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FRACTURE BASED NON LINEAR MODEL FOR REINFORCED CONCRETE BEAMS

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ABSTRACT: An analytical model based on the fracture properties of concrete and the nonlinear hinge model is proposed in this paper for the flexural behavior modelling of simply supported reinforced concrete beams. The model supposes the development of a single crack in the midsection of the beam within a zone called the hinge. The cross section of the beam is divided into a finite number of layered strips of concrete and a reinforcement bar. Each strip has a single freedom degree which is the elongation. Stress-strain relationships proposed in Euro code 2 were adopted for concrete strips under uniaxial compression and steel under tension. For concrete strips in tension zone three cases were studied: without softening effect, linear strain-softening behavior, and power-law strain softening behavior. The proposed model gives the load-deflection relationship, the development of the crack opening from cracking up to failure and the evolution of the crack height during loading. In order to validate the proposed model, the analytical results were compared with experimental ones of a set of beams selected from scientific references. Comparisons showed that the adequate prediction of flexural behavior requires the knowledge fracture properties with an adequate strain softening function beside basic mechanical properties of both concrete and steel. Moreover, the power law strain softening curve is the most suitable to model the experimental behavior of beams while linear softening function gives conservative results. The analytical results were supported by the results of 3D finite element analysis using ANSYS software.

Keywords: *Reinforced concrete, Nonlinear hinge, Cracking, Fracture properties, Flexural behavior*

1. INTRODUCTION

Concrete is the most used building material worldwide but its low tensile strength induces the cracking of structural elements even under the effect of service loads. For flexural design, standards assume that concrete tensile strength is neglected and only steel reinforcement supports tensile stresses [1], [2]. Cracking is a complex phenomenon which starts right after the setting up of concrete and is due to in service state to both environmental and mechanical loads when the tensile stresses exceed the concrete strength [3], [4] For horizontal elements such as beams and slabs, cracking affects the overall stiffness and makes the correct deflection calculation a very complicated problem whether with analytical or numerical methods [1], [2], [5]. Cracks play a key role in the durability of structures because they allow the penetration of water and the aggressive agents that lead to the steel corrosion [6].

Several works have been conducted in the literature to study the cracking of reinforced concrete beams (crack width and spacing) as well as the parameters affecting cracking ability [7]. Concerning analytical models for crack width and spacing available in the literature and design standards, it was concluded that prediction results are very dispersed due to the negligence or to the simplification of concrete tensile behavior [8].

Several models have been developed in the literature to model the flexural behavior of reinforced concrete beams using finite element method [7], [9], [10]. However, the common point of weakness between these models was the neglect of cracked behavior of concrete in traction as well as the simplification of the compressive behavior. In addition, the aforementioned models focus on the analysis of deflection without proposing methods for crack opening [10], [11].

Indeed, the tensile resistance of concrete does not vanish suddenly when the tensile strength is exceeded but gradually decreases with the crack opening and becomes zero when the opening exceeds a critical value [12]. The relationship between stress relaxation and crack opening is called strain-softening and can be modeled using several mathematical functions such as linear [12], [13], bilinear [14], [15], and power law [3], [16], [17]. The area under the strain-softening curve is called fracture energy and represents the energy dissipated upon the creation of a crack of unit surface area [12].

The present paper aims to propose an analytical model which takes into account the fracture properties of concrete in the analysis of flexural behavior of beams and their effect on crack opening.

2. OBJECTIVE AND RESEARCH SIGNIFICANCE

This paper investigates the effect of the fracture properties on the flexural behavior of reinforced concrete members by:

1. Proposition a fracture properties based analytical model,
2. Comparison between the analytical model and 3D finite element model developed using ANSYS software,
3. Validation of analytical results with experimental results of a database built from data available in the literature.

The paper highlights also the effect of the strain softening function shape on the overall behavior of RC members.

