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PROPOSED FORMULATION FOR PREDICTING DEFLECTIONS OF CFRP-RC BEAMS

Zaiimatul Ummah[1], Tavio[1]*

[1]Department of Civil Engineering, Institut Teknologi Sepuluh Nopember (ITS), Surabaya, Indonesia

*Corresponding Author, Received: 26 July 2023, Revised: 20 Sep. 2023, Accepted: 24 Sep 2023

ABSTRACT: This research describes an attempt to conduct analytical investigations on the deflection behavior of Carbon Fiber Reinforced Polymer (CFRP)-reinforced concrete beams. The primary objective of this study is to undertake a comprehensive review of formulation studies grounded in deflection equations. To accomplish this objective, a total of eleven test data points from three groups of researchers were acquired. Then, these data are compared against the deflection predictions from four deflection equations, namely, ACI 440.1R-06, ACI 440.1R-15, Bischoff and Gross, and ISIS Canada. An empirically derived model was proposed to predict the effective moment of inertia for reinforced concrete (RC) beams reinforced with CFRP, based on Branson's equation. Furthermore, to enhance the prediction of the moment-deflection relationship up to the ultimate strength, a nonlinear parameter (*k*) has been introduced in previous research for FRPs. These parameters were added to the formulation and aimed to mitigate the impact of the cracked moment of inertia on the reinforced concrete member. The accuracy of the novel formulation for analyzing deflections in CFRP-reinforced concrete beams was statistically evaluated. In a comparative study employing various design codes, the proposed model exhibited greater agreement with experimental test results. Ultimately, the proposed model demonstrated enhanced accuracy and emerged as a familiar approach for structural engineers to forecast and evaluate the deflection behavior of RC beams reinforced with CFRP.

Keywords: Carbon Fiber Reinforced Polymer, Deflection, Design Codes, Disaster Risk Reduction, RC Beams

1. INTRODUCTION

Throughout their lifespan, reinforced concrete structures require frequent modifications and enhancements to their performance. Several factors contributing to the need for these changes include shifts in usage, updated design standards, steel corrosion resulting from aggressive environments, and natural disasters such as earthquakes [1]. In these cases, two feasible arrangements are mostly referred to: replacement or reinforcement [2,3]. A complete structural replacement has several disadvantages, such as high material and labor costs, increased environmental impact, and inconvenience due to structural malfunctions, such as traffic problems. Therefore, several previous studies have suggested opting for structural repair and enhancement using reinforcement whenever it is feasible [4,5].

The reinforcement of reinforced concrete beams can be accomplished through a range of methodologies, encompassing the addition of reinforcement via jacketing, the incorporation of steel plates, the installation of trusses, the utilization of high-strength steel bars and FRP (Fiber Reinforced Polymer) materials [6]. FRP itself, as a material, is classified into several types, including Glass Fiber-Reinforced Polymer (GFRP), Steel Fiber Reinforced Polymer (SFRP), and Carbon Fiber Reinforced Polymer (CFRP).

Reinforcement using CFRP offers several advantages over conventional steel reinforcement, such as superior yield strength, lighter material composition, improved corrosion resistance, and enhanced durability. Thus, the selection of reinforcement from CFRP as a reinforcement material in reinforced concrete beam elements can be deemed a viable option. The use of CFRP in the context of building construction has experienced a steady rise. Therefore, it becomes imperative to conduct a study that focuses on the diverse design aspects of CFRP-reinforced concrete. The deflection of such a reinforced concrete structural component is a matter of concern due to their relatively diminished elastic modulus in comparison to steel bars, thereby resulting in an increased magnitude of deflection [7]. Hence, the serviceability of CFRP-reinforced beams requires further investigation to assess and ascertain their deflection behavior.

