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A COMPREHENSIVE REVIEW OF PILE STABILIZED SLOPE: ANALYSIS METHOD AND DETERMINATION OF CRITICAL DESIGN PARAMETERS

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ABSTRACT: The laterally loaded piles play a critical role in the stabilization of both engineered and natural soil slopes. Although the importance of pile in the stabilization of slopes has been recognized from a wide variety of literatures, it remains a great challenge to understand that kind of interaction mechanism between the pile and the soil. Numerous analytical methods and numerical methods have been developed and purposed to investigate its reinforcement mechanism, the performance of pile and the behavior of slope has also received much attention. To provide a solid foundation for the understanding of its reinforcement mechanism, the methods to evaluate the lateral pressure imposes on the pile have been compared. To make a comprehensive review, the soil arching mechanisms of pile stabilized slopes and group effects of pile row have been discussed thoroughly. As the pile-soil interaction is a complicated problem, the uncoupled analysis method that the response of passive piles and stability of slopes are considered separately has also been summarized and compared. The analysis methods including Finite Element Method and Limit Analysis Method have been discussed with respect to the basic concept and the utilization. The determination of the critical design parameters of pile stabilized slope is various from literature, which has been discussed and reviewed thoroughly.

Keywords: Slope stability; Laterally loaded pile; Soil arching effect; Analysis method; Critical design parameters

1. INTRODUCTION

The laterally loaded piles have been utilized widely in the stabilization of the slope, the effect on improving the slope stability has been comprehensively demonstrated [1-10]. The advantages of using laterally loaded piles include increasing shear strength and load-bearing capacity, reducing settlement, resisting lateral loadings, and enhancing stability [11]. The major function of the lateral loaded piles is to transmit the sliding force derived from the loose or weak strata to the underlying stiffer strata and provide a resistance force to increase the overall slope stability. As a preventive measure to stabilize active landslides, numerous analysis methods have been proposed to investigate the performance of the pile stabilized slope [12-15]. The existing analysis methods for pile stabilized slopes can be categorized into four main types: (1) displacement-based methods [2, 13, 16, 17]; (2) pressure-based methods [18-20]; (3) finite element methods [21-23]; (4) limit analysis methods [24-26]. Regarding displacement-based method for the design of the pile-stabilized slopes, there are three major steps included: (1) the resisting force that is necessary to maintain slope stability at the optimal level is evaluated; (2) the contribution from each pile to the resisting force against the unstable sliding is assessed; (3) the number and the type of the piles, and optimal pile installation location is determined. Regarding the first step, the additional shear force which is required for slope stability is derived from stability analysis. For second step, the pile behavior that is subjected to moving soil is analyzed. Regarding the third step, the impact of the design parameter of pile retaining structure (pile length, pile diameter, pile spacing) is evaluated [2]. It is worth noting that displacement-based method provides uncoupled solution for the interaction between pile and soil, which implies that this method considers slope stability and the pile response separately. With respect to pressure-based method, the lateral sliding force which is caused by movement of the unstable soil mass imposes on pile row is estimated through the method derived by [27]. It is assumed that the portion of the force was utilized to resist the displacement of slopes. Then, the slope stability can be obtained with the consideration of the pile presence and its impact on the formation of the potential sliding surface. When new sliding surface is derived, the structural specifications can be adopted to design the pile row. In addition, the effect of critical factors (pile location, spacing, and diameter) is also investigated. The pressure-based method is a step-by-step process for the designing of pile stabilized slopes to meet safety requirements. For finite element methods, this method adopts the coupled analysis to investigate the interaction behavior between soil and piles, which is major shortcoming of aforementioned methods. The potential critical sliding surface would not be assumed prior to the numerical analysis, and the stability of the slope and the soil-pile interaction can be evaluated simultaneously. Due to the great potential of simulating the nonlinear stress-strain behavior of the material, the finite element analysis is adopted extensively in geotechnical practices. The finite element method can represent stress-strain behavior of soil mass, especially for non-homogeneous characteristics of the soil and complicated nonlinear stress-strain behavior. Regarding limit analysis method, it adopts the theorem of upper and lower bound of plasticity theory to determine rigorous solution of upper and lower bound for the problem of stability, then the range of the true solution can be located. The range between the lower and the upper bound could be located by determining the lowest possible solution for upper bound and the highest possible solution for the lower bound. The unknown quantity related to the stability problem such as the critical height of the slope or the factor of safety, the earth pressure of retaining wall, and bearing capacity of foundation or embankment, can be effectively addressed by limit analysis method.

In practical engineering, the lateral loaded pile can be categorized into two types: active laterally loaded pile and passive laterally loaded pile. Regarding the active case, the pile is laterally pushed into surrounding soils. With respect to passive cases, the soils are pushed around or into piles. The piles are considered active when they are used to support the superstructures which impose lateral pressure on top of the pile and then transmit lateral pressure to surrounding soils. On the other hand, when the lateral loading is imposed on the pile length because of soil movements, the pile is considered as passive cases. The application of laterally loaded piles to the stabilization of the slope can be regarded as the passive case [28]. As the passive laterally loaded pile, it is expected to provide the preventive effect against the slope failures or lateral movements of the soil mass when the pile row is placed in the unstable slopes. In that case, the piles are subjected to the lateral force induced by lateral soil movements or plastic soil deformation. It is therefore necessary to have a reasonable evaluation of lateral forces imposed on laterally loaded piles, and several analysis methods have been proposed [13, 27, 29-31]. When rows of the pile with a specific spacing are subjected to the lateral forces, the phenomenon of soil arching that could be described as a transfer mechanism of the forces between yielding soil and adjacent passive piles would occur. When surrounding soil tends to yield, the redistribution process of soil stress takes place. The yielding soil mass will be restrained to the original position as much as possible by the shearing resistance, which induces the change in the adjoining soil mass stress [32]. As piles are less compressive than the soil, the large movements would occur in unstable soil mass, mobilizing shear strength of the soil which imposes increasing lateral pressures on adjacent piles and decreasing pressure within abutting soil mass. The soil arching effect is considered to impose the important impacts on the performance of pile stabilized slopes or embankments, and extensive studies with various methods have been conducted [33-38]. Moreover, the group effect within pile row including interaction between soil and pile imposes major impact on behavior of pile stabilized slopes. Several studies suggest that the analysis results of stability of pile reinforced slopes may be conservative without considering pile group effect, especially for large pile groups [29]. A reduction factor, P, which is adopted to indicate the ratio of lateral bearing capacity of the individual pile within pile row to lateral bearing capacity of single isolated pile has been proposed by [39], this factor can indicate the pile group effect reasonably.

The pile has been widely used as the stabilizing element for the slope, and has been proved as a valid slope-stabilizing measurement. Due to advantages of the pile retaining structure in terms of the structural safety, relatively low engineering cost, and the short construction period, the demands in construction of the pile retaining structure for stabilization of slopes and the road embankment are continuously growing for infrastructural development. It is therefore important to get a distinct insight into performance of the pile stabilized slope in terms of critical design parameters such as pile length, the ratio of center-to-center spacing to pile diameter, as well as optimal pile row location. These parameters impose significant impacts on the stability of the slope, the shear force or the maximum bending moment exerting on the passive piles, the movements of unstable soil mass, the formation of soil arching and pile group effects, as well as formation of critical sliding surface.

As a well-developed slope-stabilization method, various research has been carried out on pile retaining structure to study its reinforcing mechanism, pile-soil interaction, deformation pattern, numerical simulation, and computational theory. Therefore, in the current study, the method for the analysis of pile structure is reviewed and discussed firstly, with a focus on the coupled analysis method. The lateral forces impose on piles, the soil arching mechanism, and pile group effect which represent reinforcing mechanisms of pile retaining structure are then discussed and reviewed from the literature. Moreover, the influence of critical design parameters on performance of pile retaining walls is also discussed. For current study, a comprehensive review of the pile stabilized slope is conducted from various aspects, and these summarizations and discussions may enhance the understanding of the laterally loaded pile that has been adopted widely for the slope stabilization.

According to the above research background and objective, the subsequent structure of this paper is organized as follows: the significance of the research is elaborated firstly, and the analysis method for pile stabilized slope is then systematically introduced. Moreover, the reinforcing mechanisms of pile retaining structures with respect to soil arching effect and pile group effect is reviewed. Regarding the last section, the determination of critical design parameters from a wide variety of literature is compared and discussed.

2. RESEARCH SIGNIFICANCE

The laterally loaded pile plays a significant role in the stabilization of the slope, the current study systematically reviews the analysis method, reinforcement mechanisms, and the influences of critical design parameters. The discussion and summarization of characteristics of laterally loaded pile in this study can provide a scientific basis for engineering design and lays a foundation for subsequent studies on pile-soil coupling and seismic performance, which holds a significant engineering practical and academic value.

3. ANALYSIS METHODS FOR PILE STABILIZED SLOPE

3.1 Displacement-based method

The displacement-based method has been derived and refined for the analysis of slopes stabilized by the pile [1, 2, 29, 40-42]. There are three steps involved in this approach: (1) estimating the resisting force needed to improve the factor of safety of the slope to the target value; (2) estimating the maximum resisting force that single pile could provide to restrain the movements of unstable soil mass; (3) determining the type and number of the pile, and the determination of appropriate pile installation location within slopes. Regarding the first step, the analysis of the slope stability would be conducted to evaluate required shear force, and actual safety factor $\mathbf{F^a}$ can be described as following:

$$\boldsymbol{F^a} = \frac{\sum \boldsymbol{R}}{\sum \boldsymbol{F_D}} \tag{1}$$

where $\sum \boldsymbol{R}$ denotes sum of shear force from potential sliding surface; $\sum \boldsymbol{F_D}$ represents the sum of sliding force along potential slip contour. The passive piles need to provide the addition resisting force $\Delta \boldsymbol{R}$, if actual safety factor $\boldsymbol{F^a}$ not achieve desired value, $\boldsymbol{F_T}$. The target safety factor could be depicted as:

$$\boldsymbol{F_T} = \frac{\sum \boldsymbol{R} + \Delta \boldsymbol{R}}{\sum \boldsymbol{F_D}} \tag{2}$$

The pile stabilizing force is expressed by:

$$\Delta \boldsymbol{R} = \sum \boldsymbol{F_D}\,(\boldsymbol{F_T} - \boldsymbol{F^a}) \tag{3}$$

The stabilizing force provided by piles to achieve target factor of safety could be derived. For the second step, by considering the compatibility of the lateral displacement of the pile and the soil, following equation could be derived if the interaction behavior between the soil and the pile remains elastic:

$$\left[[\boldsymbol{D}] + \frac{[\boldsymbol{I}]^{-1}}{\boldsymbol{K_R}\boldsymbol{\eta}^4}\right]\{\Delta\boldsymbol{\rho}\} = \frac{[\boldsymbol{I}]^{-1}}{\boldsymbol{K_R}\boldsymbol{\eta}^4}\{\Delta\boldsymbol{\rho_e}\} \tag{4}$$

where $\boldsymbol{K_R}$ denotes the pile flexibility dimensionless factor; $[\boldsymbol{I}]^{-1}$ represents inverted matrix of the factors for soil movements; $[\boldsymbol{D}]$ denotes finite difference coefficients matrix for pile bending; $\{\Delta\boldsymbol{\rho}\}$ denotes the incremental horizontal pile deflections; and $\{\Delta\boldsymbol{\rho_e}\}$ represents the incremental horizontal soil movements. Regarding the third step, several recommendations have been made to achieve the effective stabilizing piles: (1) the pile should have relatively large diameter and relatively large stiffness; (2) the bottom of the pile should extend well below the potential failure plane; (3) the location of the pile installation should be adjacent to the center of the potential slip surface.