3. THEORETICAL STUDY

3.1 Reinforcing Steel Behavior

In this paper, the stress-strain relationship proposed in EC2 is adopted. The relation is composed of two parts [1]:

a) The first inclined branch is linear from the beginning of loading until the yielding stress f_{yk} or the design yielding stress f_{yd} . The strain corresponding to the end of this phase is equal to $f_{yd}/\boldsymbol{E_s}$.

b) The second one represents the plastic phase and may be horizontal or inclined up to the strain ε_{uk} and a maximum stress of $k.f_{yk}/\boldsymbol{\gamma_s}$ at ε_{uk} where $k = \frac{f_t}{f_y}$. The tensile strength of reinforcement is f_t and the designed strain ε_{ud} equal to $0.9\varepsilon_{uk}$.

3.2 Concrete Behavior

3.2.1 Stress-strain curve under compression

The relationship between the strain and stress can be expressed by the following expression [1]:

$$\frac{\sigma_c}{f_{cm}} = \frac{k.\eta - \eta^2}{1 + (k - 2).\eta} \tag{1}$$

With f_{cm} the concrete cylinder compressive strength, $\boldsymbol{\varepsilon_{c1}}$ the compressive strain in the concrete at the peak stress f_{cm}, $\boldsymbol{\varepsilon_{cu}}$ the ultimate compressive strain in the concrete, $\eta = \frac{\varepsilon_c}{\varepsilon_{c1}}$, $\kappa = \frac{1.05 E_{cm} \varepsilon_{c1}}{f_{cm}}$ and $\boldsymbol{E_{cm}}$ the secant modulus of elasticity.

In the absence of experimental data, all previous parameters can be evaluated as functions of the compressive strength using the following expressions:

$$E_{cm} = 22000.(\frac{f_{cm}}{10})^{0.3} \tag{2}$$

$$\varepsilon_{c1} \text{‰} = 0.7 f_{cm}^{0.31} \leq 2.8 \tag{3}$$

$$\varepsilon_{cu} \text{‰} = 3.5 \tag{4}$$

3.2.2 Tensile behavior of concrete

Fig.1.a shows the tensile behavior of concrete where the relationship between strain and stress can be divided into two parts [12]:

1. The first branch is $(\sigma - \varepsilon)$ curve and represents the behavior in the elastic phase.
2. The second nonlinear curve $(\sigma - w)$, called softening, represents the relationship between the decrease of the stress and the crack opening. The softening curve may be modelled using linear relationship [12], [13] (Fig. 1).b or using a power law function as shown in(Fig. 1.c) [3], [16]. The area under this curve represents fracture energy G_F .

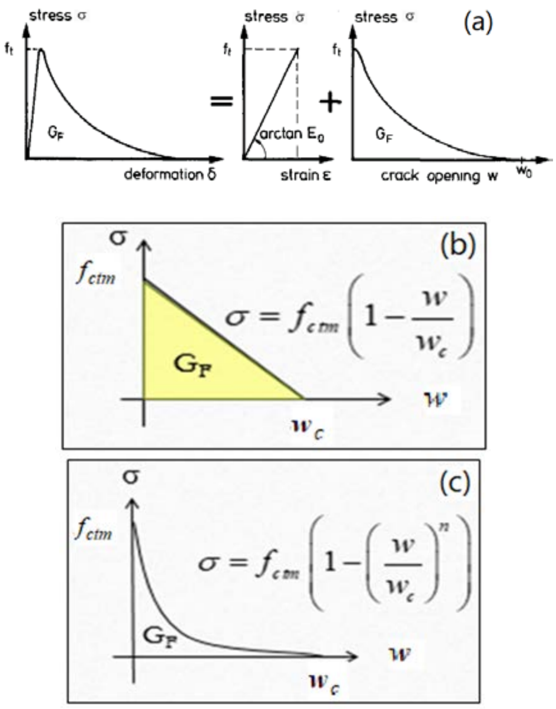

Fig.1(a) concrete behavior in uniaxial tension[12]
(b)-linear softening behavior
(c) Power-law softening behavior [3], [16].

