MICROMECHANICAL MODELING OF TENSILE STRENGTH OF SHORT RANDOM CARBON FIBER REINFORCED CONCRETE

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ABSTRACT: The accurate prediction on the response of the composite material is difficult to achieve due to the complexity of its mechanical properties. However, such complexity can be understood well using the micromechanical analysis. One of the most important goals of micromechanical analysis is to predict the failure and strength of the composite material on the basis of the geometries and properties of the matrix and the fibers. This study aims to develop a simplified micromechanical model that predicts the direct tensile strength of a randomly oriented short Carbon Fiber Reinforced Concrete (CFRC) using the modified rule of mixtures based on the assumptions that CFRC will fail by fiber failure mode and with the perfect interfacial bond between the matrix and the fibers. PAN-based High Tensile short carbon fibers distributed randomly in 3D with low fiber volume fractions (V_f) of 0.10%, 0.15%, 0.20%, 0.25% and 0.30% were used in this study. Fiber lengths (L_f) of sizes 19mm, 30mm, and 38mm were used. The designed compressive strength considered for each fiber volume fraction and fiber length was 21MPa, 28MPa, and 35MPa. There were three samples of specimens considered for each case. Each case was tested for its cylindrical compressive strength and its direct tensile strength. Test results showed that the tensile strength of CFRC was optimum at $V_f = 0.10\%$ and $L_f = 38$mm. Finally, good agreement has been observed between the experimental tensile strength and predicted tensile strength using the micromechanical model.

Keywords: Micromechanical Analysis, Tensile Strength, Short Randomly Oriented Composites, Carbon Fiber Reinforced Concrete

1. INTRODUCTION

The micromechanical analysis is developed to understand well the complex mechanical properties of the combined materials known as the composite material. Within the composite material, it deals more with the stress and strain transfer within the fibers and the matrix. The inputs in the micromechanical analysis of composites are the fiber properties, matrix properties, a geometric configuration that would result in the uniaxial strengths and fracture toughness of the composite material. However, it is believed that the central parameter in micromechanical modeling is the fiber content and the matrix [1].

The fiber's length inclusion in composite material may be classified as continuous or short. Fiber is considered short if its length is less than its critical length. For increased fiber length, the strain difference tends to become smaller until equal strain can be attained between the fibers and the matrix. At this point of equality in failure strain, the fiber length is called the critical fiber length.

Unlike the continuous fiber composites, the mechanical properties of the short-fiber composites are very different due to their fiber's orientation. The orientation of short fibers may be aligned discontinuous, off-axis aligned discontinuous, and randomly oriented discontinuous. If the fiber's orientation in a composite is random, there are two considerations that must take into account. If the fiber length is less than the thickness of the part, the fibers are randomly oriented in 3D, but if the fiber length is greater than the thickness of the part, the fibers are randomly oriented in 2D. A random discontinuous fiber composite is indeed different from those of unidirectional continuous fiber-reinforced concrete. The tensile strength of the random discontinuous fiber is at its optimum as the content of the fiber increases and then tends to decrease [2].

According to Fu and Lauke [3], the modified rule of mixtures is usually used to estimate the direct tensile strength of short-fiber composites by presuming a perfect interfacial bond among the matrix and the fibers. The modified rule of mixture is actually the extension of the simple rule of mixture in which modification focuses on factors such as fiber length and fiber orientation.

It is believed that the accurate prediction of the failure and strength of composite materials is difficult to achieve. Thus, it is necessary that the failure mechanisms both on the fiber and matrix level should be understood. Fracture of the composite happens when either the fiber or the matrix fails. There are two possible failures for typical composites as shown in Fig. 1.
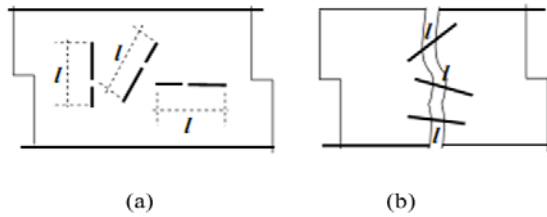
Fig. 1   Typical failures for typical composite: (a) Fiber failure mode   (b) Matrix failure mode

The first type is the fiber failure mode, where the failure strain of the matrix is larger than the failure strain of the fiber. This implies that the stress in the fiber portion at fracture must be larger than the stress in the matrix portion of the composite volume after the fracturing of all fibers so that the fibers will actually be reinforcing. On the other hand, the second type is the matrix failure mode, where the failure strain of the fiber is larger than that of the matrix. This assumes that the fiber has to carry the whole load when the matrix fails.