Cai and Ugai proposed the subgrade reaction solution for the behavior of the flexible pile subjected to landslide in which the effect of horizontal displacement of unstable soil mass on the response of piles is taken into account [43]. The proposed solution has been adopted to several flexible pile cases under the condition of lateral movements of soil. Regarding this subgrade reaction method, the subgrade reaction coefficient, $\boldsymbol{K_h}$, is introduced. The deflection of pile segment below and above critical slip surface can be described as:

$$\boldsymbol{EI}\frac{d^4 \boldsymbol{y_i}}{d\boldsymbol{x_i^4}} = -\boldsymbol{K_{hi}}\boldsymbol{y_i}, (\boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}) \tag{5}$$

where $\boldsymbol{i}$ denotes the segments of the pile, $\boldsymbol{i} = \boldsymbol{1}$ represents the segment of pile above failure plane, and $\boldsymbol{i} = \boldsymbol{2}$ denotes the segment of pile below failure plane. Then general formulation of pile deflection can be described by:

$$\boldsymbol{y_i} = \boldsymbol{e^{\beta_i x_i}}(\boldsymbol{A_i}\cos\boldsymbol{\beta_i x_i} + \boldsymbol{\beta_i}\sin\boldsymbol{\beta_i x_i}) \tag{6}$$

where the value of integral constant, $\boldsymbol{A_i}$ and $\boldsymbol{B_i}$, could be calculated by continuity condition of pile at depth of slip surface (Fig. 1). For pile segment above failure plane, the bending moment ($\boldsymbol{M_1}$) and deflection ($\boldsymbol{y_1}$) are given by:

$$\boldsymbol{M_1} = \frac{-\boldsymbol{H}}{\boldsymbol{2}}\boldsymbol{e^{\beta_1 x_1}}\left[\left(\frac{\boldsymbol{1}}{\boldsymbol{\beta_1}} + \frac{\boldsymbol{1}}{\boldsymbol{\beta_2}} + \frac{\boldsymbol{1}}{\boldsymbol{\beta_0}}\right)\cos\boldsymbol{\beta_1 x_1} - \left(\frac{\boldsymbol{1}}{\boldsymbol{\beta_1}} + \frac{\boldsymbol{1}}{\boldsymbol{\beta_2}} - \frac{\boldsymbol{1}}{\boldsymbol{\beta_0}}\right)\sin\boldsymbol{\beta_1 x_1}\right] \tag{7}$$

$$\boldsymbol{y_1} = \frac{-\boldsymbol{H}}{\boldsymbol{4EI\beta_1^2}}\boldsymbol{e^{-\beta_1 x_1}}\left[\left(\frac{\boldsymbol{1}}{\boldsymbol{\beta_1}} + \frac{\boldsymbol{1}}{\boldsymbol{\beta_2}} - \frac{\boldsymbol{1}}{\boldsymbol{\beta_0}}\right)\cos\boldsymbol{\beta_1 x_1} + \left(\frac{\boldsymbol{1}}{\boldsymbol{\beta_1}} - \frac{\boldsymbol{1}}{\boldsymbol{\beta_2}} + \frac{\boldsymbol{1}}{\boldsymbol{\beta_0}}\right)\sin\boldsymbol{\beta_1 x_1}\right] \tag{8}$$

The deflection and bending moment for pile segment below failure plane are given by:

$$\boldsymbol{M_2 = \frac{-H}{2} e^{\beta_2 x_2} \left[\left(\frac{1}{\beta_1} - \frac{1}{\beta_2} + \frac{1}{\beta_0} \right) \cos \beta_2 x_2 + \left(\frac{1}{\beta_1} + \frac{1}{\beta_2} + \frac{1}{\beta_0} \right) \sin \beta_2 x_2 \right]} \quad (9)$$

$$\boldsymbol{y_2 = \frac{-H}{4EI\beta_2^2} e^{-\beta_2 x_2} \left[\left(\frac{1}{\beta_1} + \frac{1}{\beta_2} + \frac{1}{\beta_0} \right) \cos \beta_2 x_2 + \left(\frac{1}{\beta_1} - \frac{1}{\beta_2} + \frac{1}{\beta_0} \right) \sin \beta_2 x_2 \right]} \quad (10)$$

The result from aforementioned method indicates that bending moment of pile segment embedded in firm layer increases and bending moment of the pile segment in unstable layer decreases with the increase in $\boldsymbol{\theta_0}$. Therefore, the effect of $\boldsymbol{\theta_0}$ on passive pile behavior should be considered to ensure pile stability. The value of $\boldsymbol{\theta_0}$ can be measured by the unstable soil mass displacements with an inclinometer in advance of the pile installation.

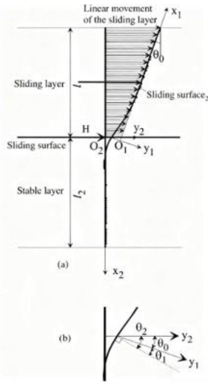

Fig. 1 (a) Concept of the studied model; (b) Continuity condition at slip surface depth [43]

3.2 Pressure-based method

The pressure-based method has been derived to analyze the pile stabilized slope [12, 18, 44]. Ito et al. proposed a pressure-based design method for slopes that stabilized by pile against landslide [12]. The pile was installed in the slope in a row and a fixed sliding surface was assumed for the slope. The impact of pile stiffness and diameter, the pile length above failure plane, the pile head condition, and the spacing between neighboring piles on slope stability have been investigated. To evaluate the stability of pile stabilized slopes, two separate analyses for pile stability and slope stability are conducted (Fig. 2). In addition, a reasonable evaluation of lateral forces exerting on the passive piles is essential for slope stability analysis. Hence, the first procedure is about the evaluation of lateral forces induced by soil mass movements exerting on passive piles. The theoretical formulation of lateral forces exerting on the passive pile has been developed by [27], which has been adopted in the current design method. Regarding the pile stability analysis, the pile stability was evaluated based on bending moment. By comparing allowable pile bending moment with the lateral soil movement induced maximum bending moment, the pile safety factor is expressed as:

$$\boldsymbol{(F_S)_{pile} = \frac{\sigma_{allow}}{\sigma_{max}}} \quad (11)$$

where $\boldsymbol{\sigma_{allow}}$ represents the allowable bending stress of the pile; $\boldsymbol{(F_S)_{pile}}$ denotes pile safety factor; $\boldsymbol{\sigma_{max}}$ is the maximum pile bending stress. It is notable that pile stability can be ensured if the factor of safety is larger than unity. With respect to slope stability analysis, the evaluation of factor of safety could be conducted by comparing driving forces from sliding soil mass and resisting forces, according to:

$$\boldsymbol{(F_S)_{slope} = \frac{F_r}{F_d} = \frac{(F_{rs} + F_{rp})}{F_d}} \quad (12)$$

where $\boldsymbol{F_{rp}}$ and $\boldsymbol{F_{rs}}$ are the resisting forces from passive pile and soil, respectively; $\boldsymbol{F_d}$ represents the driving force. It is worth noting that the $\boldsymbol{F_{rs}}$ and $\boldsymbol{F_d}$ can be derived from the ordinary slice method, and the value of the $\boldsymbol{F_{rp}}$ could be calculated by dividing pile reaction forces with the spacing of pile group. The stability of slopes would be ensured if safety factor of slopes were larger than required value.

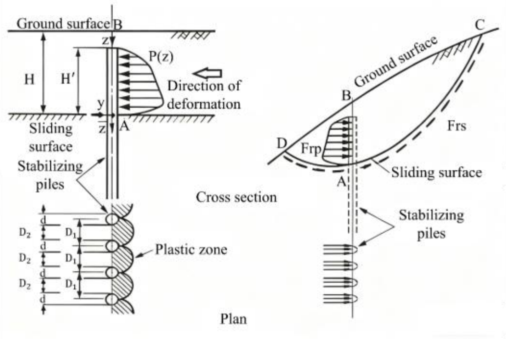

Fig. 2 Basic concept of pressure-based design method [12]

Hassiotis et al. developed the methodology for the analysis of the slope reinforced by one row of the passive pile [18]. The lateral force exerting on passive piles were determined by the method that developed based on plasticity theory. A portion of the lateral forces was assumed to be mobilized to resist displacements of the unstable soil. The existence of passive piles as well as the effect on location of critical slip surface is then considered by Taylor's method for slope stability analysis. The passive piles are designed to meet structural requirements when new potential failure plane is determined. The impact of major influencing factors on stability of pile stabilized slopes has also been studied. First, the lateral force exerting on passive piles was estimated using the methodology derived by [27]. Then, Taylor's methodology for the analysis of slope stability would be adopted to consider the existence of the pile and determine new sliding surface after installation of piles. The resistance forces from the piles, $\boldsymbol{F_p}$, are adopted in the expression of the stability number, and the new slope configuration for the analysis has been demonstrated in Fig. 3. For toe failure of pile stabilized slopes, the stability can be given by:

$$\frac{C_a}{F_c\gamma H}=\frac{E-\frac{12F_p}{\gamma H^3}\left[\frac{\cos(CEO)H}{\sin\upsilon\quad 2}\csc x\csc y\sin\varphi+OG\right]}{6\csc^2 x\csc y\sin\varphi\left[\frac{\cos x}{\sin\upsilon}+\csc(\mu-\upsilon)\cos(x-\upsilon)\right]}\tag{13}$$

For the base failure of pile stabilized slopes, the safety number is described as following:

$$\frac{C_a}{F_C\gamma H}=\frac{\left(E+6\eta^2-6\eta\sin\varphi\csc x\csc y\right)-\frac{12F_p}{\gamma H^3}A}{6\csc^2 x\csc y\sin\varphi\left[\frac{\cos x}{\sin\upsilon}+\csc(\mu-\upsilon)\cos(x-\upsilon)\right]}\tag{14}$$

where the subscript denotes the required and the available quantities; $\boldsymbol{C_a}$ represents the available shear strength; $\boldsymbol{F_C}$ denotes the safety factor with regard to cohesion; $\boldsymbol{F_p}$ represents the resistance force from piles; $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{\upsilon}$, and $\boldsymbol{\mu}$ are the angle indicated in Fig. 3; $\boldsymbol{CEO}$ is the angle between horizontal and $\boldsymbol{F_p}$; $\boldsymbol{OG}$ denotes moment arm of the $\boldsymbol{F_p}$.