The linear decrease in stress with the crack opening is given by "Eq. (5)" [12]:

$$\sigma = f_{ctm}(1 - \frac{w}{w_c}) \quad (5)$$

As the power law strain softening relationship is given by the following expression [3] **:**

$$\sigma = f_{ctm}\left[1 - (\frac{w}{w_c})^n\right] \quad (6)$$

w_c represents the critical crack width corresponding to zero tensile strength and n is the power of the law.

Based on both softening functions, the fracture energy can be defined as follows:

$$G_F = \begin{cases} \frac{1}{2} f_{ctm} w_c & \text{linear softening} \\ \frac{n}{n+1} f_{ctm} w_c & \text{power law softening} \end{cases} \quad (2)$$

Danhash [15] showed that w_c is related to G_F and f_{ctm} through "Eq. (8)" Concerning the fracture energy, G_F it depends on many factors, including the compressive strength and the maximum size of aggregates as well as the paste volume of the binder in the concrete mixture "Eq. (9)" . The power of the softening law, n , depends, according to Ghorbel and Wardeh [18], on the compressive strength as well as the mix design parameters "Eq. (10)" :

$$w_c = \frac{5G_F}{f_{ctm}} \quad (8)$$

$$G_F(\frac{N}{mm}) = 1.15(f_c)^{0.7}[0.003(1 + \frac{d_{max}}{10}) + (PV)^{5.7}] \quad (3)$$

$$n = \alpha f_{cm}\left(\gamma d_{max} + \mu(\frac{w}{B})\right) \quad (10)$$

Where $\alpha = 0.06, \beta = 0.77, \gamma = 1.29 \times 10^{-3}, \eta = 0.51$

3.3 Flexural Behavior of Reinforced Concrete Section

The model described below represents the flexural behavior of RC member in all stages of loading. The model is an extension the nonlinear hinge model which was initially developed for plain concrete [13]. The model assumes the existence of a crack in the mid span embraced in an area called the hinge. The width of the hinge is denoted by S and taken equal to the half of the beam height [13]. The cross section of the beam is considered consisting of layered strips of concrete and a reinforcement bar. Each strip has a single freedom degree, which is the elongation, and the work consists in establishing a relationship between generalized sectional forces and strains in both compression and tension.

From the beginning of loading up to failure the section will pass in two stages:

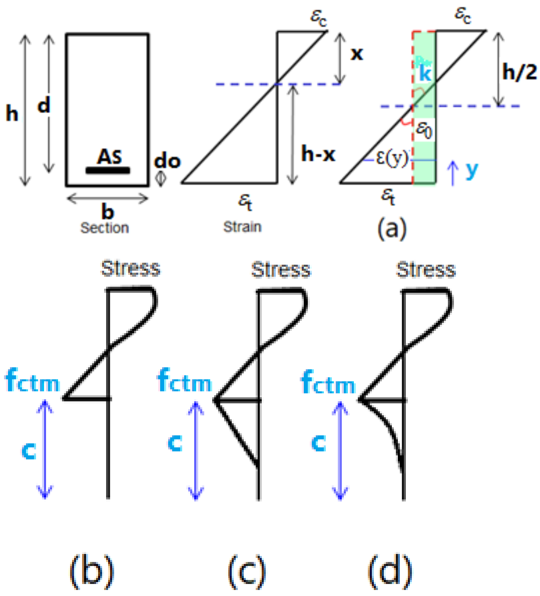

Fig.2: (a) strips strain-(b) Section behavior without softening
(c) Linear softening-(d) Power-law softening.

The linear stage:
The behavior is linear elastic while the stress at the
Lower strip of the beam is lower than the tensile strength f_{ctm}.
The cracked stage:
This stage starts after the cracking of the bottom strip until the failure of the element whether by steel yielding in tension and concrete crushing in compression. Both crack width and height change as a function of the curvature as it will be described later. The elongation of any strip in the cross section can be expressed as a function of mean normal strain at the beam axis, ε_0, and the curvature k as shown in Fig. 2.a. The strain of the strip located at a distance y from the bottom of the beam can be given using "Eq. (11)" .

