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THE MODIFIED DECOMPOSITION METHOD FOR SOLVING VOLTERRA FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS USING MAPLE

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ABSTRACT: In this paper, Vito Volterra studied the phenomenon of population growth, and new types of equations have been developed and termed as the integro–differential equation. Here we will address the Mixed Volterra Fredholm integro-differential equation. We introduce some basic idea of Modified Decomposition method (MDM) to solve the Volterra-Fredholm integro-differential equation. The modified decomposition method (MDM) is applied to solve the Volterra Fredholm integro-differential equation. Based on the proposed after treatment technique, the truncated series solution obtained by the Modified Decomposition method (MDM) can be expressed as another series in terms of the independent sine and cosine trigonometric functions. In addition, examples that illustrate the pertinent features of this method is presented, and results of the study is discussed. Also, we compare the result with exact solution, and we investigate the error between numerical solution and exact solution and draw the graph of function by using Maple18. Finally, the results reveal that the Modified Decomposition method is very effective and convenient for solving Volterra Fredholm integro-differential equation.

Keywords: Volterra Fredholm Integro-Differential equation, Modified Decomposition method, Maple18 .

1. INTRODUCTION

Integrated stems from the mathematical modeling of many complex real-life problems of differential Has been formulated many scientific .equations phenomena using integrated differential equations Solution integrative non-linear equation is .[5.6] much more difficult than the linear equation of the Therefore, the use of different types of .analytical numerical methods to obtain an approximate solution effective [1-4]. Numerical solution of Volterra integral equation of second kind using Implicit Trapezoidal [7,11]. Adomian Decomposition method of Fredholm integral equation of the second kind using Maple and MATLAB [8,10].Application of Adomian Decomposition method for solving of Fredholm Integral Equation of the Second Kind [9].

The Modified Decomposition Method for Solving Volterra Integral Equation of the Second Kind Using Maple [12].

The main goal in this article is to apply the Modified Decomposition method (MDM)to the Volterra Fredholm integro-differential equation using Maple and then compare the exact solution results with the approximate results obtained from the algorithm Maple. By showing some examples we see how this method gives the best results.

2. THE MODIFIED DECOMPOSITION METHOD

To illustrate the basic idea of this method, we consider the following general non-linear differential equation:

$$u^{(n)}(x) = f(x) + \lambda_1 \int_a^x K_1(x,t)\, u(t)dt + \lambda_2 \int_a^b K_2(x,t)\, u(t)dt. \quad (1)$$

And the mixed form

$$u^{(n)}(x) = f(x) + \lambda \int_0^x \int_a^b K(r,t)\, u(t)dtdr, \quad (2)$$

where $u^{(n)}(x) = \frac{d^n(x)}{dx^n}$.

$$u(x) = \sum_{k=0}^{n-1} \frac{1}{k!} b_k u^k + L^{-1}\big(f(x)\big) + L^{-1}\left(\lambda \int_0^x \int_a^b K(r,t)\, u(t)dtdr\right), \quad (3)$$

where L is assumed invertible and L^{-1} is an

The standard Adomian method defines the solution $u(x)$ by the series $u(x) = \sum_{n=0}^{\infty} u_n(x)$.

The modified decomposition method

$$u_0(x) = f_1(x)$$

$$u_1(x) = f_2(x) + L^{-1}\left(\lambda \int_0^x \int_a^b K(r,t)\, u_0(t)dtdr\right),$$

$$\boldsymbol{u_{n+1}(x) = L^{-1}\left(\lambda \int_0^x \int_a^b K(r,t)\, u_n(t)dtdr\right)}. \quad (4)$$

The use of the modified decomposition method not only minimizes the computations but avoids the use of the higher order Adomian polynomials for such cases.