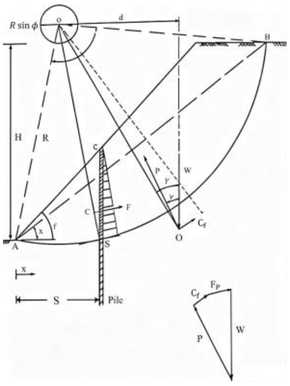

Fig. 3 Forces of pile reinforced slope [18]

Regarding the next step, the pile is designed to meet the structural specifications. The deflection of the pile segments below and above critical failure plane will be considered separately by governing equations (Fig. 4). Regarding the deflection of pile segment above potential failure plane, the closed form solution is given by:

$$\boldsymbol{EI(d^4y_1/d_z^4)=q(z),(-CE\le z\le 0)}\tag{15}$$

where $\boldsymbol{y_1}$ is deflection of pile segment above the failure plane; $\boldsymbol{EI}$ represents pile stiffness. $\boldsymbol{q(z)}$ is the force intensity, which can be calculated based on theory of soil plastic deformation and can be depicted as:

$$\boldsymbol{q(z)=q_2+(q_2-q_1/CE)z}\tag{16}$$

where $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$ is linear distribution of the $\boldsymbol{q(z)}$. The deflection of the pile segment below the failure plane, which is embedded in firm layer as a beam, is given by:

$$\boldsymbol{EI\left(\frac{d^4y_2}{d_z^4}\right)=-Ky_2,\,(z\ge 0)}\tag{17}$$

where $\boldsymbol{y_2}$ is the deflection of the pile segment below the critical slip surface.

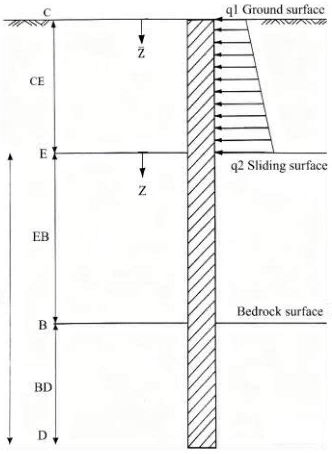

Fig. 4 Pile embedded in firm foundation [18]

A sequential process has then been derived for the analysis of passive piles and slopes. When the required value of the factor of safety and the pile installation location are determined, the desired pile and slope strength, the diameter of the pile, and the center-to-center spacing of the pile row can then be determined, representing that pile integrity and the stability of the slope can both be ensured.

3.3 Finite Element Method (FEM)

Clough and Woodward first employed the FEM in geotechnical engineering [45]. Currently, the FEM has been adopted extensively for geotechnical engineering owing to its advantages in simulating the nonlinear stress-strain behavior. Regarding limitations of FEM, the computational cost may be high and requires many computational power, especially for the three-dimensional nonlinear analysis. Regarding the FEM, the material shear strength would be progressively decreased with the increase in strength reduction factor till the occurrence of the slope failure. Using Mohr-Coulomb failure criterion as typical example to illustrate, the reduced effective shear strength is given by:

$$\boldsymbol{c_r = \frac{c'}{SRF}} \tag{18}$$

$$\boldsymbol{\varphi_r = arctan\left(\frac{tan\varphi'}{SRF}\right)} \tag{19}$$

where $\boldsymbol{c'}$ represents effective cohesion; $\boldsymbol{\varphi'}$ denotes effective friction angle; $\boldsymbol{SRF}$ represents strength reduction factor.

With respect to geotechnical engineering, one of the major issues related to the correctness of FEM simulation is appropriate definition of the initial conditions, the sequence of the loading, the boundary conditions, as well as the stress-strain constitutive relationship. For the initial conditions of the geotechnical issues, it can be measured from the site or estimated according to knowledge in soil mechanics. The determination of stress-strain constitutive model of materials is critical for the analysis by FEM as the stress-strain relationship imposes considerable impacts on the complexity of the analysis and the accuracy of the results. In addition, several basic constitutive models such as the multilinear elastic model, hyperbolic elastic model, and the linear elastic model, including several elastoplastic constitutive models, have been widely adopted to represent simplified stress-strain relationship. These constitutive models can yield reasonable predictions for specific geotechnical problems. The advantages of the FEM for slope stability analysis have been summarized as in the following [46]:

(1) There is no need to presume the location or the shape of potential failure plane in advance. The failure of slopes would occur through the soil zone in which the soil shear strength is unable to restrain the shear stress induced by unstable soil mass movements.

(2) The FEM can predict the deformations of the slope under working stress conditions.

(3) The slope failure from the initiation to overall shear failure can be progressively monitored by the FEM.

Several studies have been conducted to investigate the performance of the slope and the behavior of the pile under lateral soil movements condition with the consideration of pile-soil interaction through FEM [21, 35, 47-49]. Cai and Ugai adopted elastoplastic three-dimensional FEM to investigate the impact of the passive pile on slope stability [35] (Fig. 5). The numerical result from FEM was then compared with the result from simplified Bishop method in which lateral forces of soil mass were determined by Ito-Matsui’s formulation. The significant impact of head conditions and bending stiffness of passive piles on the slope stability were proved by finite element approach, whereas these effects cannot be demonstrated by limit equilibrium method. The numerical results indicated that the passive pile installation location imposes significant influence on slope behavior. The optimal pile row position indicated by the numerical analysis is the middle of slope to yield the maximum value of the stability number. In contrast, the limit equilibrium method denotes that suitable pile row position should be adjacent to the crest of slopes. Wei and Cheng evaluated the stability of the slope stabilized by passive piles through FEM [49]. According to numerical results, when the spacing of the pile row was small, the critical slip surface would be separated into two segments. With the rise in pile group spacing, these two separate segments would be gradually connected until the formation of a clear potential failure plane. When the slope was stabilized by the passive pile row, the potential failure plane was relatively shallower than that of slope without the passive pile row. This is distinct from previous studies that critical slip surface of the pile stabilized slope is very deep. Regarding the optimal passive pile location, it was found that installing the passive pile row at the position between middle of potential slip surface and the middle of slopes would improve the slope stability most effectively (Fig. 6). Ho conducted a numerical study of the slope stabilized by passive piles containing a thin weak soil layer utilizing the three-dimensional FEM [21] (Fig. 7). The existence of the weak thin soil layer imposed a negative influence on slope stability. The failure principles of the slope with and without stabilizing pile were compared, which indicates that the installation of the passive piles would alter critical slip surface depth and the slope failure mechanisms. The result from the numerical analysis indicates that middle of the slope would be an optimal location of passive pile row, and 40%-60% pile length should be extended below the critical failure plane. To improve slope stability more effectively, the ratio of spacing to pile diameter should be smaller than 4.0 according to finite element approach. For the slope containing a weak thin soil layer, half pile length should cross the weak thin layer bottom.

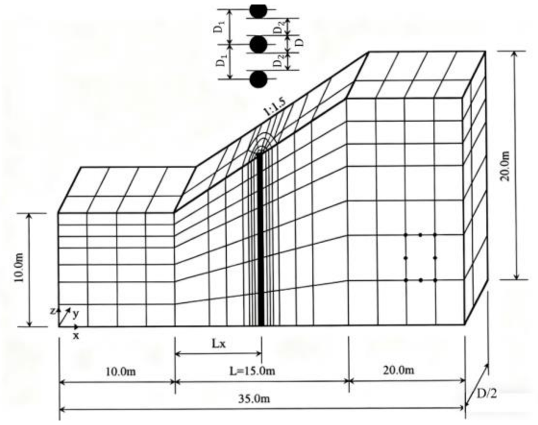

Fig. 5 Numerical modelling and arrangement of mesh [35]

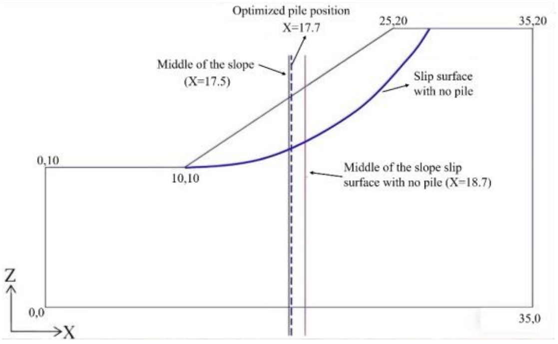

Fig. 6 Optimized position of the passive pile row [49]

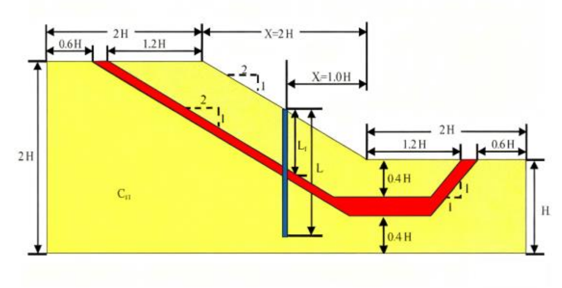

Fig. 7 Numerical model of slope has the weak thin soil layer stabilized by passive pile row [21]

3.4 Limit Analysis Method (LAM)

Limit analysis is the approach that adopts the advantages of kinematic and static theorem based on the plasticity theory to define the strict lower and upper bound for the true solution of slope stability problem. The range between lower and upper bound would be narrowed by determining the lowest possible upper bound solution and the highest possible lower bound solution. The unknown quantity related to the stability problem such as the critical height or slope safety factor, the earth pressure of retaining wall, and bearing capacity of foundation or the embankment, can be effectively addressed by the limit analysis. Regarding the LAM, the soil deformation would be presumed as plastic based on normality rule related to Mohr-Coulomb failure criterion. For lower bound solution, the stress fields are in equilibrium with body loads and the surface tractions, and yield criterion will not be violated everywhere in soil mass. For upper bound solution, the work rate done by the surface traction and the body force needs to equal the dissipation rate of internal energy, and the strain rate fields need to be compatible with imposed velocity at the soil mass boundary. Therefore, the lower bound represents the equilibrium state and upper bound represents collapse status. In general, if the stabilizing forces need to be defined, the utilization of static theorem will yield the lower bound solution; if the force induces the unstable of the soil mass, the kinematic theorem is adopted to yield upper bound of the true solution. The difficulty of LAM is to determine reasonable velocity fields and the stress fields to yield the lowest possible upper bound and the highest possible lower bound of true solution. In addition, incorporating FEM with LAM can overcome these difficulties.