$$\varepsilon(y) = \varepsilon_0 + k.(\frac{h}{2} - y) \quad (11)$$

Where, h is the cross-section height, A_S reinforcement area, and d_o is the distance between the center of reinforcement and the lower strip. If the strip is located in the compression zone, the stress is calculated according to Eq. (1) while in the tension zone the stress depends on the shape of the softening function.

$$\varepsilon_0 + k.(\frac{h}{2} - y) = \frac{f_t(1 - \frac{w}{w_c})}{E_c} + \frac{w}{s} \quad (12)$$

$$\varepsilon_0 + k.(\frac{h}{2} - y) = \frac{f_t\left[1 - \left(\frac{w}{w_c}\right)^n\right]}{E_c} + \frac{w}{s} \quad (13)$$

For the linear softening illustrated in Fig. 2.c, the deformation of the strip located at the coordinate y from the bottom of the section is given by "Eq. (12)" , while by using a power law function Fig. 2.d the strain is expressed by "Eq. (13)" .

These two equations are valid as long as the crack opening, w, is smaller the critical crack opening w_c For the strip representing the reinforcement, the stress is calculated as a function of the strain according to EC2 relationship described in the paragraph 3.1.The position of the neutral axis can be obtained using horizontal equilibrium of the section using "Eq. (14)". The solution of this nonlinear equation consists in imposing a value of the curvature, k, and seek the value of ε_0 which satisfies the internal equilibrium of the section given by Eq. (14). The equilibrium equation cannot be solved directly and required an iterative method such as Newton-Raphson method which is available in Matlab through fsolve function or which can be also implanted by the user. The solution algorithm requires the knowledge of internal force derivatives, called N', in each strip with respect to the strain ε_0 . These derivatives may be obtained numerically or analytically by deriving the constitutive laws for both steel and concrete.

$$N(\varepsilon_0, k) = b\int_0^h \sigma.dy + \sigma_S.A_S = 0 \quad (14)$$

When the internal equilibrium is reached the resisting moment can be computed using the following equation:

$$M = b\int_0^h \sigma_C.(\frac{h}{2} - y) + \sigma_S.A_S.(\frac{h}{2} - d_0) \quad (15)$$

During the second stage, the crack height (c) and the crack opening w represented in Fig.2 can be calculated as follows:

$$\varepsilon_0 + k.y_2 = \varepsilon_{ct} \Rightarrow y_2 = \frac{\varepsilon_{ct} - \varepsilon_0}{k}, c = \frac{h}{2} - y_2 \quad (16)$$

$$w = \varepsilon(y).s \quad (17)$$

The deflection in the middle of the beam is calculated by "Eq. (18)" where κ is the curvature and x represents the distance between the support and the studied point.

$$\Delta = \int_0^{L/2} x.\kappa.dx \quad (18)$$

3.4 Crack Width Determination According to EC2

Cracks appear in reinforced concrete elements when tensile stresses exceed the concrete strength, which leads to a decrease in the element stiffness. The expression for the design crick width proposed by EC2 [1] is given by "Eq. (19)".

$$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) \quad (19)$$

Where $(\varepsilon_{sm} - \varepsilon_{cm})$ may be calculated from expression:

$$\varepsilon_{sm} - \varepsilon_{cm} = \left[\sigma_S - \frac{k_t.f_{ct}}{\rho}.(1+\alpha_e.\rho)\right] / E_S$$

with $s_{r,max}$ the maximum crack spacing, σ_S the stress in the tensile steel, $\rho = A_S / A_C$ the reinforcement ratio, $\alpha_e = E_S / E_C$ the reinforcement ratio, κ_t coefficient to account for loading duration (0.6 for short term loads and 0.4 for long term loads).

The maximum crack width when semi-continuous loads are applied is given by the following relationship:

$$w_k = \beta.w_m \quad (20)$$

The value of β accounts for the section dimensions, and its range between 1.3 and 1.7**.**

3.5 Numerical Modeling Using Finite Element Method

Finite element modelling using ANSYS academic V18.2 was included in this work to validate the results of the proposed model. This software was chosen because of the simplicity of reinforced concrete modelling. Concrete is modelled using 8 nodes element SOLID65 which has with three degrees of freedom at each node being the translation in x, y, and z directions.

