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INFLUENCE OF IMPERFECTION IN LENGTH AND LOADING ON DYNAMIC RESPONSE OF TRUSSES UNDER HARMONIC LOAD

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ABSTRACT: In structural engineering, trusses are important for covering large-span structures. However, imperfections are often present in the manufacture and assembly of truss structures, particularly in terms of the length and loading. This article presents the influence of imperfection, both in length and loading, on the dynamic response of trusses under harmonic loads considering geometrical nonlinearity. To analyze trusses, the hybrid matrix of elements of the truss is established to solve dynamic equations by applying the Newmark integration and Newton–Raphson iteration methods. The authors continue to develop the previous study by investigation of the influence of two imperfect parameters of structures in terms of element length and loading. An incremental-iterative algorithm was developed, and a calculation programming routine in MATLAB software was written to illustrate the dynamic responses of trusses with imperfections under harmonic loading. The results obtained in this study verified the accuracy and effectiveness of the proposed approach in the analysis of trusses under harmonic loading. The numerical results show that when considering both length and load imperfection, the dynamic response of the trusses is significantly different in comparison to the case of consideration of length imperfection separately. With consideration of both imperfect parameters in length and loading, the critical load is significantly decreased. From there, it can be concluded that, in the dynamic analysis of trusses, all possible imperfect parameters, especially in element length and loading, must be considered.

Keywords: Dynamic Response of Trusses, Hybrid Finite Element Method, Length Imperfection of Element, Loading Imperfection, Harmonic Loading.

1. INTRODUCTION

Currently, truss structures are often used in construction to overcome large spans because of their outstanding advantages of being lightweight and slender compared with other types of structures. Therefore, truss structures have become increasingly popular and widely used. Many trusse analysis studies have been performed to demonstrate the significant concern of scientists regarding truss structures [1-7]. However, in the process of manufacturing and assembling a truss structure system, errors cannot be avoided, creating various types of imperfections. One of the common types is the length imperfection. Truss systems are highly sensitive to length imperfections [7,8]. Therefore, this factor cannot be neglected in the design and analysis process. Many studies have been conducted worldwide on the influence of imperfections on the behavior of structures, as mentioned in previous studies [8-12]. It is important to note that with the influence of length imperfections, the behavior of the truss structure under the impact of dynamic loads is always of special concern. In [13-16], a significant number of studies were concerned with the problem of imperfect element lengths in structures. The authors of these studies also pointed out that the load-bearing capacity of a space truss is significantly influenced by length imperfections. Various methods have been applied to solve specific problems. It can be observed that the nonlinear dynamic analysis of trusses subjected to dynamic loading has specific difficulties in treating initial imperfections. In recent years, researchers have been interested in the dynamic response of structures in general and trusses, particularly under harmonic and impulse loads [17-19].

To analyze the influence of length imperfections, the Finite Element Method (FEM) exhibits outstanding advantages when solving dynamic analysis problems for trusses [19,20]. In linear finite element analysis, length imperfections are replaced by equivalent external loads placed at the nodes. In [8-9], Dao and Vu introduced a method to solve the problem of nonlinear dynamic analysis of a truss. In these studies, the authors used a mathematical treatment using the Lagrange multiplier method and a penalty function to address the length imperfection of the elements. In [9], Vu et al. proposed and established a new formula based on the hybrid FEM to handle systems with length imperfections. With this application, the mathematical difficulties have been overcome relatively completely. The authors have used the formulation of a Hybrid FEM in several studies. Using Hybrid FEM to solve the dynamic analysis problem for trusses subjected to dynamic loads, a system of dynamic balance equations for truss elements was established by adding inertial and damping forces to the static balance equation based on D'Alembert's principle. Simultaneously, Newmark's integration method and Newton-Raphson iteration method were used to solve the problem more thoroughly. Similarly, in [10,11], Vu et al. applied the mixed finite element method to solve the problem of nonlinear dynamic analysis of trusses. From these studies, it can be concluded that the mixed finite element method has both advantages and disadvantages.

In this article, to overcome the mathematical complexity in handling the initial member length imperfection, the authors proposed an approach based on the hybrid finite element formula, in addition to known methods, to solve the problem of analyzing the nonlinear dynamics of the truss under the effect of harmonic load. A hybrid formula was applied to establish an equilibrium nonlinear dynamic equation of the truss using the static potential energy principle. The proposed hybrid finite element of the truss with imperfect length was initially constructed based on the hybrid transformation formulation, considering large truss deformations. In addition, this study applies an incremental iteration algorithm based on a combination of Newmark's integration and Newton–Raphson iteration methods. Based on the algorithm proposed by the authors, the calculation process was established and written in MATLAB to illustrate the dynamic response of trusses with initial length imperfections under the effect of harmonic loading. The numerical analysis results demonstrate the effectiveness of the hybrid finite element formula in solving the nonlinear dynamic problem of a truss system with length imperfections under the effect of a dynamic load. The results obtained from this study demonstrate that when the number of length defects approaches zero, the solution converges to the perfect length case.