Some theories for unidirectional fiber-reinforced composites were derived from micromechanics simple rule of mixtures. However, the simple rule of mixtures does not apply for short randomly oriented fiber composites because it does not consider the fiber-matrix interaction effects and the fiber orientation at a given dimension (2D or 3D). From these considerations, a researcher like Curtis et al. modified the rule of mixtures by introducing factors such as fiber length efficiency factor and fiber orientation factor. The ultimate tensile strength of a short discontinuous and random composite can be calculated using the modified rule of mixtures developed by Curtis et al. [4].

2. METHODOLOGY

2.1 Materials

The composite material used in this study is Carbon Fiber Reinforced Concrete (CFRC). The carbon fiber type used was 0.111mm thick chopped PAN-based high tensile (HT) strength. The fiber lengths used in this study were 19mm, 30mm, and 38mm. The tensile strength (F_t), modulus of elasticity (E), specific gravity (SG) and other properties of the PAN-based HT carbon fibers considered are shown in Table 1.

Table 1   Properties of PAN-based HT carbon fibers

L_f (mm)	F_t (MPa)	E (MPa)	SG	Width (mm)
	4510	231000	1.8	3.0

The concrete matrix is consists of cement, fine aggregates, coarse aggregates, superplasticizer, and water. A Portland cement that meets ASTM standard specification C595 [5] was used. Crushed coarse aggregates with a maximum size of 19mm having an absorption of 1.01% and a mass density of 1592 kg/m^3 were used. Fine aggregates with a mass density of 1551 kg/m^3, and having a 3.01% water absorption and fineness modulus of 2.40 were used. A superplasticizer was added to ensure the workability of the fresh CFRC mix. The water used was of good quality.

2.2 Specimens and Testing

The different cases of fiber volume fractions (V_f) and fiber lengths (L_f) combination used in this study are presented in Table 2. Three specimens were prepared for each case of CFRC composites both in compression and tension. All these CFRC specimens were tested and compared to the plain concrete specimen.

Table 2   Cases of the CFRC specimens

Case No.	L_f, mm	V_f, %	Case No.	L_f, mm	V_f, %
1	19	0.10	24	38	0.20
2	30	0.10	25	19	0.25
3	38	0.10	26	30	0.25
4	19	0.15	27	38	0.25
5	30	0.15	28	19	0.30
6	38	0.15	29	30	0.30
7	19	0.20	30	38	0.30
8	30	0.20	31	19	0.10
9	38	0.20	32	30	0.10
10	19	0.25	33	38	0.10
11	30	0.25	34	19	0.15
12	38	0.25	35	30	0.15
13	19	0.30	36	38	0.15
14	30	0.30	37	19	0.20
15	38	0.30	38	30	0.20
16	19	0.10	39	38	0.20
17	30	0.10	40	19	0.25
18	38	0.10	41	30	0.25
19	19	0.15	42	38	0.25
20	30	0.15	43	19	0.30
21	38	0.15	44	30	0.30
22	19	0.20	45	38	0.30
23	30	0.20			

On the other hand, the carbon fibers' bond strength relative to its concrete matrix and its tensile strength was also tested and are shown in Fig. 2.

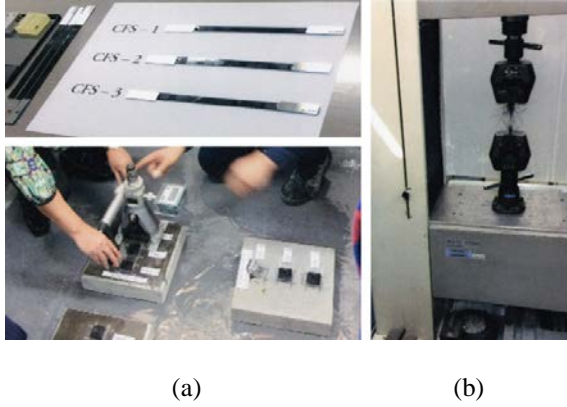(a)   (b)

Fig. 2   Tests on carbon fibers   (a) Bond strength (b) Tensile strength

The design mix of the CFRC composite was based on the compressive strengths 21MPa (Cases 1-15), 28MPa (Cases 16-30), and 35MPa (Cases 31-45) with a water-cement ratio of 0.68, 0.57 and 0.48, respectively considering the slump requirement that ranges from 25mm to 100mm, and 2% entrapped air. Moreover, partial replacements of the sand by the carbon fibers are made in the design mix. Then the workability of each mix was checked using the slump test in accordance with ASTM C143 [6].