Ausilio *et al*. adopted the kinematic theorem to determine stabilizing force contributed by the pile retaining structure to improve the slope stability composed by homogeneous soil to the target safety factor value [50]. Moreover, the effect of pore-water pressure on slope stability was not investigated. The kinematic theorem was adopted for the slope without the passive pile first to evaluate its stability, and then applied to investigate the stability of slopes stabilized by passive piles. Regarding kinematic mechanism, it considers the potential failure plane as log-spiral in which the unstable soil mass will rotate like a rigid body (Fig. 8). To consider the existence of the passive pile within the slope, the presumed moment and the lateral forces were adopted at the critical slip surface depth. The result derived from the proposed approach well agree with the result obtained by other methods including the Bishop simplified method. The optimal pile row installation position was found to be near the slope toe where the stabilizing force required from the pile is minimum. Between middle and toe of the slope, the installation of pile row can also improve the slope stability effectively. However, with the rise in the target safety factor value, this optimal region for the installation of the pile row will be narrowed. Nian *et al*. investigated the slope stability composed of nonhomogeneous and anisotropic soil reinforced by the passive piles using LAM incorporating the strength reduction technique [24]. Numerical studies are then carried out to define suitable installation location of passive piles and to investigate the influence of the anisotropy and non-homogeneity of the soil on stabilizing force contributed by the passive piles. A simplified procedure for designing the pile stabilized slopes against landslides was then proposed. The LAM incorporating with the strength reduction technique has been confirmed as a valid approach for the analysis of the stability of pile stabilized slopes.

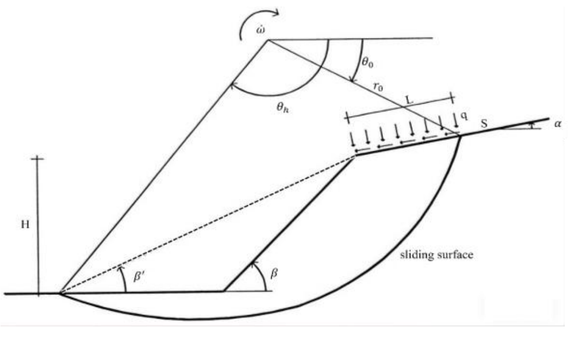

Fig. 8 Log-spiral sliding surface [50]

4. REINFORCING MECHANISMS OF PILE RETAINING STRUCTURE

4.1 Soil arching mechanisms

Soil arching mechanisms exist in a wide variety of geotechnical engineering practices, which can be described as a transfer mechanism of the forces between adjacent passive piles and yielding soil mass. When surrounding soils has a tendency to yield, the stress redistribution process within the soil mass takes place. For the pile stabilized slope, the unstable soil layer tends to move downward, the shear forces against the soil movements will act upward, and the stress at the bottom of the yielded soil mass will be reduced. Moreover, as the pile is less compressive than the soil, the large movements will occur in unstable soil layer, mobilizing shear strength which imposes increasing lateral forces on adjacent piles and decreasing pressure of abutting soil mass. In general, soil arching mechanisms are considered to impose an important effect on the behavior of the pile stabilized slopes or embankment, which therefore needs a thorough investigation.

To study the potential soil arching mechanisms of pile stabilization slope, Adachi *et al.* carried out a model test in two-dimensional as shown in Fig. 9 [33]. In the model test, the parallel arrangement of pile group with uniform spacing and zigzag arrangement have been adopted. The pile is rigid with $\mathbf{50}\ \boldsymbol{mm}$ length of and $\mathbf{3}\ \boldsymbol{mm}$ diameter. The sandy soils are adopted to place around the rigid pile groups. The pile behavior has been monitored during the movement of soil mass induced by the downward movement of bottom plate. The soil arching development from two-dimensional model test has been explained in Fig. 10. When soil behind pile row moves downward, the soil at point D will move downward first and the soil at points B and C will also move downward, dragged by the soil at point D. With continuous soil movement at pile spacing, the soil at point A has the tendency to move downward, but the shear resistance from the adjacent soil particles keeps the soil at point A at its original position. In contrast, the shear resistance at points B, C, and D is not mobilized due to the disconnection with the neighboring soil particles. Therefore, an equilateral triangular soil arch will be completely formed behind the pile group. In addition, with the consideration of the soil arching and the symmetry of loading condition, the lateral forces acting on each pile were estimated as two times the hatched area (Fig. 10). According to Fig. 10, in Zone (1) the soil particles can move along the side surfaces of the circular pile. This soil zone may induce the instability of the foothold of soil arching between two piles. It has also been found that lateral forces exerting on circular piles tend to be smaller than lateral forces imposed on rectangular passive pile by approximately $\mathbf{15}\%$. Moreover, comparing the effect of different arrangements of the pile group adopted in this two-dimensional model test, it was found that the arrangement of the pile group in the zigzag pattern is more effective than the arrangement in the parallel pattern due to the double-soil arching mechanism.

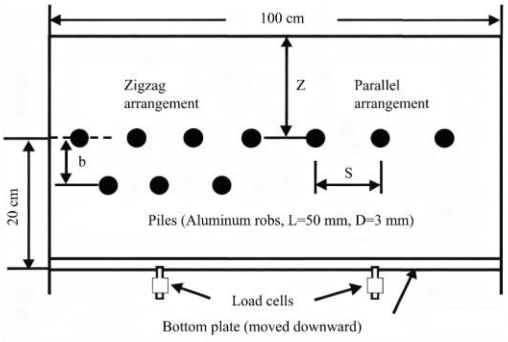

Fig. 9 Experiment apparatus and pile group arrangement [33]

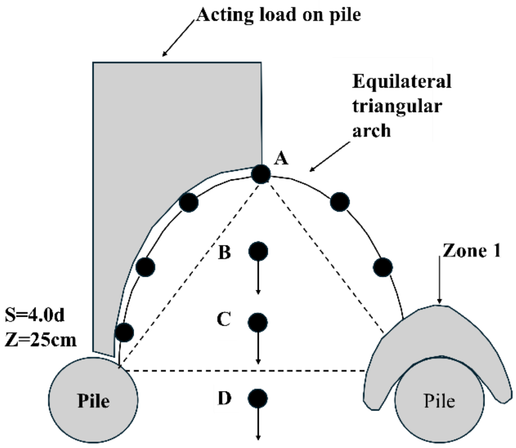

Fig. 10. Soil arching development mechanisms [36]

Chen and Martin conducted a numerical study on the mechanisms of the shearing resistance derived from pile row under the horizontal soil mass movement condition based on the standpoint of soil arching effect [36]. The FLAC, a Finite Difference code, was adopted to examine existence of soil arching zone abutting passive pile groups installed in the fine-grained or the granular soils. Combining soil arching mechanisms with load-displacement curves of piles, the stress transfer mechanisms from soil mass to passive piles was investigated. A parametric study was carried out to analyze the impact of the soil dilatancy, interface properties, Young's modulus, initial stress field, and the failure criteria of the soil on interaction behavior between soil and pile. The rigid piles with a spacing of $\mathbf{4}\ \boldsymbol{m}$ and a diameter of $\mathbf{1}\ \boldsymbol{m}$ is adopted to model the soil plastic flow abutting the pile (Fig. 11). The numerical result indicated that soil dilatancy imposes important impact on soil arching development, as larger soil dilatancy would induce the rise in the soil volume around piles. Moreover, the soil arching development has been indicated by the displacement contour in soil movement direction as shown in Fig. 12.

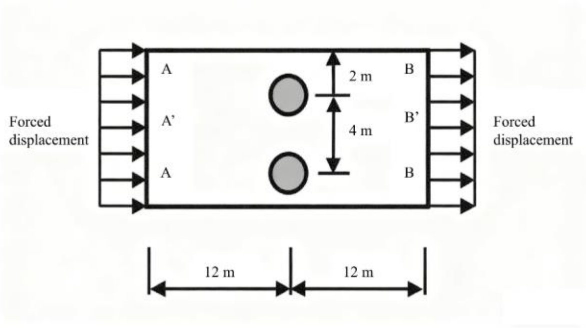

Fig. 11. Basic concept of two-dimensional numerical simulation [36]

Liang and Zeng performed the FEM numerical study to quantitatively investigate soil arching effect for cohesionless and cohesion soil [51]. The results from numerical analysis indicate that slope stability can be significantly improved when the passive piles are inserted in the firm layer. As expected, the soil that has higher internal friction angle can develop soil arching with higher magnitude. Chen *et al.* carried out **15** tests to investigate behavior of pile stabilized embankment with the emphasis on soil arching mechanism [37]. The result from model test implies that a higher ratio of pile width to spacing would lead to the larger stress concentration ratio, and thereafter the stronger soil arching. Moreover, it is worth noting that soil arching magnitude is significantly dependent on relative displacements between soil and pile.