In addition, this study continued to develop the topic mentioned in a previous study by Dao et al. [8]. In [8], Dao et al. addressed the research issue of the influence of length imperfections on the dynamic response of space trusses while considering only length imperfections. In this study, we investigated the influence of both imperfect parameters in terms of length and loading. The obtained results illustrate the necessity of considering all the possible imperfections. Therefore, in the next section, the authors will present the method and numerical investigation to present the influence of both imperfection parameters in length and loading to dynamic response of truss under harmonic load.

2. RESEARCH SIGNIFICANCE

The influence of both imperfect parameters on truss structures under harmonic loading was investigated. The approach proposed in this study shows certain advantages in solving the dynamic analysis of trusses, particularly in cases where the trusses have length and loading imperfections. In addition, the results obtained in this study show that it is necessary to consider more parameters of imperfections than only one parameter in the dynamic analysis of trusses.

3. ANALYSIS METHOD

3.1 Theoretical background

3.1.1. Equilibrium equation for truss element with length imperfection

In Fig. 1, the nodal coordinates of truss elements before deformation are $\{X_1, Y_1\}$ and $\{X_2, Y_2\}$, L_e, Δ_e, are initial element length and imperfection parameter; L_0 and L are distances between nodes before and after loading; A, E, N are cross section area, elastic modulus and axial load of element; f_i, u_i, m_i (i=1– 4) are nodal forces, displacements and lumped masses; f_I, f_D, p(t) are inertia, damped and external forces. In hybrid formulation, the authors considered e_I and e_{II} are perfect and imperfect elements [8].

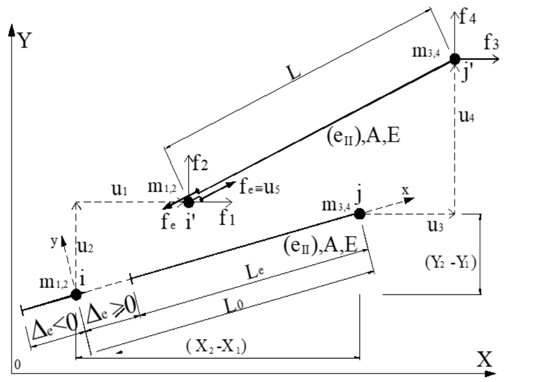

Fig. 1 Truss element with length imperfection

From condition of equilibrium, it is assumed that, $u_5=f_e=N$, in which u_5 and f_e are force unknown and external force in the initial node after loading. The deformations of perfect and imperfect elements are determined as:

$$\varepsilon_x^{(e_I)} = \frac{\Delta L^{(e_I)}}{L_0}; \varepsilon_x^{(e_{II})} = \frac{\Delta L^{(e_{II})}}{L_e}; \tag{1}$$

$$\Delta L^{(e_I)} = L - \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}; \quad \Delta L^{(e_{II})} = L - \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2} + \Delta_e. \tag{2}$$

The internal and external virtual work for perfect elements $\delta W_{int}^{(e_I)}$, $\delta W_{ext}^{(e_I)}$ and imperfect elements $\delta W_{int}^{(e_{II})}$, $\delta W_{ext}^{(e_{II})}$ can be calculated as follows [8]:

$$\delta W_{int}^{(e_I)} = -N\left\{\sum_{i=1}^{4} \frac{\partial \Delta L}{\partial u_i}\delta u_i\right\}; \quad \delta W_{int}^{(e_{II})} = -N\left\{\sum_{i=1}^{4} \frac{\partial \Delta L}{\partial u_i}\delta u_i + \frac{\partial \Delta L}{\partial \Delta_e}\delta \Delta_e\right\}. \tag{3}$$

$$\delta W_{ext}^{(e_I)} = \sum_{i=1}^{4} f_i \ \delta u_i;$$
$$\delta W_{ext}^{(e_{II})} = \sum_{i=1}^{4} f_i \ \delta u_i + f_e \delta \Delta_e. \quad (4)$$

The total work of the system for two types of elements is given by following:

$$\delta W_{int}^{(e_I)} + \delta W_{ext}^{(e_I)} = \sum_{i=1}^{4} \left\{ -N \frac{\partial \Delta L}{\partial u_i} + f_i \right\} \delta u_i = 0;$$
$$\delta W_{int}^{(e_{II})} + \delta W_{ext}^{(e_{II})} = \sum_{i=1}^{4} \left\{ -N \frac{\partial \Delta L}{\partial u_i} + f_i \right\} \delta u_i \quad (5)$$
$$+ \left\{ -N \frac{\partial \Delta L}{\partial \Delta_e} + f_e \right\} \delta \Delta_e = 0.$$

