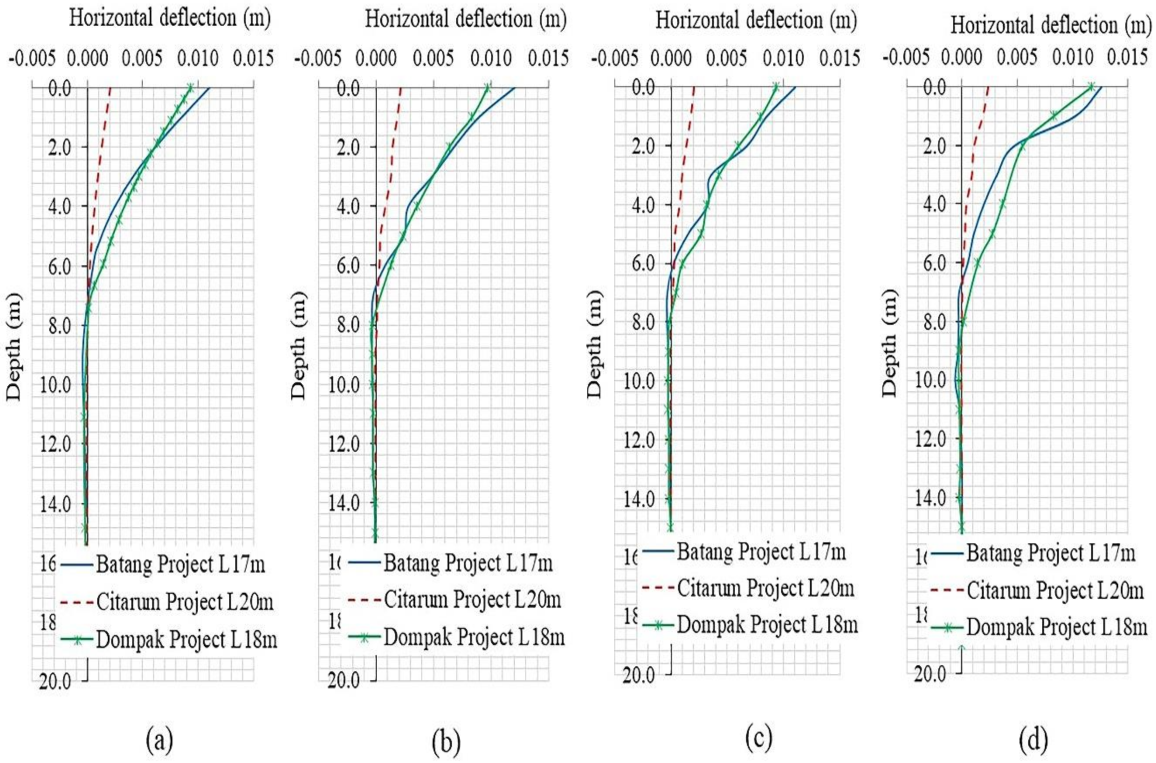

Fig. 8 Comparison of the horizontal deflection, (a) by Reese Matlock (1960), $\boldsymbol{y_{RM}}$, (b) layering, $\boldsymbol{y_L}$, (c) average, $\boldsymbol{y_R}$, (d) by Winkler (1867), $\boldsymbol{y_{Wk}}$

6. ACKNOWLEDGMENTS

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