Element LINK180 is used for steel reinforcement modelling. It is a 3D spar element with 3 degrees of freedom at each node being translations in x, y and z directions. For rebar reinforcement an elastic-perfect plastic material model was adopted. Poisson's ratio was set to 0.3 whereas

elastic modulus and yield stress were set equal to experimental values.
For the supports and the loading plates, SOLID185 element was used. Eight nodes having three degrees of freedom at each node being translations in the nodal x, y, and z directions define it.
The total load was applied through a series of load steps and the analysis type was set to small displacement static. The sparse direct solver based on a direct elimination of equations was used in order to solve the model. The iterative process of Newton-Raphson method was adopted to solve the nonlinear equations of the model, along with the line search tool. The program do equilibrium iterations until the convergence criteria are satisfied.

4. EXPERIMENTAL DATABASE

Simply supported beams tested in 4 points bending were collected from bibliographical references [5], [19]–[24]. For all selected beams, the concrete mix design parameters and fracture properties calculated through "Eq. (8-10)". It should be noted that all these beams are made of classical natural aggregate concretes.

5. RESULTS AND DISCUSSIONS

The beam shown in (Fig.3), tested in 4 points bending, was selected from Ajdukiewicz and Kliszczewicz [19] for the sake of comparison with analytical and numerical modelling using ANSYS since these authors gave all the expected results including the cracking card.

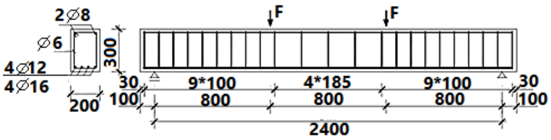

Fig.3: The studied beam[19]

The behavior of the beam is linear elastic up to a load of 20 KN with a corresponding deflection of 0.4 then the slope of the curve changes and the beam enters the cracked phase up to a load Fy=63.5 kN and a corresponding deflection Δy=7mm. The third phase is related to the steel yielding until the beam failure under a load Fy=64.8 kN and a related deflection Δy=58mm.
In addition, the cracks height at the end of the experiment ranged between (210mm and 250mm), with a crack spacing comprised between 180mm and 200mm.
To evaluate the effect of fracture properties on the behavior of the beam, three cases were considered:

a) Fracture properties are neglected (without tensioning) this means that tensile behavior is neglected after reaching the concrete tensile strength.
b) Linear softening behavior where the fracture energy is calculated using "equation (9)" and the critical crack opening value w_c is taken equal to $2G_F/f_t$.
c) Power-Law softening behavior and the critical crack opening value, w_c, was equal to $5G_F/f_t$.

"Equation (14-18)" were solved according to the algorithm and the results are presented in Fig.4. From this figure it can be observed that:
- The linear softening law does not give a satisfactory prediction of the behavior in the post-cracking phase. This model predicts the smallest deflection compared to the experimental one and this is related to the critical crack opening, w_c, which is higher than the value adopted for the power law softening model.
- A sudden drop in the predicted load occurs once the section cracks when the fracture properties are neglected. After this drop, the section returns to behave in a manner similar to the experimental one.
- The power law softening "Eq. (6)" gives the closest prediction of the experimental behavior.
- When steel yields, all models converge and predict the same ductile behavior.

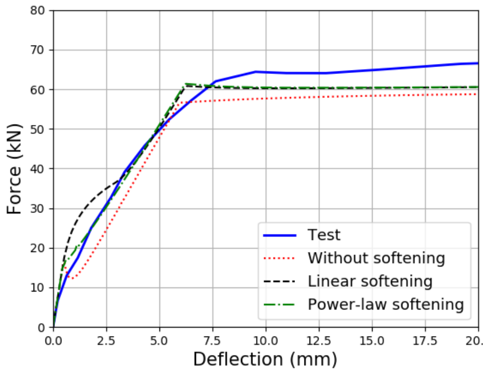

Fig.4: Predicted and experimental load-deflection curves.