To improve planning effectiveness, conducting a comprehensive examination of the regulations and standards governing the planning of concrete buildings in Indonesia is crucial [8]. Specifically, the ones that are related to reinforcing and strengthening concrete structures using CFRP. Moreover, various countries have established regulations and standards with different calculation methods. Based on the aforementioned issues, the authors concentrate on investigating the formulation of deflection in CFRP-reinforced concrete beams using ACI 4401R-06, ACI 4401R-15, Bischoff and Gross's equation, and ISIS Canada. The procedure for calculating the deflection of reinforced concrete beams, as outlined in regulations and formulations, will be examined through a set of diverse literature sources. From the results of this comparative study, the accuracy of each standard procedure employed for predicting deflection in CFRP-reinforced concrete beams can be observed.

2. RESEARCH SIGNIFICANCE

The accuracy and reliability of equations that are used to predict the deflection of CFRP beams require further improvements. This study proposes a modified effective moment of inertia and conducts a comparative analysis of the deflection behavior of CFRP-strengthened RC beams, utilizing experimental tests. The proposed model to predict the effective moment of inertia for RC beam with CFRP strips was investigated empirically, based on Branson's equation to have better accuracy and a familiar approach to a structural engineer. This study analyzes several deflection equations from the standards and previous research by combining some parameters derived from other equations related to the cases. The analytical calculations are validated by comparing them with experimental data obtained from previous research.

3. REVIEW OF EXISTING AND PROPOSED FORMULA FOR CFRP–REINFORCED CONCRETE BEAMS

The calculation of the maximum deflection at the center of a beam within a four-point load system can be determined through the following equation:

$$\Delta_i=\frac{Pa}{48E_cI_e}\left(3L^2\text{-}4a^2\right) \quad (1)$$

Eq. (1) signifies that Δ_I is the mid-span deflection (mm), P is the value of force/load (N), a is the shear span (mm), E_c is the modulus of elasticity of concrete (MPa), and L is the span (mm). Additionally, the effective moment of inertia of the beam (I_e) will be thoroughly explained and evaluated for CFRP-reinforced concrete beams, that adhere to the following set of rules.

According to ACI Committee 440.1R-06 [9], the effective moment of inertia of the beam cross-section can be determined using the formula presented in Eq. (2). This calculation is an adapted version of Branson's equation [10], with the modification involving the introduction of the parameter, β_d.

$$I_e=(\frac{M_{cr}}{M_a})^3\,\beta_d I_g+[1\text{-}\left(\frac{M_{cr}}{M_a}\right)^3]I_{cr}\leq I_g \quad (2)$$

The parameter β_d, as described in Eq. (3), takes into account the relationship between FRP bars and concrete, including factors such as the elastic modulus of FRP bars.

$$\beta_d=\frac{1}{5}\frac{\rho_f}{\rho_{fb}}\leq 1 \quad (3)$$

Eq. (3) interprets ρ_{fb} as the balanced reinforcement ratio. The aforementioned ratio can be derived through a calculation as explained in Eq. (4).

$$\rho_{fb}=\frac{0.85\beta_1 f'_c}{f_{fu}}\frac{0.003E_f}{0.003E_f+f_{fu}} \quad (4)$$

The parameter f_{fu} represents the tensile strength of FRP bars, while E_f denotes the elastic modulus of FRP bars. Then, the value of β_1 can be determined using Eq. (5) as follows.

$$\beta_1=0.85\text{-}0.05\frac{f'_c\text{-}27.6}{6.7}\geq 0.65 \quad (5)$$

According to ACI Committee 440.1R-15 [11], the deflection prediction can be estimated by calculating the effective moment of inertia, I_e , which is taken from Bischoff et al. [12]. This formula now incorporates the influence of γ , as explained in Eq. (7) to accommodate the varying stiffness of the member along its length. The prediction of deflection necessitates the estimation of a consistent moment of inertia along the entire length of the beam.