3. EXAMPLE

Example1. Consider the Volterra Fredholm integro differential equation

$$u'(x) = \frac{1}{4}x^2 - e^{-x} + \int_0^x \int_0^1 rtu(t)dtdr, \quad u(0) = 0$$

Applying the Modified Decomposition Method using Maple we find

Table 1 Numerical results and exact solution of Volterra Fredholm integro differential equation for example 1

x	$u(x)$#	$Exact = 1 - e^x$	$Error$
0.10000	-0.1051709	-0.1051710	0.0000001
0.20000	-0.2214028	-0.2214035	0.0000007
0.30000	-0.3498588	-0.3498613	0.0000025
0.40000	-0.4918247	-0.4918306	0.0000059
0.50000	-0.6487213	-0.6487328	0.0000116
0.60000	-0.8221188	-0.8221388	0.0000200
0.70000	-1.0137527	-1.0137845	0.0000318
0.80000	-1.2255409	-1.2255883	0.0000474
0.90000	-1.4596031	-1.4596706	0.0000675
1.00000	-1.7182818	-1.7183744	0.0000926

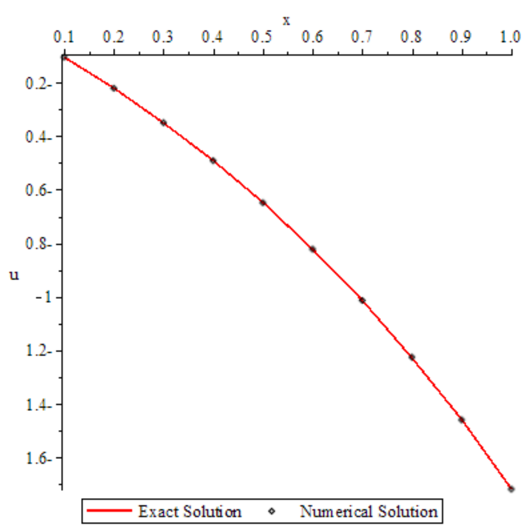

Fig.1 Plot 2D of the exact solutions result Of Volterra Fredholm integro differential equation for example 1.#

Example2. Consider the Volterra integral equation of second kind

$$u''(x) = -15x + \int_0^x \int_0^1 rtu(t)dtdr,$$

$$u(0) = 1, u'(0) = 0.$$

#

Applying the Modified Decomposition Method using Maple we find

Table 2 Numerical results and exact solution of Volterra Fredholm integro differential equation for example 2

x	$u(x)$#	$Exact = 1 - (\frac{5}{2})x^3$	$Error$
0.10000	0.9975000	0.9975000	0.0000000
0.20000	0.9800000	0.9800000‡	0.0000000
0.30000	0.9325000‡	0.9325000	0.0000000
0.40000	0.8400000	0.8400000	0.0000000
0.50000	0.6875000	0.6875000	0.0000000
0.60000	0.4600000	0.4600000	0.0000000
0.70000	0.1425000	0.1425000	0.0000000
0.80000	-0.2800000	-0.2800000	0.0000000
0.90000	-0.8225000	-0.8225000	0.0000000
1.00000	-1.5000000	-1.5000000	0.0000000

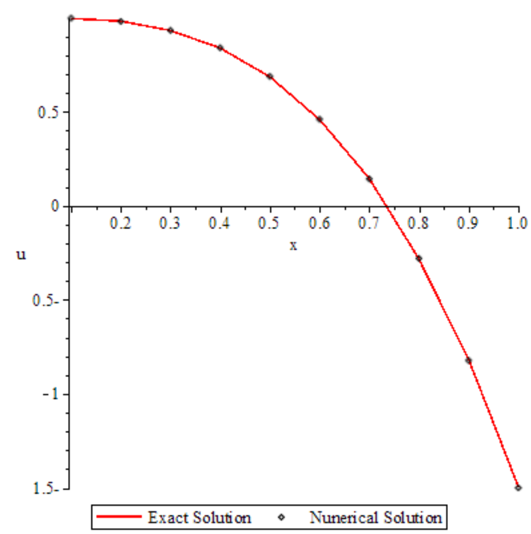

Fig. 2 Plot 2D of the exact solutions result Of Volterra Fredholm integro differential equation for example 2.