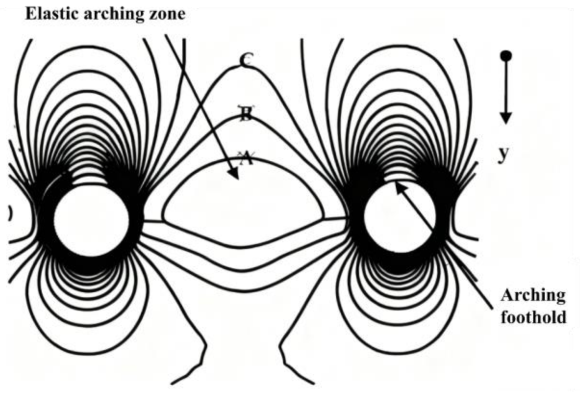

Fig. 12. Illustration of elastic arching zone and foothold of arching by displacement contour [36]

4.2 Group effects of pile row

The response of single pile within pile row would be affected by the presence of neighboring piles. Under these circumstances, group effects that can be regarded as pile-soil-pile interaction should be considered. The group effect of pile row is considered to impose significant impacts on performance of pile stabilized slopes. Moreover, the effect of laterally loaded piles may be overly conservative without the consideration of group effect. A reduction parameter $\boldsymbol{P}$ is proposed by [39], which can be adopted to indicate the ratio of lateral load-carrying capacity of individual pile within pile group to lateral load-carrying capacity of single isolated pile. A summary of the case history data of pile group inserted in different soil types was regressed as shown in Fig. 13 [39]. It is notable that the pile group effect on reducing lateral soil pressure exerting on passive piles basically disappears when ratio of spacing to pile diameter exceeds **3**.

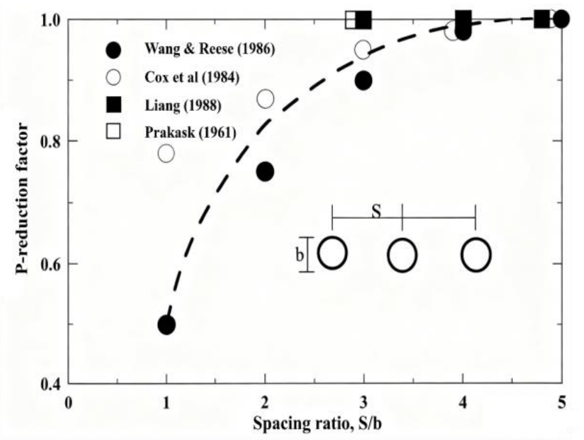

Fig. 13. Pile group impact on reduction of lateral earth pressure exerting on piles [39]

Chen and Martin also studied the group effect of one passive pile row under the drained condition [36]. The behavior of the pile subjected to the superstructure loading (active pile) has been compared with the behavior of the pile subjected to the horizontal soil displacements (passive pile) with the consideration of pile group effects. The pressure-relative displacement ($\boldsymbol{p}$-$\boldsymbol{\delta}$) curve of passive pile and the pressure-pile deflection ($\boldsymbol{p}$-$\boldsymbol{y}$) curve of active piles is depicted in Fig. 14. It is notable that lateral soil pressure exerting on passive piles is larger than lateral pressure acting on active piles, which indicates that responses from the passive piles are relatively stiffer. This phenomenon could be attributed to strong group effects within passive pile row. Under undrained conditions, it has been found that horizontal soil forces on piles are higher than drained condition, and pile group effect disappears when spacing of pile is greater than **4** times the diameter of pile. In general, the pile group effects are more considerable under the drained condition rather than the undrained condition.

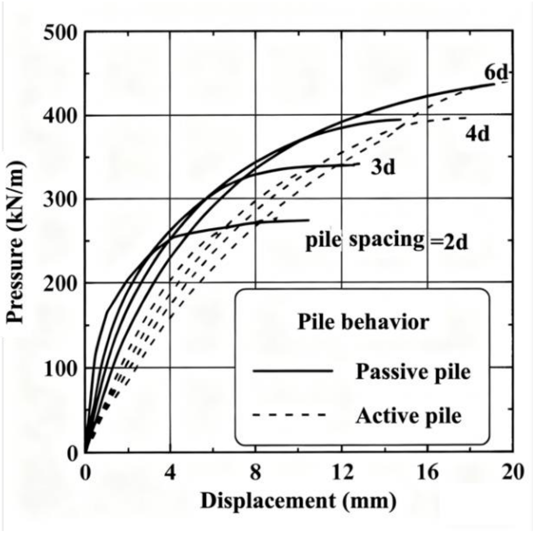

Fig. 14. Pile group effects in granular soil with different spacings [36]

Jeong *et al*. conducted the numerical analysis of pile stabilized slope subjected to horizontal soil displacements [41]. Through the Finite Element analysis, the interaction factor between soil and pile is proposed and the response of piles as a group was also studied. It is notable that the group effect can be assessed by the lateral pressure, the deflections, and bending moment of piles. For this study, the group effects are evaluated mainly based on the pile bending moment. The interaction factor that compares the maximum pile bending moment within the group with that of single isolated pile under similar lateral soil movements, was defined by:

$$\mu = \frac{M_{G(max)}}{M_{S(max)}} \tag{20}$$

where μ denotes the interaction factor of group effect; $M_{G(max)}$ represents the maximum pile bending moment within the group; $M_{S(max)}$ is the maximum bending moment of isolated pile. Moreover, it is noticeable that pile group effects mainly depend on pile group spacing.

5. DETERMINATION OF CRITICAL DESIGN PARAMETERS

5.1 Effect of pile length

Zheng *et al*. carried out the limit analysis to optimize the design of the embedded pile with consideration of embedded pile length [52]. It was observed that embedded length of piles imposes important influence on the formation of critical slip surface. The effect of embedded pile length shows a similar trend on safety factor of the slope. The safety factor of slopes increases with the increase in embedded length of pile, and if the embedded length of pile larger than threshold value of $\boldsymbol{9\ m}$, further rise in embedded pile length did not improve the slope stability. Moreover, Griffiths *et al.* first conducted several modifications for the finite element code to take the effect of pile reinforcement into account [53]. The result from that modified finite element program was then compared with finite difference approach for the purpose of validation. The impact of pile length and pile location on slope stability were investigated through a series of parametric study. In parametric study, the length of pile ranges from $\boldsymbol{6\ m}$ to $\boldsymbol{16\ m}$. For shorter piles, in which the length varies from $\boldsymbol{6\ m}$ to $\boldsymbol{8\ m}$, the factor of safety of slopes reached the maximum value when piles were installed in lower slope part. For length of pile greater than $\boldsymbol{10\ m}$, the slope stability would be increased most effectively when passive pile was installed at slope middle. The numerical result represented that the effect of the pile length also depends on the pile location. When piles are installed at midpoint of slope, the effect of increase in the length of pile on the improvement of the slope stability would be more significant. It was also found that the critical slip surface depth increases with the increase in pile length, whereas potential failure plane becomes much shallower if the length of pile surpasses the threshold value of $\boldsymbol{14\ m}$ in the study. Kourkoulis *et al*. developed a numerical approach for the analysis of the passive pile stabilized slope [5] (Fig. 15). The impact of major design factors including pile length was investigated by parametric studies with finite element analysis. For short pile length, the failure mode is rigid-body rotation with insufficient flexural deformation. Although pile head deflection is large, the substantial bending moment is not developed within the pile and the soil strength is not mobilized sufficiently. This failure mechanism thus can be regarded as short pile failure mode. The numerical result also indicated that the effective embedment pile length in firm layer should not less than the unstable soil layer height to avoid short pile failure mode. Yang *et al*. conducted a numerical analysis by shear strength reduction approach to study the impact of embedded length of passive pile on slope behavior [54]. As anticipated, the safety factor of slopes increases with an increase in embedded length of pile. When length of pile surpasses the critical embedded length of pile, the slope stability gradually approaches a constant state, and the positive pressure exerting on upper pile also has a tendency to reach a constant value. It was also found that the deflection of the pile along the depth is basically linear when embedded pile length is less than the threshold value, this phenomenon represents that less resisting force is provided from the bottom of the pile. Shooshpasha and Amirdehi conducted a numerical analysis with assistance of strength reduction approach to study the behavior of the slope reinforced through the pile row [55].

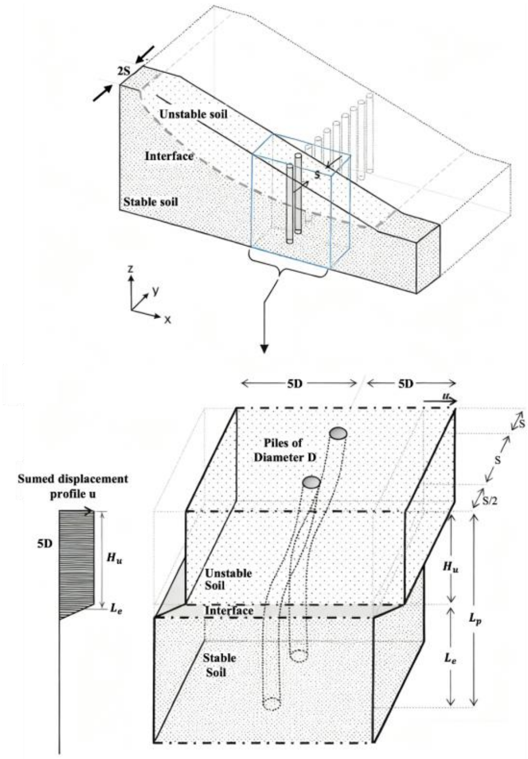

Fig. 15. Illustration of hybrid methodology for pile design [5]

In this study, the ratio of critical slip surface depth to pile length was introduced. The numerical results indicated that the slope stability increases with the rise in length of pile, whereas there is a threshold value of pile length beyond which the slope stability will not be imporoved further. This is because the sufficient resisting force was provided to slope stability when length of pile reaches critical value. It was also found that optimal pile length depends on slope angle, pile bending stiffness, and pile spacing. Moreover, with the rise in slope angle, the influence of passive pile on improving stability of the slope decreases. Ho couducted the numerical analysis to study the performance of pile stabilized slope [56]. For that study, the ratio of length of pile to the height of slope was adopted to investigate the impact of the pile length on slope behavior. The range of the ratio was between $\mathbf{0.5}$-$\mathbf{0.95}$. Regarding piles with fixed-head conditions, the optimal ratio of length of pile to height of slope is found to be around $\mathbf{0.7}$. In addition, the contact pressure distribution along piles has also been extracted to figure out the failure mechanism (Fig. 16). Moerover, Gong *et al*. introduced a design procedure for pile stabilzied slope against the landslide with the assistance of finite difference approach, and effectiveness of passive piles as well as the cost efficiency were considered [57]. In the parametric study, eight values of pile length are adopted including $\boldsymbol{6\ m}$, $\boldsymbol{8\ m}$, $\boldsymbol{10\ m}$, $\boldsymbol{12\ m}$, $\boldsymbol{14\ m}$, $\boldsymbol{16\ m}$, $\boldsymbol{18\ m}$, and $\boldsymbol{20\ m}$. The results from the numerical amalysis indicated that the length of the passive piles should be relatively large, otherwise the lateral force from unstable soil mass cannot be efficiently transmitted to stable foundation, the lateral soil mass movements will not be resisted by the pile either, and consequently the slope stability enhanced by the piles will be minor. Yang *et al*. carried out the three-dimensional numerical study to investigate the performance of the pile that installed in slopes [58]. Regarding parametric studies, the adopted value of pile length are $\boldsymbol{1.5\ m}$, $\boldsymbol{1.0\ m}$, and $\boldsymbol{0.7\ m}$. The numerical results indicated that for the passive pile that has same length, the relation between maximum bearing capacity and distance from piles to slope toe is bilinear. The maximum bearing capacity of piles with different lengths linearly increase with increase in distance from pile cross section to slope toe. Moreover, the effect of pile length on head deflection has also been shown in Fig. 17. The resistance supplied by piles is less if the pile is short, and the head deflection rises with the increase in slope angle. These aforementioned studies has been summarized in Table 1.