The dynamic equation of equilibrium for perfect and imperfect elements can be presented as follows [8]:

$$\begin{cases} m_i \ddot{u}_i + c_i \dot{u}_i + q_i(\mathbf{u}^{(e_I)}) = P_i \ ; i = 1-4; \\ \mathbf{u}^{(e_I)} = \{u_1, u_2, u_3, u_4\}^T. \end{cases}$$
$$\begin{cases} m_k \ddot{u}_k + c_k \dot{u}_k + q_k(\mathbf{u}^{(e_{II})}, \Delta_e) = P_k \ ; k = 1-5; \\ \mathbf{u}^{(e_{II})} = \{u_1, u_2, u_3, u_4, u_5 \equiv f_e\}^T. \end{cases} \quad (6)$$

In which, $\mathbf{u} \equiv \{\mathbf{u}^{(eI)}, \mathbf{u}^{(eII)}\}$ is vector of unknowns.
In matrix form, the dynamic equilibrium equation for truss element can be written as

$$\mathbf{M}\ddot{\boldsymbol{u}} + \mathbf{C}\dot{\boldsymbol{u}} + \mathbf{q}(\boldsymbol{u}, \Delta_e) = \mathbf{P} + \mathbf{P}_{imperfect} \quad (7)$$

The dynamic equation of equilibrium (7) is a second-order non-linear differential equation.
In this study, the authors applied the formula of Taylor series to expand function of Eq. (7):

$$\mathbf{M}\delta\ddot{\boldsymbol{u}} + \mathbf{C}\delta\dot{\boldsymbol{u}} + \frac{\partial \mathbf{q}(\boldsymbol{u}, \Delta_e)}{\partial \boldsymbol{u}} \delta \boldsymbol{u} = \Delta\mathbf{P} + \Delta\mathbf{P}_{imperfect} \quad (8)$$
$$\text{or } \mathbf{M}\delta\ddot{\boldsymbol{u}} + \mathbf{C}\delta\dot{\boldsymbol{u}} + \boldsymbol{k}^{(e_I;e_{II})}(\boldsymbol{u}, \Delta_e)\delta\boldsymbol{u} = \Delta\mathbf{P} + \Delta\mathbf{P}_{imperfect}.$$

In which,

$$\boldsymbol{k}^{(e_I)}(\boldsymbol{u}) = \frac{\partial \boldsymbol{q}(\boldsymbol{u}, \Delta_e)}{\partial \boldsymbol{u}^{(e_I)}}; \boldsymbol{k}^{(e_{II})}(\boldsymbol{u}, \Delta_e) = \frac{\partial \boldsymbol{q}(\boldsymbol{u}, \Delta_e)}{\partial \boldsymbol{u}^{(e_{II})}} \quad (9)$$

M, C, ΔP, ΔP$_{imperfect}$ are the mass and damping matrices and vectors of incremental dynamic load and incremental imperfect dynamic, respectively,
$\delta\ddot{\boldsymbol{u}}$, $\delta\dot{\boldsymbol{u}}$, $\delta\mathbf{u}$ are the vectors of incremental acceleration, velocity and displacements.

3.1.2. Formulation of equation for truss system with length and loading imperfection

From Eqs. (7), (8), assemble all element matrices to global matrices of the truss system. The dynamic equilibrium equation can be express as following:

$$\mathbf{M}\ddot{\boldsymbol{u}} + \mathbf{C}\dot{\boldsymbol{u}} + \mathbf{q}(\boldsymbol{u}, \Delta_e) = \mathbf{P} + \mathbf{P}_{imperfect} \quad (10)$$
$$\mathbf{M}\delta\ddot{\boldsymbol{u}} + \mathbf{C}\delta\dot{\boldsymbol{u}} + \boldsymbol{K}(\boldsymbol{u}, \Delta_e)\delta\boldsymbol{u} = \Delta\mathbf{P} + \Delta\mathbf{P}_{imperfect} \quad (11)$$