The compressive strength specimens were tested using 100mm x 200mm cylinders in accordance with ASTM C39-05 [7] after 28 days of curing period. Likewise, the tensile strength specimens employing a dumbbell form with a critical section of 75mm x 50mm, and a gauge length of 300mm were also tested on the same curing period. The set-up for the tensile strength test and its corresponding failure mode at the critical section are shown in Fig. 3.
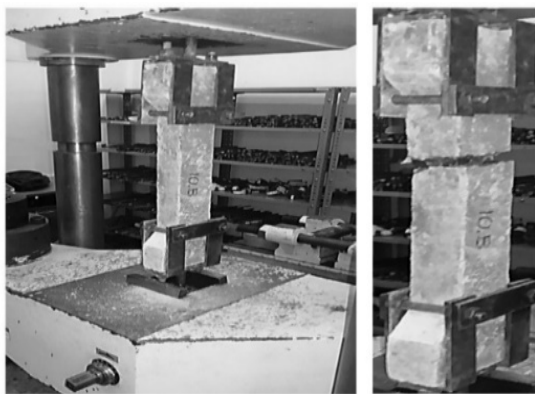

Fig. 3   Tensile strength test set up and failure mode

2.3 Micromechanical Analysis

Since the composite material involved in this study is CFRC, its ultimate tensile stress is best determined by micromechanics equation. In this study, the fiber failure mode is considered because short carbon fibers were used for CFRC. Short fibers favor a fiber failure where the matrix failure strain is assumed to be greater than the failure strain of the fiber. From a practical point of view, fiber failure means composite failure. Such fiber failure mode is based primarily on the assumption that since PAN-based carbon fibers filament has a very small diameter despite their very high tensile strength, they tend to break first before the concrete matrix does upon the tensile strength test.

The modified rule of mixtures by Curtis et al. is generally expressed as,

$$\sigma_{ult} = \chi_1 \chi_2 V_f \sigma_{fu} + \sigma_{mu} V_m \qquad (1)$$

where χ_1 denotes the effect of fiber's orientation, χ_2 denotes the effect of fiber's length. V_f is the volume of fraction percentage of fiber's inclusion, V_m is the volume of fraction percentage of the matrix, σ_{fu} is the fiber's ultimate tensile stress, and σ_{mu} is the matrix ultimate tensile stress. For three-dimensional (3D) random fiber orientation, the value of χ_1 is 0.20 [8]. While χ_2 is computed as,

$$\chi_2 = 1 - \frac{s_c}{2s} \qquad \text{when } s > s_c \qquad (2)$$

$$\chi_2 = \frac{s}{2s_c} \qquad \text{when } s < s_c \qquad (3)$$

where s denotes the fiber aspect ratio and s_c denotes the critical aspect ratio. The critical aspect ratio is the ratio between the critical fiber length l_c and the fiber diameter d. The critical fiber length can be computed as,

$$l_c = \frac{\sigma_{fu} d}{2\tau_y} \qquad (4)$$

where σ_{fu} is the fiber's tensile stress and τ_y denotes the interfacial shear stress. The diameter, d of the chopped PAN-based carbon fiber in tow is computed using the equivalent diameter equation [9] given by,