5. 1 Crack Width Evolution

For each studied case the crack width is computed and the results are compared to the crack width value calculated according to EC2 [1] using "Eq. (20)". The parameters of the aforementioned equation were taken as follows:

$k_t = 0.4$, $f_{ctm} = 3.5MPa$, , $\rho = 7.54\times10^{-3}$

$s_{rm} = 200mm$, $\alpha_e = 6.3$, $E_s = 2.1\times10^{5}MPa$

It should be mentioned that the value of steel stress σ_s was taken from analytical results.
The results presented in Fig.5 show that EC2 underestimates the value of the crack width. It can be also observed that the crack width estimated using a linear strain softening curve is lower than other cases up to a load of 40 KN. The crack opening was approximately 40% smaller than values calculated without softening and with a power law softening behavior. Based on these observations, only the power law softening curve was chosen for the other beams of the database.

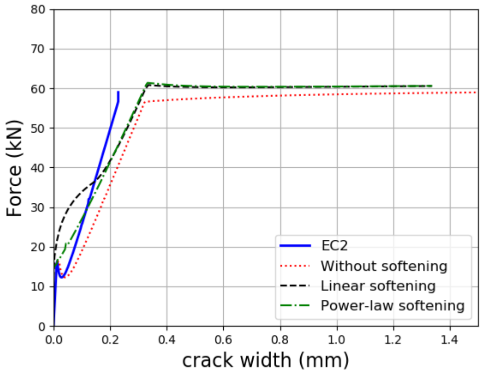

Fig.5: Analytical and EC2 crack width evolution

5. 2 Comparison between Analytical and Experimental Results

Fig.6 shows a comparison between the analytical and experimental loads at the beginning of steel yielding called F2. It can be observed that the predicted results using a power law softening behavior are close to experimental ones. The correlation coefficient R_2 is equal to 0.985 and is close to one, which means the efficiency of the proposed model in calculating the value of the yielding force F2. (Fig. 7) illustrates a comparison between the analytical and experimental deflection corresponding to F2 where a good correlation is found excepting some results. The correlation coefficient R_2=0.706 and the divergence between experimental and predicted values may be due to the scattering in the elastic modulus values tested by each researcher as well as to the inaccuracy in evaluating the fracture properties using relationships "Eq. (8-10) ".

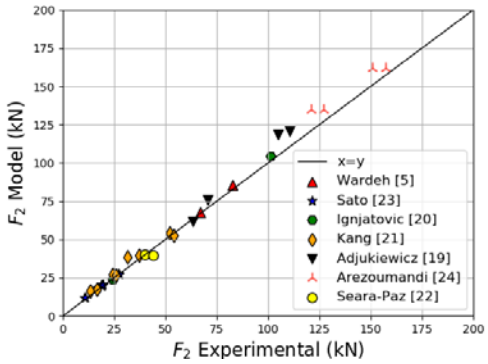

Fig.6: Experimental versus predicted yielding loads

The comparison between the crack heights measured on the crack maps and the calculated values given in Fig. 8 shows also a good agreement. However, the limited number of experimental results about crack widths did not allow for a satisfactory comparison.