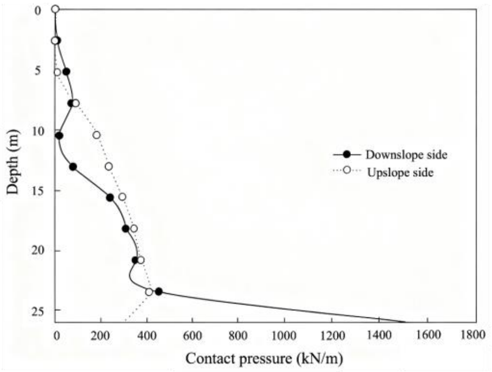

Fig. 16. The contact pressure distribution along pile [56]

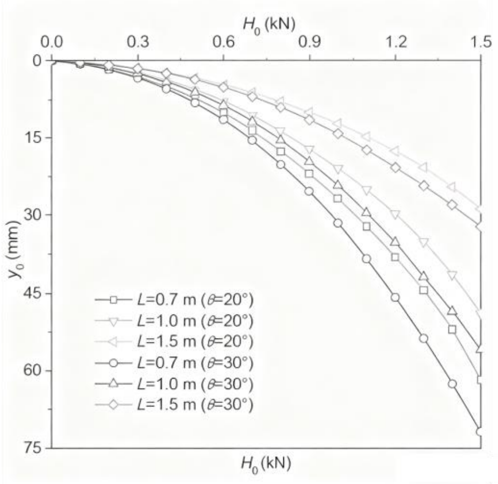

Fig. 17. Pile length effect on head deflection [58]

Table 1. Summary of numerical studies of the pile length

No.	1	2	3	4	5
Authors	[52]	[53]	[5]	[54]	[55]
Numerical methods	FEM	FEM	FEM	FDM	FEM
Software	PLAXIS	p_63s	Abaqus	FLAC	Abaqus
2D or 3D	2D	2D	3D	3D	3D
$\boldsymbol{L_p\ (m)}$	**7-21.22**	**6-16**	**10.2-15**	**6-26**	**5-22.5**
$\boldsymbol{S/D}$	-	-	**2-4**	**2-6**	**2-6**
Optimal location	-	Halfway down the slope	-	-	Middle-slightly upper part of the slope
SCM	Perfectly elastoplastic model with MC failure criteria	Perfectly elastoplastic model with MC failure criteria	Elastoplastic model with MC failure criteria	Perfectly elastoplastic model with MC failure criteria	Perfectly elastoplastic model with MC failure criteria
PCM	Linear elastic model	Beam-rob element	Nonlinear/linear elastic model	Linear elastic model	Linear elastic model
PSI	MC model	-	-	MC model	Friction coefficient

Table 1. Summary of numerical studies of the pile length (Continued)

No.	6	7	8
Authors	[56]	[57]	[58]
Numerical methods	FEM	FDM	FEM
Software	Abaqus	FLAC	Abaqus
2D or 3D	3D	2D	3D
$\boldsymbol{L_p\ (m)}$	**20-38**	**6-20**	**0.7-1.5**
$\boldsymbol{S/D}$	**4**	**1-4**	-
Optimal location	Middle portion of the slope	Middle part of the slope	-
SCM	Perfectly elastoplastic model with MC failure criteria	Perfectly elastoplastic model with MC failure criteria	Elastoplastic model with MC failure criteria
PCM	Linear elastic model	Beam elements	Linear elastic model
PSI	Friction coefficient	Interface elements	Coulomb friction

Note: MC: Mohr-Coulomb; FEM: finite element method; FDM: finite difference method; $\boldsymbol{L_p}$: length of pile; $\boldsymbol{D}$: pile diameter; $\boldsymbol{S}$: center-to-center spacing; SCM: soil constitutive model; PCM: pile constitutive model; PSI: pile/soil interface.

5.2 Effect of ratio of center-to-center spacing (S) to pile diameter (D)

Cai and Ugai investigated the impact of the pile spacing on slope stability through three-dimensional elastoplastic shear strength reduction FEM [35]. The value of $\boldsymbol{S/D}$ ratio adopted in the parametric study is $\boldsymbol{2}$, $\boldsymbol{3}$, $\boldsymbol{4}$, $\boldsymbol{5}$, and $\boldsymbol{6}$. As expected, the slope stability decreases with the rise in the $\boldsymbol{S/D}$ ratio as shown in Fig. 18. When pile spacing decreases, it was found that pile row becomes more similar to the continuous barricade and soil arching magnitude assumes greater significance. Although results predicted by Bishop's simplified approach are comparatively more conservative than FEM, the change rate in safety factor of slopes induced by decrease in pile spacing would be similar. Chae *et al.* carried out the FEM analysis of rigid piles placed abutting slope crest [59]. In that study, the emphasis is about behavior of each pile within the pile group and pile group efficiency with consideration of pile spacing. Two $\boldsymbol{S/D}$ ratio values, $\boldsymbol{2}$ and $\boldsymbol{4}$, were used in analysis. The numerical results indicated that efficiency of pile group increases with the rise in pile row spacing. In addition, if pile spacing was wider, the horizontal deflection of piles becomes smaller. Comparing pile behavior between single pile within the group of pile and the solitary pile, it it also observed that depth of rotation point (point with zero horizontal deflection) is smaller for solitary pile than pile within group, and the location of rotation point of the pile group will not be influenced by the pile spacing. Kourkoulis *et al*. also utilized that developed hybrid approach to investigate impact of axis-to-axis pile spacing on slope behavior [5]. According to Fig. 19, the resisting force from pile per unit width rises with decrease in the spacing of pile. Nonetheless, the effectiveness of passive pile row decreases with decrease in pile group effect (pile-to-pile interaction). It is determined that reaction force of passive piles subjected to horizontal soil movements largely depends on the pile spacing and is also influenced by sliding soil mass thickness. When thickness of sliding soil mass is relatively shallow ($\boldsymbol{4\ m}$ adopted in the current study), the pile behavior would be similar to rigid pile due to the interface depth is small, therefore, the pile group impact would be minor, and pile reaction force would be independent on the pile spacing. However, when unstable soil mass thickness was relatively large ($\boldsymbol{8\ m}$), the pile behavior is more flexible and the pile group effect is more considerable, the pile reaction force would be more sensitive to pile spacing. Therefore, the flexibility of passive piles does not only rely on the spacing of the pile but also is determined by unstable soil layer thickness. Moreover, Li *et al*. carried out the strength reduction finite difference approach to investigate the impact of pile spacing on pile stabilized slope [60]. The numerical result demonstrated that with increase in the spacing of the pile row, the slope failure mechanism gradually alters from local failure to the global failure. In addition, if piles were installed adjacent to crest or toe of slopes or the $\boldsymbol{S/D}$ ratio was greater than $\boldsymbol{3}$, the impact of pile spacing on slope stability gradually becomes less.

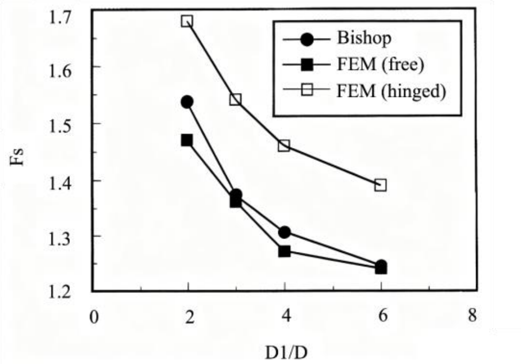

Fig. 18. Effect of spacing on slope stability [35]

Yang *et al*. also analyzed the impact of the spacing of pile on the slope behavior by FEM approach [54]. Regarding parametric study related to pile spacing, the $\boldsymbol{S/D}$ ratio values are $\boldsymbol{2}$, $\boldsymbol{3}$, $\boldsymbol{4}$, and $\boldsymbol{6}$.