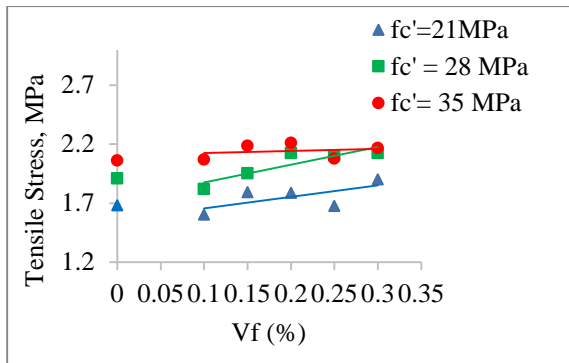$$D_e = \sqrt{\frac{4m_f}{\pi l \rho_f}} \qquad (5)$$

where m_f = mass of the chopped carbon fibers, ρ_f = mass density of chopped carbon fiber, l = total length of the sample carbon fiber used.
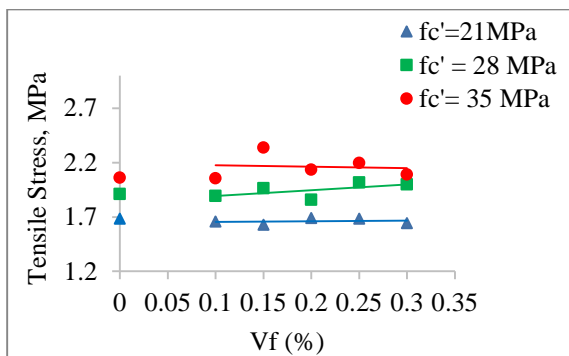
3. RESULTS AND DISCUSSIONS

3.1 Influence of Fiber Volume Fraction and Fiber Length to Tensile Strength of CFRC Composite

Generally, the fiber volume fraction strongly affects the tensile strength of the CFRC. Most of the
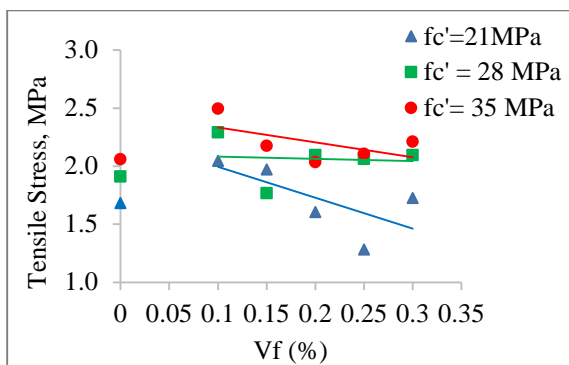
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cases of V_f and L_f mix have satisfied and significantly increased the tensile strength of CFRC in relation to their control specimens. The effect of fiber volume fraction to the tensile strength of CFRC at cases L_f = 19mm, L_f = 30mm, and L_f = 38mm are shown in Fig. 4(a), Fig. 4(b), and Fig. 4(c), respectively.

(a)

(b)

(c)

Fig. 4 Influence of fiber volume content to tensile stress of CFRC at (a) L_f =19mm (b) L_f = 30mm (c) L_f = 38mm

The addition of 19mm fiber length in concrete as shown in Fig. 4(a), the tensile strength tends to increase with increased fiber volume fraction particularly at f_c'=21MPa and at f_c'=28MPa.

With the addition of L_f =38mm as shown in Fig. 4(c), the tensile stress tends to decrease at increased fiber volume fraction in all cases of designed compressive strengths, while no trend was observed in the 30mm fiber length as shown in Fig. 4(b). However, 38mm fiber length at 0.10% fiber volume fraction has its tensile strength increased significantly to 2.047 MPa, 2.293 MPa, and 2.498 MPa by 21.63%, 19.99%, and 21.14% respectively as compared to their corresponding control specimens.

It is also noticeable from the findings that only the addition of L_f = 19mm at V_f = 0.30% and the addition of L_f = 38mm at V_f = 0.10% satisfy the compressive strength of CFRC, while most of the V_f cases for L_f = 19mm and V_f cases for L_f =38mm satisfy and significantly increased the tensile strength of CFRC. This indicates that the compressive strength of CFRC is relatively less affected by the addition of fibers as compared to the tensile strength of CFRC. This finding agrees to the report of ACI 544 [10] where the recommended fiber length of carbon fiber may vary from 5mm to 50mm, but preferably on 19mm or 38mm. In addition, Tezuka et al. [11] claimed that to achieve high strength in CFRC, use a chopped strand of carbon fibers with an average length based on the maximum aggregate size to be used as a minimum, but, more preferably not less than twice the maximum aggregate size. This refers to the fiber lengths 19mm and 38mm since the maximum aggregate size used in this study was 19mm.