In which,

$$\begin{cases} \boldsymbol{u} \equiv \{u_1, u_2, ..., u_n\}^T; \delta\boldsymbol{u} \equiv \{\delta u_1, \delta u_2, ..., \delta u_n\}^T; \\ \mathbf{M}_{i,j} = \sum_{e=1}^{m} \mathbf{M}_{i,j}^{(e)}; \mathbf{C}_{i,j} = \sum_{e=1}^{m} \mathbf{C}_{i,j}^{(e)}; \\ \mathbf{K}_{i,j}(\boldsymbol{u}, \Delta_e) = \sum_{e=1}^{m} \boldsymbol{k}_{i,j}^{(e)}(\boldsymbol{u}, \Delta_e), \ (i, j = 1-n); \\ \mathbf{q}(\boldsymbol{u}, \Delta_e) \equiv \{\boldsymbol{q}_1(\boldsymbol{u}, \Delta_e), \boldsymbol{q}_2(\boldsymbol{u}, \Delta_e), ..., \boldsymbol{q}_n(\boldsymbol{u}, \Delta_e)\}^T; \\ \mathbf{P} \equiv \{\boldsymbol{P}_1, \boldsymbol{P}_2, ..., \boldsymbol{P}_n\}^T; \Delta\boldsymbol{P} \equiv \{\Delta\boldsymbol{P}_1, \Delta\boldsymbol{P}_2, ..., \Delta\boldsymbol{P}_n\}^T; \\ \boldsymbol{P}_i = \sum_{e=1}^{m} \boldsymbol{P}_i^{(e)}; \ \Delta\boldsymbol{P}_i = \sum_{e=1}^{m} \Delta\boldsymbol{P}_i^{(e)}. \end{cases} \quad (12)$$

To solve the Eqs. (7), (8), in this study, the Newmark's method was applied. The dynamic equation can be expressed in the incremental form:

$$\begin{cases} \delta\ddot{\boldsymbol{u}} = \frac{1}{\beta\Delta t^2}\delta\boldsymbol{u} - \frac{1}{\beta\Delta t}\dot{\boldsymbol{u}} - \frac{1}{2\beta}\ddot{\boldsymbol{u}}; \\ \delta\dot{\boldsymbol{u}} = \frac{\gamma}{\beta\Delta t}\delta\boldsymbol{u} - \frac{\gamma}{\beta}\dot{\boldsymbol{u}} - \left(\frac{\gamma}{2\beta} - 1\right)\Delta t \ddot{\boldsymbol{u}}. \end{cases} \quad (13)$$

Thus, $\delta\ddot{\boldsymbol{u}}, \delta\dot{\boldsymbol{u}}$ are added from Eq. (8) to Eq. (7):

$$\underbrace{\left[\mathbf{K}(\boldsymbol{u}, \Delta_e) + \frac{\mathbf{M}}{\beta\Delta t^2} + \frac{\gamma\mathbf{C}}{\beta\Delta t}\right]}_{\bar{\mathbf{K}}(\boldsymbol{u}, \Delta_e)} \delta\boldsymbol{u}$$
$$= \underbrace{\Delta\mathbf{P} + \mathbf{M}\left(\frac{1}{\beta\Delta t}\dot{\boldsymbol{u}} + \frac{1}{2\beta}\ddot{\boldsymbol{u}}\right) + \mathbf{C}\left[\frac{\gamma}{\beta}\dot{\boldsymbol{u}} + \left(\frac{\gamma}{2\beta} - 1\right)\Delta t \ddot{\boldsymbol{u}}\right]}_{\Delta\bar{\mathbf{P}}} \quad (14)$$

Eq. (14) can be written in compact form as:

$$\bar{\mathbf{K}}(\boldsymbol{u}, \Delta_e)\delta\boldsymbol{u} = \Delta\bar{\mathbf{P}} \quad (15)$$

The authors applied Newton-Raphson iterative method to find the solution of Eq. (15).

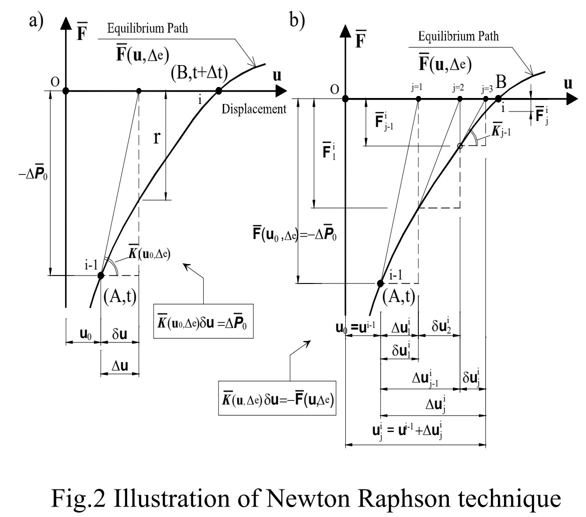

Fig.2 Illustration of Newton Raphson technique

3.2 Solving procedure for dynamic analysis of truss system with length imperfection

The iterative algorithm of Newton-Raphson approach [8-11, 21] is applied for solving the system of equation nonlinear dynamic of the trusses Eq. (14)

as shown in Fig. 2a.

The adjacent equilibrium point (B, t+Δt) is determined from the equilibrium point (A, t). For every iteration, the increments from residual force are great, to eliminate the residual force and implement in many iterations as shown in Fig 2b. The authors built an iterative Eq. (16), Eq. (15) is a particular case corresponds to the first iteration of Eq. (16). The iterative algorithm established based on mentioned methods is applied to write a program to calculate the dynamic parameters of truss systems with imperfection.