$$I_e=\frac{I_{cr}}{1\text{-}\gamma\left(\frac{M_{cr}}{M_a}\right)^2\left(1\text{-}\frac{I_{cr}}{I_g}\right)}\leq I_g \quad (6)$$

where $M_a \geq M_{cr}$

Eq. (6) describes I_{cr} as the moment of inertia of the transformed section (mm^4). M_{cr} refers to the cracking moment (N.m), which can be obtained by following the calculation in Eq. (8) and Eq. (9). In the latter equation, the variable y_t represents half of the beam thickness. Returning to Eq. 6, M_a signifies the maximum service load moment in member (N.m). The parameter I_g indicates the gross moment of inertia of the uncracked section (mm^4), f_r expresses the concrete tensile strength, and f'_c refers to the concrete compressive strength (MPa).

$$\gamma=1.72\text{-}0.72\frac{M_{cr}}{M_a} \quad (7)$$

$$M_{cr}=\frac{f_r\times I_g}{y_t} \quad (8)$$

$$f_r=0.62\sqrt{f'_c} \quad (9)$$

By employing a genetic algorithm approach and experimental findings, Mousavi [13] proposed an alternative semiempirical model that modifies Branson's equation [10]. Empirical evidence was incorporated to enhance the accuracy of prediction. The model, characterized by two multiplying factors and an exponential factor denoted as 'm' in Eq. (11), was examined using experimental data obtained from FRP-reinforced concrete beams to establish the load-deflection relationship. The influence of the elastic modulus of FRP bars, reinforcement ratio, and level of loading on the exponent 'm' in Branson's equation is accounted for in Eq. (10) as follows [13]:

$$I_e=0.13\left(\frac{M_{cr}}{M_a}\right)^m I_g+0.89[1-\left(\frac{M_{cr}}{M_a}\right)^m]I_{cr} \quad (10)$$

$$m=-0.24\frac{\rho_f}{\rho_{fb}}+5.35\frac{M_{cr}}{M_a}+2.28\frac{E_f}{E_s} \quad (11)$$

Eq. (12), introduced by Intelligent Sensing for Innovative Structures [14], presents the formulation for calculating the effective moment of inertia in FRP bar-reinforced concrete beam [15]. This equation incorporates additional corrective terms into a modified version of Branson's equation by incorporating more experimental data. The parameters mentioned in Eq. (12) have been previously introduced and explained, along with their corresponding notations.

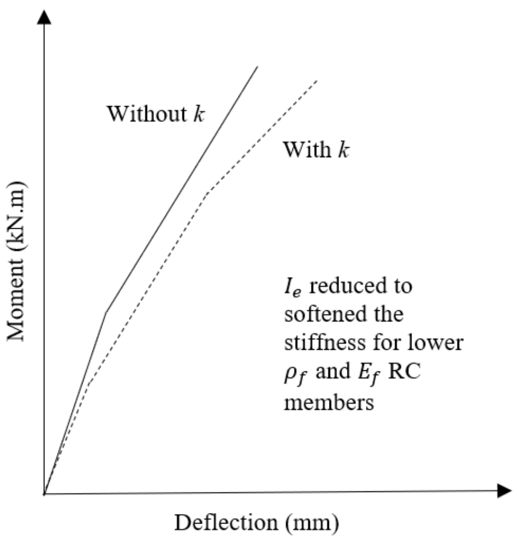

Fig.1 Illustration of the basic concept of k (Minkwan Ju, 2016)

$$I_e=\frac{I_g I_{cr}}{I_{cr}+(1-0.5\left(\frac{M_{cr}}{M_a}\right)^2)(I_g-I_{cr})} \quad (12)$$