3.2 Micromechanical Modeling

3.2.1 Assumptions

The micromechanics modeling of the ultimate tensile strength of the CFRC composites is based on the assumptions that the CFRC base material will fail by fiber failure mode during the tensile test and is linearly elastic until failure. It is also assumed that the fibers' strength is uniform, and the fibers and matrix are perfectly bonded at their interface. The errors are assumed to distribute normally with zero mean and variance.

In this study, the ultimate tensile strength of the CFRC composites was calculated by micromechanics analysis using the modified rule of mixtures. Although the ultimate tensile strength of a composite is difficult to achieve accurately, the modified rule of mixture is selected because it is the most appropriate for composite material at fiber failure mode. Figure 5 shows the fiber failure mode of different tensile strength specimens used in this study. During the direct tensile test, no bridging effect has been observed to any dumbbell specimens. It can be seen that most of the chopped carbon fiber breaks when the

ultimate tensile strength of the CFRC specimens is reached.

Fig. 5 Fiber failure mode of tensile strength specimens

3.2.2 Modified Rule of Mixtures

The observed tensile strength was plotted against the cases of the specimens and compared with the modified rule of mixtures equation by Curtis et al. as shown in Fig. 6. The ultimate tensile stress of the CFRC composites using this model was computed by Eq. (1). Varying parameters are χ_2 and s. χ_2 is computed using Eq. (3), and s is the ratio of L_f and d. While L_f, V_f, V_m, and σ_{mu} are the experimental data. Constant parameters such as $\chi_1 = 0.20$, $\tau_y = 2.3$ MPa, $\sigma_{fu} = 4510$MPa, $l_{fc} = 755.52$mm, d = 0.771mm were also used as part of the micromechanics modeling. The value of χ_1 is based on 3D random fiber orientation, $\tau_y = 2.3$MPa and $\sigma_{fu} = 4510$MPa are the carbon fiber properties found in Table 1, and the l_c and d values are computed using Eq. (4) and Eq. (5), respectively.
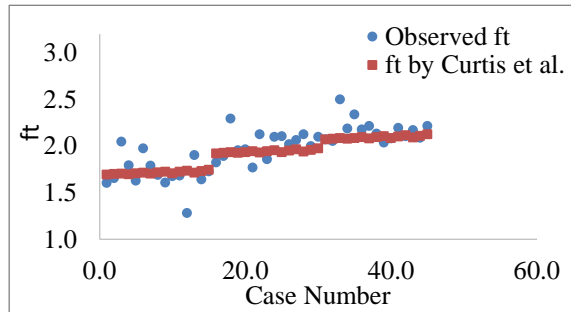

Fig. 6 Scatter plot of ultimate tensile stress for the observed and modified rule of mixtures model against cases of specimen

The Curtis et al. model was compared to the observed tensile stress as presented in the scatter plot above. Although there are cases where the tensile stress is extreme, its scatter is acceptable at the coefficient of variance (CV) of 12.23% which is less than the required maximum CV = 15%. Generally, the model appeared to have a good agreement. This only implies that the modified rule of mixture by Curtis et al. appeared to be an appropriate model for predicting the tensile strength of CFRC base material.

3.2.3 Modified Curtis et al. model

Despite good agreement observed from the plot as shown in Fig. 6, there are still some observed values that deviate largely from the modified rule of mixtures. However, most of these values increased significantly the tensile strength of CFRC. To address this increased effect, a tensile modification factor (k_m) that relates the observed and predicted values for tensile strength of CFRC was determined by

$$k_m = ft / \sigma_{ult} \qquad (6)$$

In this study, the computed tensile strength of CFRC composite is denoted by f_{tc} and is expressed as

$$f_{tc} = k_m \sigma_{ult} \qquad (7)$$

Equation 7 is considered in this study as the Modified Curtis et al. model, where k_m is the modification factor for tensile strength computed by Eq. (6) and σ_{ult} is the tensile strength derived from the modified rule of mixtures by Curtis et al. However, due to the difficulty in testing the direct tensile strength of the concrete matrix σ_{mu}, it is best expressed as a function of its compressive strength. The relation of the direct tensile strength to the compressive strength of the control specimens (f_{cm}) was denoted by k_t and expressed as

$$k_t = \sigma_{mu} \Big/ \sqrt{f_c} \qquad (8)$$

The observed (σ_{mu}) values of the direct tensile strength were utilized to calculate the numerical value of k_t as shown in Table 3.