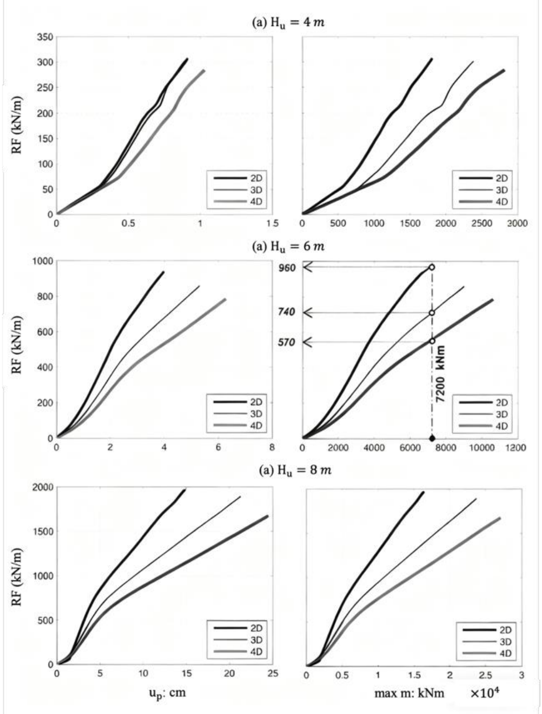

Fig. 19. The impact of pile row spacing on resisting force provided by pile with respect to bending moment and pile head deflection [5]

The pile behavior with various spacing has been depicted in Fig. 20. According to Fig. 20, the bearing capacity of the pile rises with the increase in spacing. This phenomenon is due to the loading zone of each pile within the pile row is expanded with the rise in the pile spacing. As anticipated, the numerical results also indicated that the pile retaining structure becomes closer in similarity to the uninterrupted wall and integrity of pile and soil becomes greater with decrease in pile spacing. Kahyaoglu *et al*. adopted finite element analysis to investigate load transfer mechanism of the passive pile in cohesionless soil [61]. For parametric study, five $\boldsymbol{S/D}$ ratios were adopted and they are $\boldsymbol{2}$, $\boldsymbol{4}$, $\boldsymbol{6}$, $\boldsymbol{8}$, and $\boldsymbol{10}$. The numerical results indicated that with rise in pile spacing, the horizontal force exerting on the passive piles decease. If $\boldsymbol{S/D}$ ratio equal to $\boldsymbol{2}$, approximate by $\boldsymbol{80}\%$ of horizontal force was transmitted to passive piles; when $\boldsymbol{S/D}$ ratio equals $\boldsymbol{6}$, around $\boldsymbol{40}\%$ of horizontal force was transferred to pile row; with further increase in $\boldsymbol{S/D}$ ratio to $\boldsymbol{8}$, the pile within pile row behaves similar to single isolated pile. Wang and Zhang established the numerical model to study the influence of slope inclination, restriction pattern of pile head, the pile location, and pile spacing on response of the pile-stabilized slopes [62]. Two basic concepts, the shear effect and the compression effect, are used to describe the mechanism of reinforcement, indicating that the pile decreases the shear strain and increases the compression stress of slopes compared to slopes without reinforcement. For pa parametric study related to pile spacing, the pile spacing increases from 3D (diameter of pile) to 7D. The numerical results indicated that with a decrease in the spacing of pile, the lateral movements of slopes also decrease. Regarding different pile spacing, the basic rule of lateral movements of the slope was consistent. The response of pile stabilized slopes was considerably affected by pile spacing and other factors, whereas the mechanism of reinforcement is consistent for influencing parameters investigated in current research. Pirone and Urciuoli conducted two-dimensional and three-dimensional numerical computation to study ultimate limit state of the discontinuous and continuous passive piles placed in the slope composed of frictional soil [48] (Fig. 21).

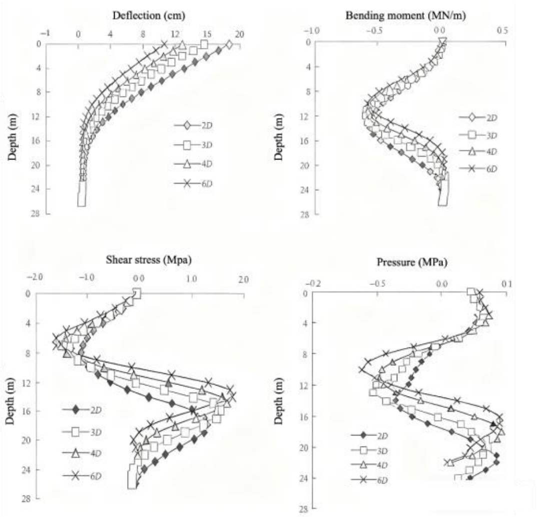

Fig. 20. The pile behavior with different spacings [54]

The three-dimensional parametric study was carried out to study the effect of the pile spacing on the soil arching effect and the limit lateral load exerted on piles. For the parametric study of pile spacing, the pile spacing adopted ranges between 2 and. For the condition that the embedded pile length equals the pile length in the unstable layer, the threshold value of pile spacing approximately equals **3**. In addition, the soil arching effect normally occurs when pile spacing ranges between **3 - 6** . The aforementioned numerical analysis on $\boldsymbol{S/D}$ ratio of pile group has been summarized in Table 2.

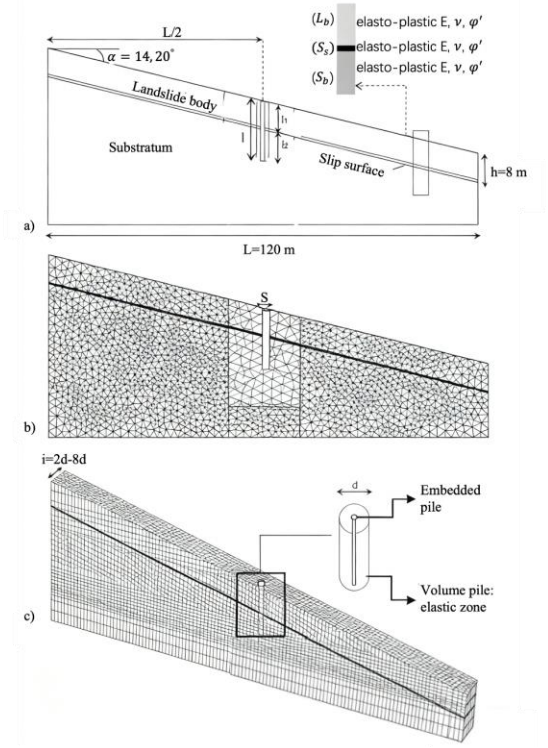

Fig. 21. Geometry and mesh for two-dimensional and three-dimensional finite element analysis [48]

Table 2. Summary of numerical studies of the $\boldsymbol{S/D}$ ratio

No.	1	2	3	4	5
Authors	[35]	[59]	[5]	[60]	[54]
Numerical methods	FEM	FEM	Hybrid method	FDM	FDM
Software	-	-	Abaqus	FLAC	FLAC
2D or 3D	3D	3D	3D	2D	3D
$\boldsymbol{L_p\ (m)}$	**15**	**10**	**10.2-15**	Full length	**6-26**
$\boldsymbol{S/D}$	**2-6**	**2-4**	**2-4**	**2.5-4**	**2-6**
Optimal location	Middle portion of slope	-	-	Middle-slightly upper part of the slope	-
SCM	Elastoplastic model with MC failure criteria	MC-DP constitutive model with non-associative flow rule	Elastoplastic model with MC failure criteria	Linear elastic-perfectly plastic model with MC failure criteria	Perfectly elastoplastic model with MC failure criteria
PCM	Linear elastic solid material	Linearly elastic constitutive model	Nonlinear/linear elastic model	Elastic material	Linear elastic model
PSI	Elastoplastic relation (large elastic stiffness)	Frictional brick elements	-	Spring-slider system	MC model

Table 2. Summary of numerical studies of the S/D ratio (continued)

No.	6	7	8
Authors	[61]	[62]	[48]
Numerical methods	FEM	FEM	FEM
Software	PLAXIS	TOSS3D	PLAXIS
2D or 3D	3D	3D	2D&3D
L_p (m)	$\mathbf{0.5}$	$\mathbf{0.7}$	**9-20**
S/D	**2-10**	**3-7**	**2-8**
Optimal location	-	Upper portion of slope	-
SCM	Perfectly elastoplastic model with MC failure criteria	Elastoplastic constitutive model	Linear elastic-perfectly plastic constitutive model with MC failure criteria
PCM	Linearly elastic constitutive model	Linear elastic model	Perfectly elastic constitutive model
PSI	Coulomb's frictional law	Elastoplastic damage model	Rigid interface

Note: MC: Mohr-Coulomb; DP: Drucker-Prager; FEM: finite element method; FDM: finite difference method; L_p: length of pile; D: pile diameter; S: center-to-center spacing; SCM: soil constitutive model; PCM: pile constitutive model; PSI: pile/soil interface.

5.3 Optimal location of piles in the slope

For optimal position of the passive pile in unstable slopes, several investigations are conducted [35, 53, 55, 56, 60, 62]. The optimal position of passive piles in slopes is summarized and discussed as follows. Lee *et al*. investigated the optimal position of passive piles within slopes under various conditions: (1) homogeneous slope; (2) a slope composed of two layers in which soft upper layer is underlain by stiff layer; (3) a slope composed of two layers in which soft lower soil layer is overlain by stiff layer [1]. For the first condition, it was found that installing piles at crest or toe of slopes results in higher improvement ratio than the pile installed in midpoint of slopes. Regarding second condition, the recommended installation position of passive piles is between the crest to midpoint of slopes. For third condition, the optimal location of passive piles is similar to homogeneous slope. Hassiotis *et al*. also investigated optimal position in slopes for installation of passive piles, and found that the appropriate position for passive piles was in the range of the upper middle part, which is similar to the second slope condition investigated by Lee *et al*. [1, 18] (Fig. 22). With the increase in the slope inclination (from horizontal), the optimal position of passive piles in slopes gradually moves to slope crest. Cai and Ugai also investigated the impact of passive pile position on slope stability by elastoplastic FEM approach [35]. The numerical results indicated that if passive piles were installed at midpoint of slopes, the improvement in slope safety factor would be the largest. However, the result from the Bishop's simplified method indicates that slope safety factor rises with the increase in distance between the pile position and slope toe, until pile position is considerably closer to slope crest. Ausilio *et al*. also conducted limit analysis to investigate optimal position for pile installation within slopes [50]. The results from the limit analysis indicated that the best installation position of passive piles should be closer to slope toe, which is different from the result obtained by [18, 27]. The slope midpoint could be an optimal position for passive piles, whereas when the higher improvement ratio value was required, the passive pile should be placed as close as possible to slope toe.

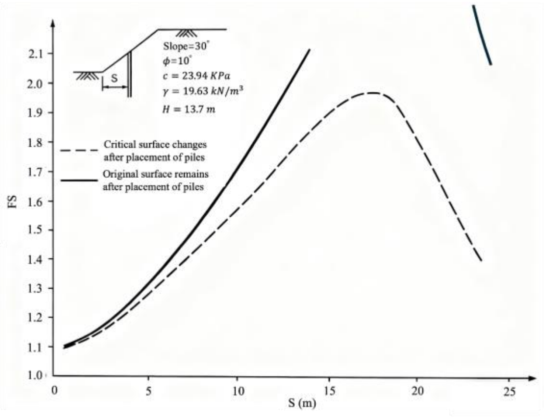

Fig. 22. Effect of pile position on slope stability [18]

Nian *et al*. combined the strength reduction technique with limit analysis to investigate the optimal position of passive piles in slopes [24].