According to Newton-Raphson iterative algorithm [8], the increments of are expressed as follows:

$$\frac{\partial \bar{\mathbf{F}}(\boldsymbol{u},\Delta_e)}{\partial \boldsymbol{u}}\delta\boldsymbol{u} = -\bar{\mathbf{F}}(\boldsymbol{u},\Delta_e) \tag{16}$$

Where, $\bar{\boldsymbol{F}}(\boldsymbol{u},\Delta_e)$ is found from the condition:

$$\begin{cases} \dfrac{\partial \bar{\mathbf{F}}(\boldsymbol{u},\Delta_e)}{\partial \boldsymbol{u}} = \bar{\boldsymbol{K}}(\boldsymbol{u},\Delta_e); \\ -\bar{\mathbf{F}}(\boldsymbol{u}_0,\Delta_e) = \Delta\bar{\boldsymbol{P}}_0 \equiv \Delta\bar{\boldsymbol{P}}\big|_{\mathbf{u}=\mathbf{u}_0}. \end{cases} \tag{17}$$

From Eq. (16) and refers to Eqs. (9), (14), (15), vector $\bar{\boldsymbol{F}}(\boldsymbol{u},\Delta_e)$ can be obtained as following:

$$\begin{aligned} \bar{\mathbf{F}}(\boldsymbol{u},\Delta_e) &= \left\{ \boldsymbol{q}(\boldsymbol{u},\Delta_e) + \frac{\boldsymbol{M}}{\beta\Delta t^2}\boldsymbol{u} + \frac{\gamma.\boldsymbol{C}}{\beta\Delta t}\boldsymbol{u} \right\} \\ &- \left\{ \boldsymbol{q}(\boldsymbol{u},\Delta_e)\big|_{\mathbf{u}=\mathbf{u}_0} + \frac{\boldsymbol{M}}{\beta\Delta t^2}\boldsymbol{u}_0 + \frac{\gamma.\boldsymbol{C}}{\beta\Delta t}\boldsymbol{u}_0 \right\} \\ &- \left\{ \begin{array}{l} \Delta\boldsymbol{P} + \boldsymbol{M}\left(\dfrac{1}{\beta\Delta t}\dot{\boldsymbol{u}}_0 + \dfrac{1}{2\beta}\ddot{\boldsymbol{u}}_0 \right) \\ +\boldsymbol{C}\left[\dfrac{\gamma}{\beta}\dot{\boldsymbol{u}}_0 + \left(\dfrac{\gamma}{2\beta} - 1 \right)\Delta t\,\ddot{\boldsymbol{u}}_0 \right] \end{array} \right\}. \end{aligned} \tag{18}$$

Eq. (16) can be re-written as follows:

$$\bar{\boldsymbol{K}}(\boldsymbol{u},\Delta_e)\delta\boldsymbol{u} = -\bar{\mathbf{F}}(\boldsymbol{u},\Delta_e) \tag{19}$$

After finding the displacement and the increment of displacement (u+Δu) at the point B(t+Δt) in Fig. 2, using Newmark's formula [21], the corresponding velocity and acceleration will be found as Eq. (20):

$$\begin{cases} \ddot{\boldsymbol{u}} = \ddot{\boldsymbol{u}}_0 + \Delta\ddot{\boldsymbol{u}} = \ddot{\boldsymbol{u}}_0 + \dfrac{1}{\beta\Delta t^2}\Delta\boldsymbol{u} - \dfrac{1}{\beta\Delta t}\dot{\boldsymbol{u}}_0 - \dfrac{1}{2\beta}\ddot{\boldsymbol{u}}_0 \\ \dot{\boldsymbol{u}} = \dot{\boldsymbol{u}}_0 + \Delta\dot{\boldsymbol{u}} = \dfrac{\gamma}{\beta\Delta t}\Delta\boldsymbol{u} - \dfrac{\gamma}{\beta}\dot{\boldsymbol{u}}_0 - \left(\dfrac{\gamma}{2\beta} - 1 \right)\Delta t\,\ddot{\boldsymbol{u}}_0. \end{cases} \tag{20}$$

4. NUMERICAL EXAMPLE

The 35-element plane truss in the form of an arch was investigated under harmonic loading (in form of sinusoidal load) and static concentrated P at node 10, as shown in Fig. 3. in which considering imperfections in both the length of the element and loading as shown in Fig. 4. The elements had the same elastic modulus, $E=68.964\times10^6$ kN/m^2. The node coordinates of the truss and cross-sectional area of the elements are presented in Tables 1 and 2, respectively. The truss with an imperfect length of elements subjected to imperfect dynamic loads is shown in Fig. 4. The imperfection factor of loading is e=0.01. The dynamic load in the form of sinusoidal load (T_d = 0.3sec). For each truss node with a lumped mass m=50 kg (except for two nodes in the support), the influence of damping was neglected, and the concentrated P was applied at node 10.