Based on Branson’s equation [10] and the modification method that Toutanji and Saafi [15] used in their empirical approach, a semiempirical prognosis model for the effective moment of inertia of FRP beams is initiated. The deflection of the RC beam with CFRP was impacted by the reinforcement ratio of CFRP strips along with the elastic modulus of the CFRP strips. Minkwan Ju [16] introduced an empirically derived boundary, denoted as k (Figure 1), to account for the nonlinear behavior of RC beams reinforced with CFRP strips. This boundary is intended to ensure a satisfactory alignment with experimental results. This factor was employed to mitigate the impact of the cracked moment of inertia on the reinforced concrete member by incorporating a lower reinforcement ratio and modulus of elasticity for CFRP strips. This idea of modifying the equation by means of adding several parameters was derived empirically by examining the moment-deflection relationship described by the equations under consideration. The findings indicated that the stiff curve exhibited a bilinear behavior until the failure of the test specimen. Nonetheless, the test result shows a nonlinear behavior up to the failure. This finding suggests the need for a reducing factor to achieve a proper curve fitting to the experimental results. Consequently, this modification led to a reduction in the effective moment of inertia, so then it can result in an increase of the calculated deflection in accordance with the escalating applied loading.[16] For an RC member with a lower reinforcement ratio and modulus of elasticity material, this analytical concept may be more suitable:

$$I_e=\left(\frac{M_{cr}}{M_a}\right)^m I_g+[1-\left(\frac{M_{cr}}{M_a}\right)^m-k]I_{cr}\leq I_g \quad (13)$$

$$m=6-13\rho_{fb}\frac{E_f}{E_s} \quad (14)$$

$$k=[\frac{1}{11}(\frac{M_{cr}}{M_a})]^4 \quad (15)$$

4. SPECIMENS AND PARAMETERS

In this work, three experimental studies conducted by distinct research groups were adopted. These investigations comprised a total of 11 test specimens of RC beams reinforced with CFRP at a specific ratio. The first investigation was conducted by Hosen et al. [17] who performed a series of experimental tests on seven beams using the SNSM (Side Near Surface Mounted) technique, with varying numbers of CFRP strips and applied loads. The tests involved varying numbers of CFRP strips and applied loads. However, only three sets of test data (B1, B2, and B3) were selected to illustrate the behavior of CFRP-strengthened reinforced concrete beams with horizontal strips. The second experimental investigation was extracted from Rafi et al. [18]. Their research made a valuable contribution by presenting the test results of concrete beams reinforced with carbon FRP (CFRP) tension bars. Each specimen had a rectangular cross-section measuring 120 x 200 mm and a length of 2000 mm. For the characterization of CFRP-reinforced concrete beams, two test data were utilized (i.e., B4 and B5). The third experimental test data traces back to the work presented by Qaisi et al [19]. The research group conducted laboratory testing on a total of thirteen specimens exhibiting flexural deficiency and various configurations, including side Near Surface Mounted (NSM), bottom NSM, and a combination of both. However, for the purpose of describing RC beams reinforced with CFRP strips, only six test data (B6 - B11) were utilized. Table 1 presents a comprehensive overview of the specimen details. Furthermore, Figures 2 and 3 provide visual representations of the experimental setup, loading configuration, and the cross-sectional profile of the CFRP beam, respectively.

Table 1 Details of the experimental database adopted in this work

Beam	b	d	L	f'_c	f_y	E_f	Δ
		(mm)		(MPa)		(GPa)	(mm)
B1	125	250	2300	60	550	165	27.5
B2	125	250	2300	60	550	165	28
B3	125	250	2300	60	550	165	19
B4	120	200	2000	48	566	234	35.26
B5	120	200	2000	47	566	234	35.5
B6	150	250	1600	32	491	230	12.95
B7	150	250	1600	32	491	230	14.88
B8	150	250	1600	32	491	230	12.68
B9	150	250	1600	32	491	230	12.56
B10	150	250	1600	32	491	230	13.16
B11	150	250	1600	32	491	230	14.19

Note: b = beam width (mm); d = beam depth (mm); L = beam length (mm); f'_c = concrete compressive strength (MPa); f_y = steel yield strength (MPa); E_f = modulus of elasticity CFRP (GPa); and Δ = the experimental deflection results (mm).