Table 3 Results of the estimated coefficient for direct tensile strength in plain concrete

Control Case	σ_{mu}	f_{cm}	k_t
0-21	1.6833	20.17	0.375
0-28	1.9111	27.82	0.363
0-35	2.0622	35.78	0.345
		mean	0.361

Consequently, the direct tensile strength of the concrete matrix is then expressed as

$$\sigma_{mu} = 0.361\sqrt{f_{cm}} \qquad (9)$$

This estimated coefficient of compressive strength to determine the direct tensile strength of normal plain concrete is within the range cited in ACI Code 318-

89 [12], where the coefficient of $\sqrt{f_c}$ ranges 0.247 to 0.412 for f_c in MPa.

With the inclusion of Eq. (9) to the Eq. (1), it becomes

$$\sigma_{ult} = \left[\chi_1 \left(\frac{L_f}{d} \right) \tau_y V_f + 0.361\sqrt{f_{cm}} V_m \right] \qquad (10)$$

Substituting Eq. (10) to Eq. (7), the Curtis et al. model is modified to

$$f_{tc} = k_m \left[\chi_1 \left(\frac{L_f}{d} \right) \tau_y V_f + 0.361\sqrt{f_{cm}} V_m \right] \qquad (11)$$

where $k_m = 1.0245$ computed by Eq. (6) using the data from Fig. 5, $\chi_1 = 0.20$ for 3D fiber orientation, $\tau_y = 2.3$ MPa for UT70-20 PAN-based carbon fibers, and d = 0.771 mm computed from Eq. (5), and $V_m = 1-V_f$.

3.2.4 Residual Analysis

The residual output of tensile strength using the Modified Curtis et al. model is presented in Fig. 7. It is shown here the error between the observed tensile strength and predicted tensile strength computed from the Modified Curtis et al. model given by Eq. (11). Using this modified model the CV was reduced from 12.23% to 8.24%. This means that the scatter for tensile strength using the modified Curtis et al. model is acceptable.
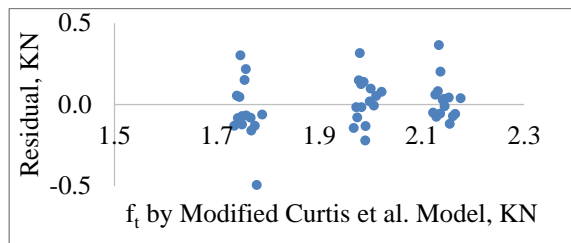

Fig. 7   Residual plot of tensile strength using a modified Curtis et al. model

The residual plot for predicted tensile strength of CFRC composites shown in Fig. 7 are clustered into three groups based on the three designed compressive strengths (21, 28, 35MPa) considered in this study, but the pattern appears to be fairly random. This means that in general, the errors are approximately normally distributed with constant variance. This implies that the modified Curtis et al. model use to predict the tensile strength of the CFRC composite follows a normal distribution and fits the observed data satisfactorily. It is noticeable that the residuals are ranging only from -0.495 to 0.364. This range of

error is obviously small. This indicates that the tensile strength of CFRC composites can be predicted satisfactorily under given limitations of the Modified Curtis et al. model.

4. CONCLUSION

Test results show that the compressive strength of CFRC in each case of designed compressive strength are consistently satisfied with the addition of $L_f =$ 38mm at $V_f = 0.10\%$. Test results also show that the tensile strength of CFRC is optimum at $V_f = 0.10\%$ and $L_f = 38$mm.

A Modified Curtis et al. model to predict the tensile strength of CFRC composite is developed based on the micromechanics equation of Modified Rule of Mixtures by Curtis et al. model. This model is significantly acceptable and can be used to predict the tensile strength of the CFRC composites, but it is applicable only for UT70-20 PAN-based chopped carbon fibers distributed randomly in 3D with low fiber volume content ranging from 0.10 % to 0.30 %. This model also applies to CFRC composites with normal compressive strength. The fiber length in CFRC composite must be limited to 19mm, 30mm, and 38mm.

5. ACKNOWLEDGMENTS

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