The amount of imperfection in the length of the elements is considered in elements (1) to (10) and is assumed to be one-thousandth the length of the shortest element (in this example, the shortest element is 381 cm in length). Thus, $\Delta_e^{(1)\div(10)}$=0.381cm and other remain elements are assumed perfect $\Delta_e^{(11)\div(35)}$=0cm.

The obtained results are compared with the case of imperfection in term of loading and length separately, as shown in Fig. 5, Fig. 6 and Fig. 7.

Table 1. Coordinates of each node of the arch-truss

Nodal number	*X* - Coordinate (cm)	*Y*- Coordinate (cm)
19, 1	± 3429.0	0.00
18, 2	± 3048.0	50.65
17, 3	± 2667.0	34.75
16, 4	± 2286.0	83.82
15, 5	± 1905.0	65.30
14, 6	± 1524.0	110.85
13, 7	± 1143.0	87.99
12, 8	± 762.0	128.50
11, 9	± 381.0	100.65
10	0.0	134.62

Table 2. Cross-section areas of each member of the arch-truss

Member's number	Cross-section area (cm^2)
1–10, 35	51.61
11, 12	64.52
13–16	83.87
17, 18	96.77
19–22	103.23
23, 24	161.29
25, 26	193.55
27, 28	258.06
29–32	290.32
33, 34	309.68

To develop the research in [8] of Dao et al., in this article, the 35-element plane arch-truss will be studied in the following cases:

Case 1: A truss with all perfect length of elements subjected to imperfect harmonic dynamic load is shown in Fig. 3. The imperfection factor of loading e=0.01. The dynamic load in the sinusoidal form (with period of loading T_d = 0.3sec). For each truss node with a lumped mass m=50 kg (except for two nodes in the support), the influence of damping was neglected, and the concentrated P was applied at node 10.

Case 2: A truss with imperfect length of elements and subjected to perfect harmonic dynamic load, as shown in Fig. 3. The amount of imperfection in the length of the elements is considered in elements (1) to (10) and is assumed to be one-thousandth the length of the shortest element (in this example, the shortest element is 0.381 cm in length). Thus, $\Delta_e^{(1)\div(10)}$=0.381cm and other remain elements are assumed perfect $\Delta_e^{(11)\div(35)}$=0cm. The form of dynamic is sinusoidal, the period of dynamic load is T_d=0.3s.

Case 3: A truss with an imperfect length of elements subjected to imperfect harmonic dynamic load, as shown in Fig. 3. The amount of imperfection in the length of the elements is considered in elements (1) to (10) and is assumed to be one-thousandth the length of the shortest element (in this example, the shortest element is 0.381 cm in length). Thus, $\Delta_e^{(1)\div(10)}$=0.381cm and other remain elements are assumed perfect $\Delta_e^{(11)\div(35)}$=0cm. The imperfection factor of loading e=0.01. The dynamic load in the sinusoidal form (T_d = 0.3sec). For each truss node with a lumped mass m=50 kg (except for two nodes in the support), the influence of damping was neglected, and the concentrated P was applied at node 10.

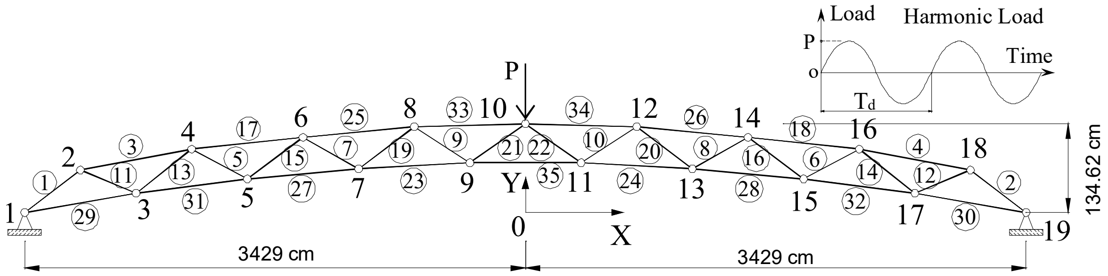

Fig. 3 Arch-truss structure

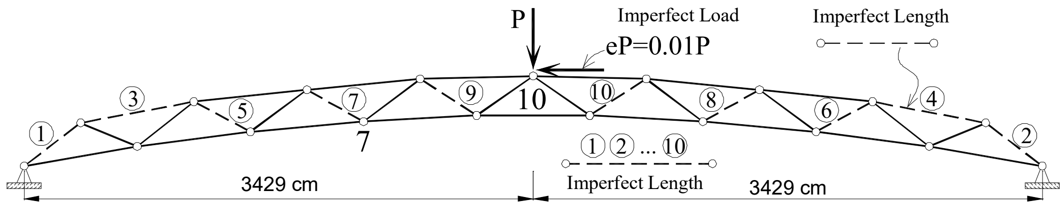

Fig. 4 The arch-truss structure under ‘imperfect length and load’