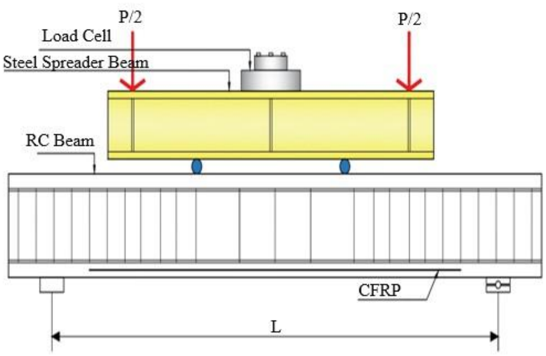

Fig.2 Illustration of the test setup and loading beam of CFRP

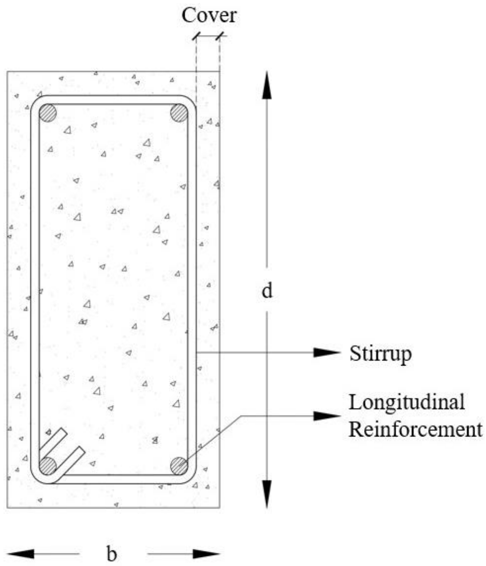

Fig.3 Cross section of the beam

5. RESULTS AND DISCUSSION

The ultimate deflection values for all CFRP samples in the database are computed using the proposed equation. Comparisons were made between the experimental deflection divided by the analytical deflection, then the average ratio (Avg), standard deviation (SD), and coefficient of variation (CoV) were obtained. The average ratio indicates the mean of the data. The standard deviation measures how far the average value lies from the mean, whereas the coefficient of variation measures the ratio of the standard deviation to the mean. Those statistical parameters, which reflect the accuracy of the deflection prediction comparison and the nodal graph, are presented in Table 2 and Figure 4, respectively.

Table 2 Deflection summary for all design codes

No.	Code	Statistical Result $\Delta_{exp}/\Delta_{calc}$		
		Avg	SD	CoV
1.	ACI 440.1R-06	1.16	0.22	0.19
2.	ACI 440.1R-15	1.17	0.22	0.19
3.	Bischoff and Gross	1.15	0.21	0.18
4.	ISIS Canada	1.14	0.19	0.17
5.	Proposed Model	1.10	0.18	0.16

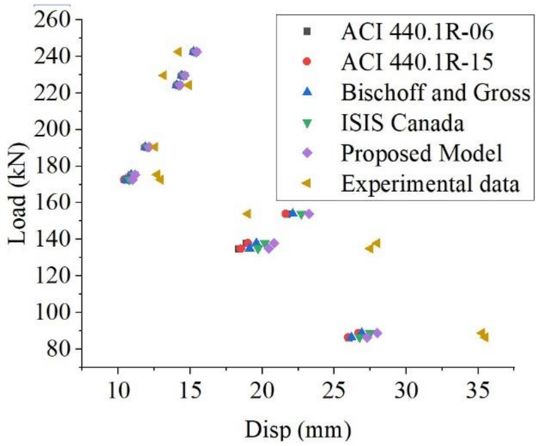

Fig.4 Comparison of analytical deflections with experimental results

In the majority of cases, the ratio between the maximum deflection obtained from experimental data and the values calculated by different design codes and equations falls within the range of 0.8 to 1.5. Specifically, the calculations based on ACI 440.1R-06 and ACI 440.1R-15 exhibit an average ratio of 1.16 and 1.17, respectively, with a coefficient of variation (CoV) of 19%. It is worth noting that the CoV value indicates a slightly higher level of variability compared to the deflection predictions of other design approaches.