Example 3. Consider the Volterra Fredholm integro differential equation

$$u''(x) = 2x^3 - \frac{1}{3}x^2 + \int_0^x \int_{-1}^1 (rt^2 - r^2 t)u(t)dtdr,$$

$$u(0) = 1, u'(0) = 1.\#$$

Applying the Modified Decomposition Method using Maple we find

Table 3 Numerical results and exact solution of Volterra Fredholm integro differential equation for example 3.

x	$u(x)\#$	$Exact = 1 + 9x$	$Error$
0.10000	1.9000000	1.9000000	0.0000000
0.20000	2.8000000	2.8000000	0.0000000
0.30000	3.7000000‡	3.7000000‡	0.0000000
0.40000	4.6000000	4.6000000	0.0000000
0.50000	5.5000000	5.5000000	0.0000000
0.60000	6.4000000	6.4000000	0.0000000
0.70000	7.3000000	7.3000000	0.0000000
0.80000	8.2000000	8.2000000	0.0000000
0.90000	9.1000000	9.1000000	0.0000000
1.00000	10.0000000	10.0000000	0.0000000

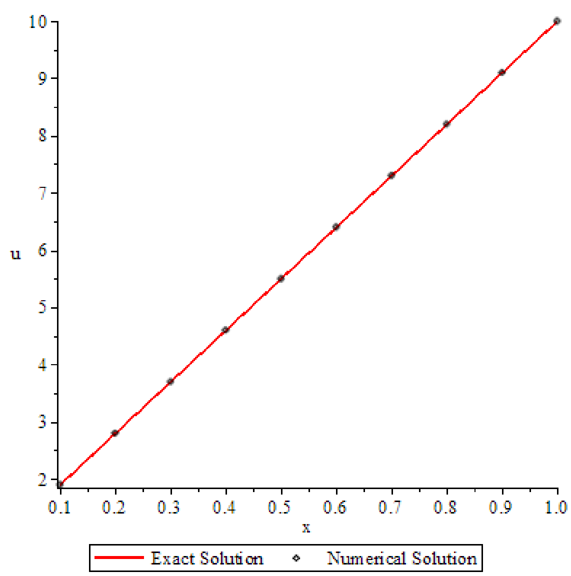

Fig. 3 Plot 2D of the exact solutions result Of Volterra Fredholm integro differential equation for example 3.

Example 4. Consider the Volterra Fredholm integro differential equation

$$u'(x) = 8x + \frac{5}{4}x^2 + \int_0^x \int_0^1 (1 - rt)u(t)dtdr,$$
$$u(0) = 2.\#$$

Applying the Modified Decomposition Method using Maple we find

Table 4 Numerical results and exact solution of Volterra Fredholm integro differential equation for example 4.

x	$u(x)\#$	$Exact = 2 + 6x^2$	$Error$
0.10000	2.0600000	2.0600000	0.0000000
0.20000	2.2400000	2.2400000	0.0000000
0.30000	2.5400000	2.5399999	0.0000001
0.40000	2.9600000	2.9599998	0.0000002
0.50000	3.5000000	3.4999997	0.0000003
0.60000	4.1600000	4.1599996	0.0000004
0.70000	4.9400000	4.9399995	0.0000005
0.80000	5.8400000	5.8399994	0.0000006
0.90000	6.8600000	6.8599992	0.0000008
1.00000	8.0000000	7.9999991	0.0000009