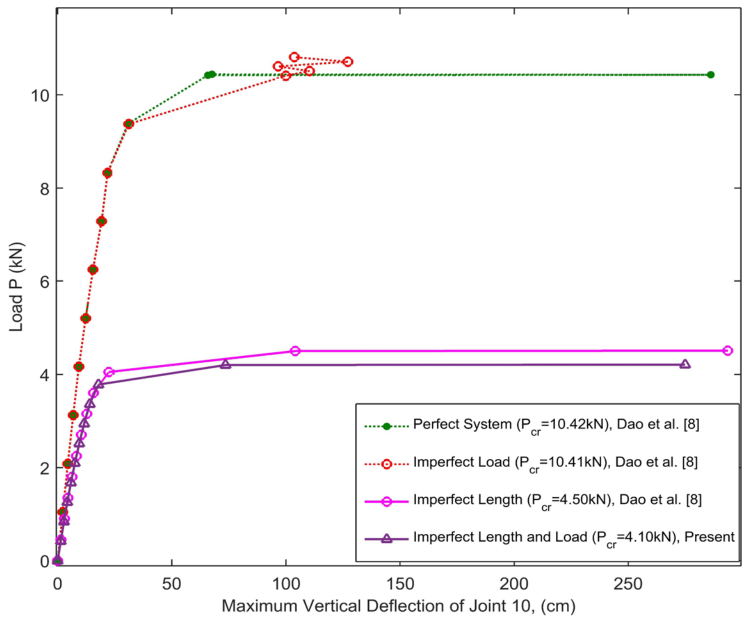

Fig. 5 Maximum vertical displacements at node 10 under harmonic load

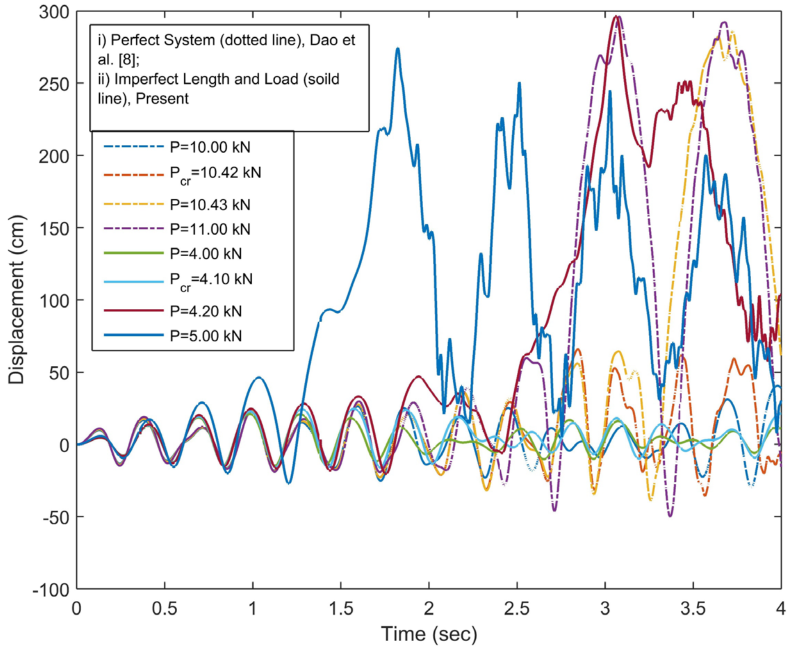

Fig. 6 Transient responses of the vertical displacement of the node 10 for different magnitudes of sinusoidal load ('Imperfect Length and Loading' and 'Perfect System')