The deflection predictions obtained from the equations proposed by Bischoff and Gross, as well as the ISIS Canada method, exhibit a comparatively lower degree of conservatism. In particular, the average ratio between the test results and the corresponding calculations for Bischoff and Gross and ISIS Canada equations are 1.15 and 1.14, with CoV values of 18% and 17%, respectively. These results indicate that the ISIS Canada equation demonstrates greater accuracy in comparison to the other methods considered.

Ultimately, the results of this study demonstrate that the proposed model exhibits a higher degree of accuracy compared to the other models evaluated because the value of the average (avg) and the standard deviation (SD) of the proposed model is the value closest to 1. The statistical analysis of the modified equation reveals a strong correlation with the experimental tests conducted on the reference specimens. Specifically, the average ratio of the test results to the calculated values is 1.10, indicating a close correspondence between the predicted and observed values. Additionally, the CoV value of 16% suggests a relatively low level of variability, further supporting the similarity between the experimental and calculated results. These findings highlight the robustness and reliability of the proposed model in predicting the behavior of the tested specimens.

Figures 5 to 9 indicate the relationship of deflection between the experiment and analytical calculation by several equations and the proposed model, in which, these graph results are taken from Origin statistic software. This shows that overall analysis tends to be conservative because most of the data lies above the threshold line. All of the graphs may show similar results because of the small difference but still, it shows the conservatism of all the data taken.

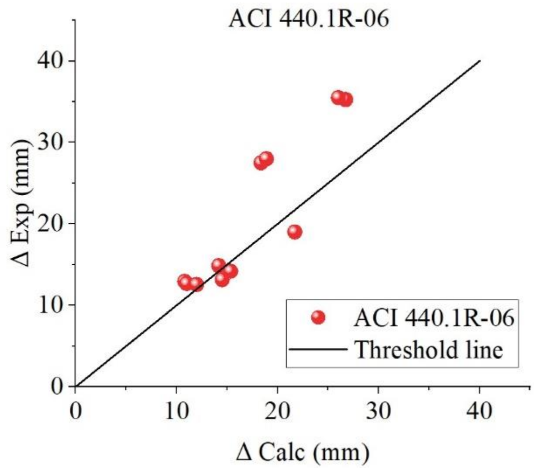

Fig.5 Relationship of deflection between experiment and analytical calculations by ACI 440.1R-06

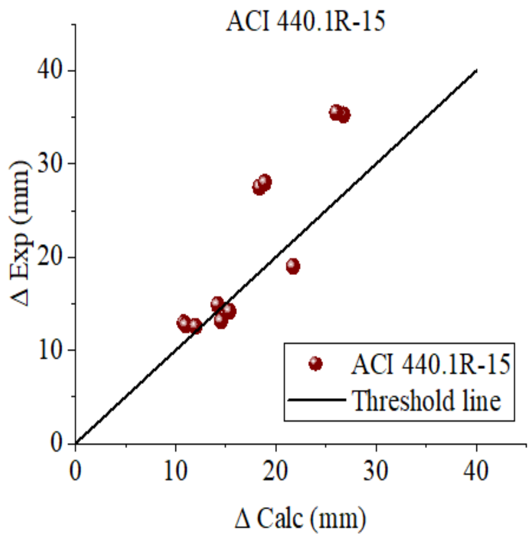

Fig.6 Relationship of deflection between experiment and analytical calculations by ACI 440.1R-15

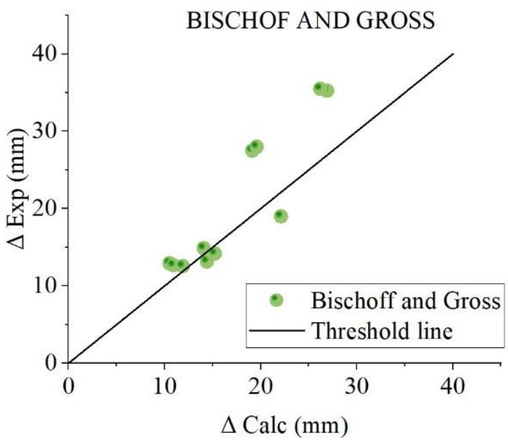

Fig.7 Relationship of deflection between experiment and analytical calculations by Bischoff and Gross equation