Regarding all conditions analyzed by the limit analysis approach, the suitable position of the passive pile has been determined as slope toe where resisting forces supplied by passive piles to improve slope stability to design value would be the lowest. In addition, it was also found that when the slopes are steep and composed of the nonhomogeneous and anisotropic soil, the suitable position of the installation of passive piles would still be closer to slope toe. Griffiths *et al*. conducted several modifications for one finite element slope stability program and adopted this program to assess the impact of passive pile position on the slope stability [53]. They found that when pile length was relatively shorter, the suitable passive pile location is in the lower half of slopes to reach the largest safety factor value. With rise in length of pile, the optimal pile installation location gradually moves to slope midpoint. It could be concluded that the most effective pile position mainly concentrates on lower half of slopes. Li *et al.* conducted finite difference analysis to study the impact of the pile position on behavior of slopes stabilized by one pile row [60]. Through a series of parametric study, it was noticed that the optimal passive pile installation location in slopes yielding the maximum safety factor value was in the range from midpoint to slightly upper part of slopes as shown in Fig. 23. This finding is contrary to the result obtained from Griffiths *et al.* related to the appropriate pile location in slopes [53]. It was also found that with the rise in pile spacing, the optimal pile position within slopes moves gradually to slope crest. Ho performed numerical analysis to study the optimal passive pile location to improve slope stability [56]. A non-dimensional factor, the ratio of the distance between pile and slope toe to slope length, was adopted to indicate passive pile location (Fig. 24). The numerical results indicated that the highest slope safety factor value always takes place when piles are inserted at middle portion of slopes (Fig. 25). In addition, the lowest slope safety factor occurs when passive piles were installed at slope crest. The numerical result also indicated that pile location governs the slope global stability, as variation of pile position changes critical slip surface depth. Moreover, it is found that if passive piles are placed at middle, toe, and crest of slopes, the slope failure mechanism will be governed by both soil masses, upslope soil masses, and downslope soil masses, respectively. The aforementioned numerical analysis for optimal passive pile location has been summarized in Table 3.

6. CONCLUSIONS

The laterally loaded pile has been adopted extensively for the stabilization of the slope and has been proved as an effective measurement to prevent geohazards. Therefore, various analysis approaches have been derived to investigate the effect of the pile on stabilizing soil slopes. Regarding pressure-based method and displacement-based method, the displacement of the soil and potential failure plane need to be presumed and then response of pile and the slope would be evaluated. After the first employment of Finite Element Method for geotechnical engineering, this kind of analysis method has been developed continuously.

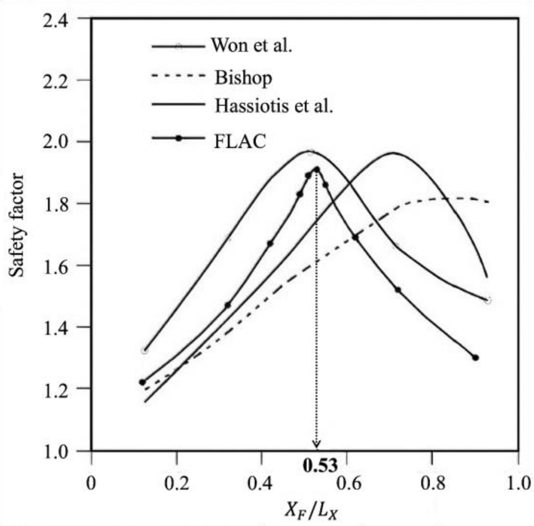

Fig. 23. Effect of pile position on slope stability [60]

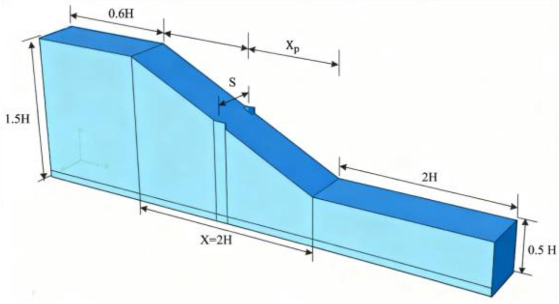

Fig. 24. Illustration of the numerical pile-slope model (Ho 2017)

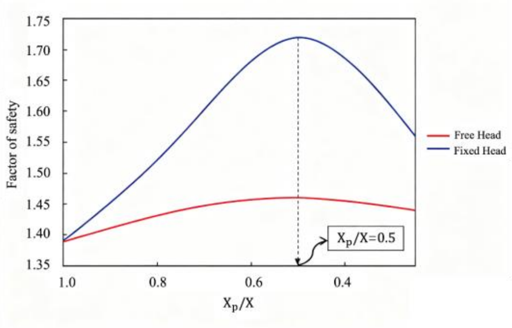

Fig. 25. The relation between slope stability and pile position (Ho 2017)

Table 3 Summary of recommended optimal location of piles

No.	1	2	3	4	5
Authors	[1]	[18]	[35]	[50]	[24]
Numerical methods	Simplified circle approach	-	FEM	LA	LA
Software	SLOPIL	Developed procedure	-	Developed procedure	Developed code
2D or 3D	2D	2D	3D	2D	2D
L_p (m)	Full length	**20**	**15**	**25**	-
S/D	**1.5-5**	**2.5**	**2-6**	-	-
Optimal location	Crest and toe *or* between crest and middle (depend on the relative strength of upper and lower soil)	Upper middle part of slope; slope inclination increases, optimal location moves towards crest of slope	Midpoint of slope, regardless of the pile head condition	Near slope toe, the range between toe and middle can also make piles effective	Near slope toe, regardless of the slope composed of nonhomogeneous or anisotropic soil
SCM	Perfectly elastoplastic model with MC failure criteria	MC constitutive model	Elastoplastic model with MC failure criteria	Perfectly elastoplastic model with MC failure criteria	MC constitutive model
PCM	Linear elastic model	Elastic material	Linear elastic solid material	Linear elastic solid material	Elastic material
PSI	Elastic continuum with nonuniform variation of strength and stiffness	-	Elastoplastic relation with considerably large elastic stiffness	-	-

Table 3 Summary of recommended optimal location of piles (continued)

No.	6	7	8	9
Authors	[53]	[60]	[55]	[56]
Numerical methods	FEM	FDM	FEM	FEM
Software	p_63s	FLAC	Abaqus	Abaqus
2D or 3D	2D	2D	3D	3D
L_p (m)	**6-16**	Full length	**5-22.5**	**20-38**
S/D	-	**2.5-4**	**2-6**	**4**
Optimal location	For shorter piles, the lower slope part; for longer piles, the slope midpoint	Middle-slightly upper portion of slope to obtain the maximum factor of safety	Middle-slightly upper part of the slope	Middle portion of slope for both fixed head and free head piles
SCM	Perfectly elastoplastic model with MC failure criteria	Linear elastic-perfectly plastic constitutive model with MC failure criteria	Perfectly elastoplastic model with MC failure criteria	Perfectly elastoplastic model with MC failure criteria
PCM	Beam-rob element	Elastic material	Linear elastic model	Linear elastic model
PSI	-	Spring-slider system	Friction coefficient	Friction coefficient

Note: MC: Mohr-Coulomb; DP: Drucker-Prager; LA: limit analysis; FEM: finite element method; FDM: finite difference method; **L_p**: length of pile; **D**: pile diameter; **S**: center-to-center spacing; SCM: soil constitutive model; PCM: pile constitutive model; PSI: pile/soil interface.

This has been used widely due to its own advantages, such as no need to presume the location or shape of the critical sliding plane, and can predict slope deformations under working stress conditions. Regarding another analysis method discussed in the current study, the Limit Analysis Method has also been used widely in slope stability-related issues as the rigorous lower and upper bound of slope stability problem could be narrowed down in which the true solution exists. Moreover, the searching of optimal velocity and stress fields by limit analysis method is a relatively complicated trial-and-error procedure, the finite element discretization is therefore used to establish the fields of stress and velocity and determine strict lower and upper bound solution. Regarding reinforcing mechanisms of the pile retaining structure, the soil arching mechanism that exists in a wide variety of geotechnical engineering practices plays an important role. When the unstable soil mass tends to yield, the stress is transmitted to abutting stable mass, and this kind of stress redistribution process can be regarded as soil arching mechanism. In current study, the basic concept of soil arching mechanism has been illustrated, the impact of pile group arrangement on soil arching magnitude has been investigated, and influence of soil factors on development of soil arching has also been discussed and reviewed. In addition, the performance of single pile within the pile row would be influenced by presence of neighboring pile, and that phenomenon could be regarded as pile group effect in which pile-soil-pile interaction would be taken into account. The ratio of lateral earth pressure exerting on single pile within group to the horizontal earth pressure acting on the single isolated pile would be the reasonable indication of pile group effect. The parameters that impose impacts on the pile group effect magnitude and assessment method of group effect magnitude have also been reviewed and discussed.

From the practical geotechnical perspective, the design parameter of pile retaining wall plays a significant role in slope response. Regarding length of the pile, several studies introduced ratio of embedded pile length to unstable soil layer height, and purposed that embedded pile length at least equals unstable soil height to avoid rotation failure mode. Moreover, the effect of pile length on critical failure plane formation has also been discussed. The ratio of spacing to diameter also imposes significant impact on response of pile stabilized slope. The pile group spacing is believed to have a close relationship with soil arching magnitude, and soil arching magnitude would become less if ratio of spacing to diameter exceeds threshold value. With decrease in pile group spacing, the integrity of pile and soil becomes greater and the pile retaining structure becomes more similar to continuous wall; however, with the increase in pile group spacing, the lateral forces exerting on pile increases, as soil arching and group effect becomes less. Regarding optimal pile position in slopes, the recommended pile installation position is various. The optimal pile location relies on various parameters, such as soil characteristics, pile head conditions, slope inclination, pile length, and other influencing factors, therefore, obtained optimal pile position from different studies is various.

In general, this study has presented a comprehensive review on pile reinforced slopes in terms of analysis methodology, reinforcing mechanism, and determination of the critical design parameters. Recent developments in Limit Analysis Method and Finite Element Method for the stability of slope reinforced by pile retaining structure have also been reviewed. The reinforcing mechanism of pile retaining wall and the effect of critical design parameters have also been discussed thoroughly. According to this review, the pile stabilized slope has received substantial attention from literature. Regarding the study related to the pile stabilized slope in the future, the coupling interaction between soil and pile needs to be investigated further. Moreover, more emphasis can be placed on the seismic capability of the pile retaining structure and the impact of unsaturated soil condition on pile stabilized slope behavior.

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