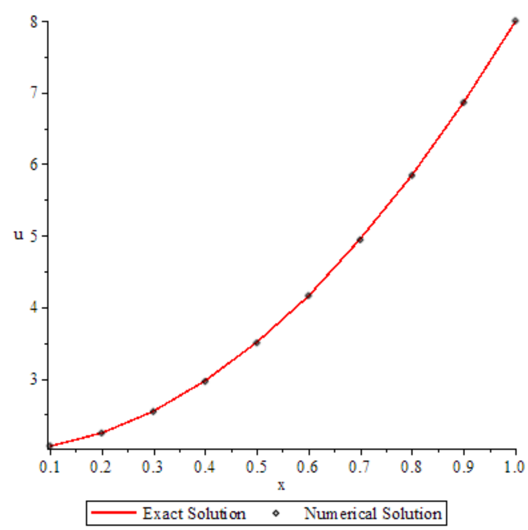

Fig. 4 Plot 2D of the exact solutions result Of Volterra Fredholm integro differential equation for example 4.

Example 5. Consider the Volterra Fredholm integro differential equation

$$u'(x) = 1 - 2x - \frac{43}{15}x^2 + \int_0^x \int_{-1}^1 (1 - rt)u(t)dtdr,$$
$$u(0) = 1.\#$$
#

Applying the Modified Decomposition Method using Maple we find

Table 5 Numerical results and exact solution of Volterra Fredholm integro differential equation for example 5.

x	$u(x)\#$	$Exact = 1 + x - x^3$	$Error$
0.10000	1.0990000	1.0990000	0.0000000
0.20000	1.1920000	1.1920000	0.0000000
0.30000	1.2730000	1.2730000	0.0000000
0.40000	1.3360000	1.3360000	0.0000000
0.50000	1.3750000	1.3750000	0.0000000
0.60000	1.3840000	1.3840000	0.0000000
0.70000	1.3570000	1.3570000	0.0000000
0.80000	1.2880000	1.2880000	0.0000000
0.90000	1.1710000	1.1710000	0.0000000
1.00000	1.0000000	1.0000000	0.0000000

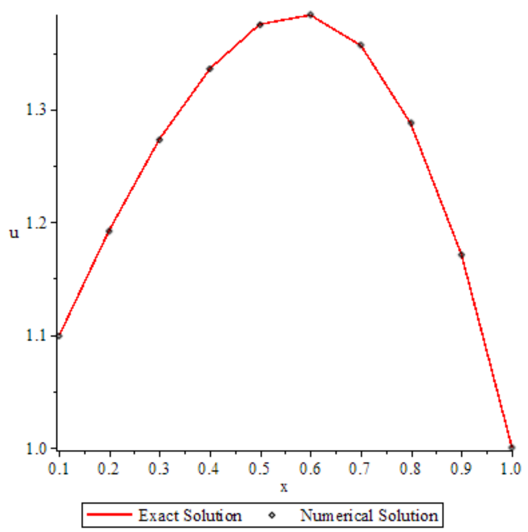

Fig. **5** Plot 2D of the exact solutions result Of Volterra Fredholm integro differential equation for example **5**.

Example 6. Consider the Volterra Fredholm integro differential equation

$$u'(x) = \frac{1}{2}x^2 - e^{-x} + \int_0^x \int_{-1}^0 rtu(t)dtdr,$$
$$u(0) = 1.\#$$

Applying the Modified Decomposition Method using Maple we find

Table 6 Numerical results and exact solution of Volterra Fredholm integro differential equation for example 6.

x	$u(x)\#$	$Exact = e^{-x}$	$Error$
0.10000	0.9048374	0.9048374	0.0000000
0.20000	0.8187308	0.8187308	0.0000000
0.30000	0.7408182	0.7408182	0.0000000
0.40000	0.6703200	0.6703200	0.0000000
0.50000	0.6065307	0.6065307	0.0000000
0.60000	0.5488116	0.5488116	0.0000000
0.70000	0.4965853	0.4965853	0.0000000
0.80000	0.4493290	0.4493290	0.0000000
0.90000	0.4065697	0.4065697	0.0000000
1.00000	0.3678794	0.3678794	0.0000000