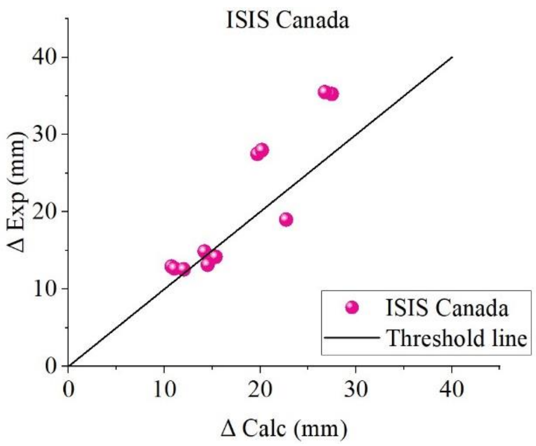

Fig.8 Relationship of deflection between experiment and analytical calculations by ISIS Canada

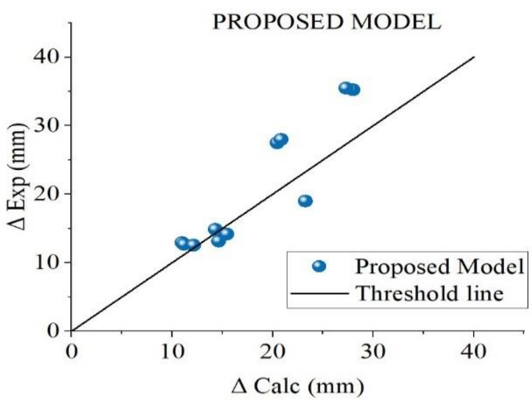

Fig.9 Relationship of deflection between experiment and analytical calculations by Proposed Model

There are several parameters that can have an impact on the behavior of concrete beams reinforced with FRP bars, including the properties of the concrete itself, the size effect, and the type of bar used, which affects the bonding properties. In particular, the bond performance of FRP bars in concrete beams is found to be more influenced by flexural loads compared to uniaxial tensile loads, mainly due to differences in surface treatments and chemical adhesion properties [16]. Therefore, it is necessary to conduct thorough experimental and analytical studies to investigate these factors more precisely.

6. CONCLUSIONS

The objective of this research was to compare the experimental and analytical deflections of RC beams reinforced with CFRP. A semi-empirical equation was developed to represent the moment-deflection relationship for RC beams reinforced with CFRP, aiming for an objective estimation of the effective moment of inertia. To achieve this, a variety of influential parameters were investigated, and equations suitable for deflection prediction were proposed. Through a comparative analysis of the values obtained using different codes and the proposed equations, several conclusions can be drawn:

1. The proposed equation for the effective moment of inertia of RC beams with CFRP was derived by modifying Branson's equation [10]. The value of parameter *m* was obtained from Toutanji and Saafi's research [15], and the nonlinearity parameter *k* was also taken into account.
2. In accordance with the experimental results, the reinforcement ratio and elastic modulus of CFRP are identified as significant factors influencing the deflection of RC beams. Both of these variables have been thoroughly considered in the proposed equation. Next to this, it is observed that the deflection predicted by applying the ACI 440.1R-06 code is more accurate compared to the ACI 440.1R-15 code, owing to the incorporation of these influential variables.
3. The proposed equation reports the most influential parameters, such as the modulus of elasticity of FRP bars and the relative reinforcement ratio, to accurately calculate the deflection. The proposed model has shown enhanced accuracy and has emerged as the most reliable approach for structural engineers to forecast and evaluate the deflection behavior of RC beams reinforced with CFRP.

4. Although the proposed equation exhibits a smaller discrepancy in results compared to other equations, further research is called upon to minimize this difference and achieve statistical convergence with a larger dataset and a broader range of equations.

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