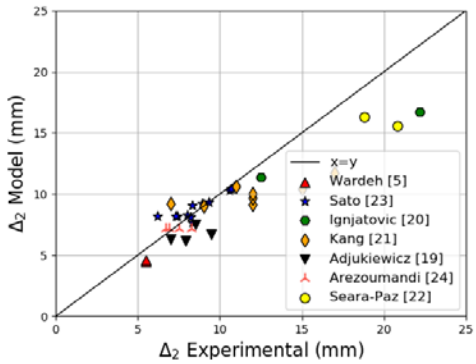

Fig.7: Experimental versus calculated yielding deflections

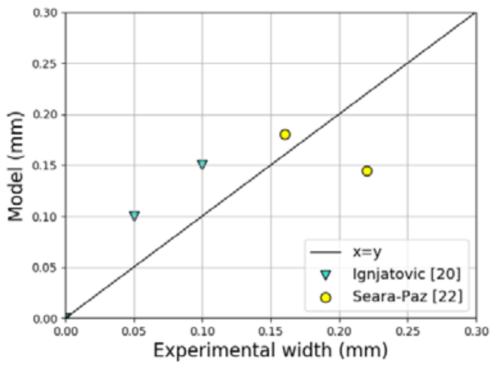

Fig.8: Experimental versus analytical crack heights

The dispersion of the results presented in figure 8 could be due to two factors. The first one is experimental and related to the difference of the measuring methods between the two references adopted. Crack widths were measured by comparing the lines of different widths drawn on the transparent foil in the work of Ignjatovic et al [20] while it was estimated based on the experimental crack spacing in the work of Seara-Paz et al. [22]. The second effect is related to the uncertainty of critical crack width which is calculated and not measured in the present work.

5. 3 Comparison between Analytical and Numerical Results

Two beams selected from Ajdukiewicz [19] and Kang [21] were modeled using ANSYS software according to the procedure described in 5.3 and the results were compared to both experimental and analytical ones. Both the proposed model and ANSYS give behavior similar to experimental one for the beam selected from Adjukiewicz [19] (Fig. 9.a). Nevertheless, a significant difference exists in the yielding stage between experimental, analytical and numerical curves. This difference also persists when a horizontal step is chosen for steel yielding.

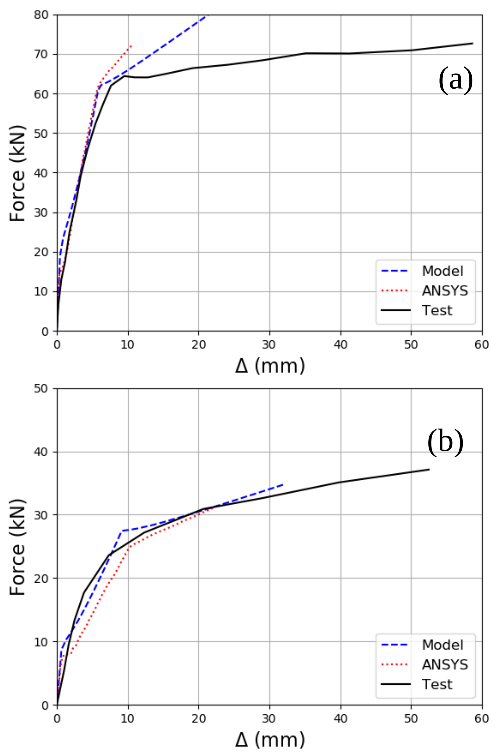

Fig.9: a) Load-deflection curves for Ajdukiewicz [19], b) Load deflection curve for Kang [21]

The results reported in Fig. 9.b concerning the beam selected from the work of Kang [21] shows a slight dispersion between ANSYS results and the test. Moreover only the inclined branch for the constitutive law of steel allows modeling the yielding phase. The results of the analytical model by adopting a power law softening curve are closer to the experimental curve.

6. CONCLUSIONS

In the present work a fracture properties based model is proposed to simulate the flexural behavior of reinforced concrete elements. The model has been validated by comparing the analytical results with the results of experimental tests selected from works available in the literature. The outputs of this model are load-deflection curve, crack opening evolution, crack height evolution as well as concrete and reinforcement stresses.
Based on the obtained results it can be concluded that the adequate prediction of flexural behavior requires the knowledge of concrete fracture properties with an adequate strain softening function beside basic mechanical properties of both concrete and steel. The power law strain softening curve is the most suitable to model the experimental behavior of beams while linear softening function gives conservative results. Moreover, the results obtained without considering concrete softening were far from the experimental results. Concerning EC2, it is too conservative in term of crack width estimation.
Further work is actually conducted to provide an efficient 1D finite element that takes into account the fracture properties of concrete. Furthermore, analytical expression for deflection and cracking control proposed by EC2 are being revised.

7. ACKNOWLEDGEMENTS

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