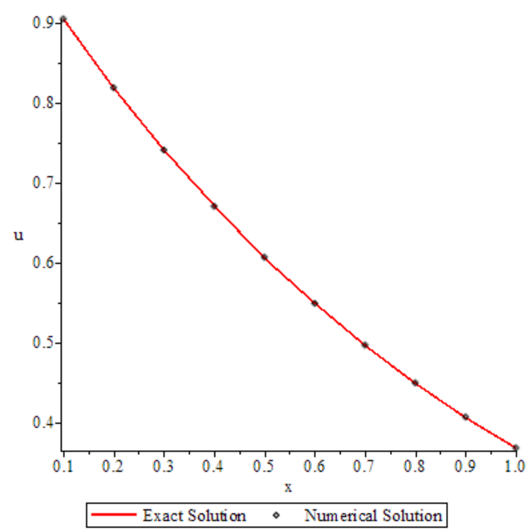

Fig. 6 Plot 2D of the exact solutions result Of Volterra Fredholm integro differential equation for example 6.

Example 7. Consider the Volterra Fredholm integro differential equation

$$\boldsymbol{u''(x) = \frac{-10}{3} + \frac{2}{9}x^3 + \int_0^x \int_{-1}^1 (rt^2 - r^2t)u(t)dtdr,}$$
$$\boldsymbol{u(0) = 1, u'(0) = 1.}\#$$

#
Applying the Modified Decomposition Method using Maple we find

Table 7 Numerical results and exact solution of Volterra Fredholm integro differential equation for example 7.

x	$u(x)\#$	$Exact = 1 + x - (5/3)x^2$	$Error$
0.10000	1.0833333	1.0833333	0.0000000
0.20000	1.1333333	1.1333333	0.0000000
0.30000	1.1500000	1.1500000	0.0000000
0.40000	1.1333333	1.1333333	0.0000000
0.50000	1.0833333	1.0833333	0.0000000
0.60000	1.0000000	1.0000000	0.0000000
0.70000	0.8833333	0.8833333	0.0000000
0.80000	0.7333333	0.7333333	0.0000000
0.90000	0.5500000	0.5500000	0.0000000
1.00000	0.3333333	0.3333333	0.0000000

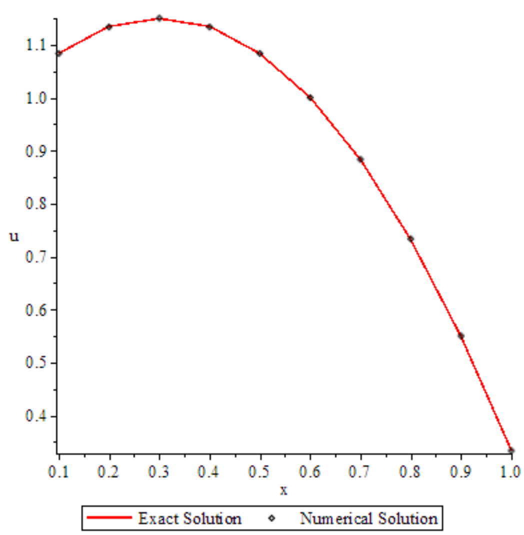

Fig. 7 Plot 2D of the exact solutions result Of Volterra Fredholm integro differential equation for example 7.

4. CONCLUSION

In this paper, the Modified Decomposition Method to the solution of Volterra Fredholm integro differential equation numerical results demonstrate that our method is an accurate and reliable numerical technique for solving Volterra Fredholm integro differential equation. Note that the results of examples 1, 2 and 3 show that the exact solution is very close to the approximate solution and that the error rate between them is very small, which proves the efficiency of the method used. This is shown in the tables and graphics from 1-3. Finally, The Modified Decomposition Method using Maple can be easily extended and applied to linear or nonlinear Fredholm and Volterra integral equations of the first or second kind.

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