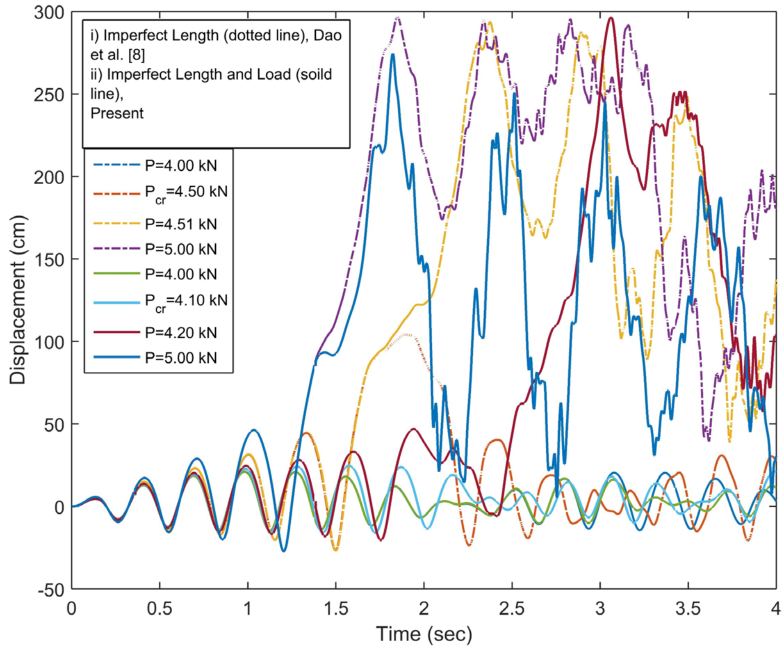

Fig. 7 Transient responses of the vertical displacement of the node 10 for different magnitudes of sinusoidal load ('Imperfect Length and Loading' and 'Imperfect Length')

When analyzing cases 1, 2, 3, the results are compared with the case of 'Perfect System' in term in length and loading and compared between cases to clearly observe how the imperfection in length of elements and imperfection in loading affects the static behavior as well as the dynamic behavior of the truss.

Fig. 5 shows the relationship between load P and the maximum vertical deflection at node 10. The dynamic critical load value obtained in the case of a perfect system is 10.42kN, which converges with the value published in [8] by Dao et al. is 10.42kN. In Case 1, the imperfection in length was investigated, and the dynamic critical value obtained was 4.5kN. In case 2, with imperfections in loading, the dynamic critical value was 10.41kN, and in case 3, when we considered both imperfect parameters in length and loading, the dynamic critical value obtained was only 4.1kN. Thus, the critical value in case 3, which considered both imperfect parameters in length and loading, was 39.39% compared to case 2, which considered only length imperfection and 39.42% for the perfect system. Therefore, it can be seen that the influence of length imperfections is greater than that of imperfections in loading. In other words, it was demonstrated that imperfections in the loading insignificantly influenced the critical load value. When both imperfection parameters are considered, the influence is significantly increased. Thus, it can be concluded that it is necessary to consider all possible imperfection parameters.

The harmonic dynamic load in the form of a sinusoidal load, and the dynamic response of the truss varies in time corresponding to different values of load amplitude, was investigated for the cases of consideration of the imperfect length and both imperfect parameters in length and loading.

Fig. 6 presents a comparison of the dynamic response of the investigated truss between two cases: the perfect system and the system with perfection in both length and loading with different values of loads: a) the load value is less than the critical load value, b) the load value is equal to the critical load, and c) the load value is greater than the critical load. Fig. 6 shows the bifurcation of the vibration that occurs when the load value is greater than the critical value. The time-displacement response curves clearly bifurcated with increasing time.

Fig. 7 presents a comparison of the dynamic response of the investigated truss between two cases: a) a truss with imperfect length and a truss with imperfection in both length and loading with different load values: a) the load value is less than the critical load value, b) the load value is equal to the critical load, and c) the load value is greater than the critical load. Fig. 7 demonstrates that in the case of a sinusoidal load, corresponding to the load values of 4.2kN and the time-displacement curves bifurcated at time 03s. This demonstrates the accuracy of the proposed method and the necessity of considering all possible imperfection parameters.

5. CONCLUSIONS

The following conclusions were drawn from the results of this study:

The mathematical model based on the hybrid finite element formulation to solve the nonlinear dynamic problems of trusses with imperfect parameters in both length and loading shows outstanding advantages compared to the mathematical model based on displacement and mixed finite element formulation.

The hybrid finite element formulation allows the application of displacements and forces as unknowns, which allows the imperfection parameters of length and loading into the hybrid matrix of truss elements and simplifies the calculation algorithm for nonlinear dynamic analysis of trusses under harmonic loading.

The Hybrid FEM method proposed in this study was applied to perform the calculation and analysis of the truss, considering the factors of imperfections in the length of the elements and the load. The convergence of the numerically obtained results of this study compared to those of previous studies demonstrates the reliability and efficiency of the Hybrid FEM method proposed by the authors.

The dynamic forced function is considered as a sinusoidal load. The 'Load-Maximum deflection of Joint' curves for this load cases are presented and the influence of both imperfect parameters of length of element as well as loading on the critical load value of the truss has been analyzed.

The numerical results show that the imperfect length of the elements significantly affects the dynamic response as well as the magnitude of the critical load, specifically,

When considering the imperfection in both the length of the elements and loading, the critical load value will decrease in comparison with the case of the truss considering only one imperfection factor.

Therefore, imperfections in the length and loading of the elements must be considered in the practical analysis of trusses. The numerical results presented in this study show a certain significance level in developing a further understanding of the buckling behavior of truss structures under dynamic loading and the imperfection factors of the length of elements as well as of loads.

6. CONFLICT OF INTEREST STATEMENT

On behalf of all the authors, the corresponding author states that there are no conflicts of interest.

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