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GEOMETRIC NONLINEAR ANALYSIS OF TWO-LAYER TIMBER COMPOSITE BEAMS WITH ELASTIC CONNECTIONS USING THE γ-METHOD IN MATLAB

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ABSTRACT: In sustainable construction, two-layer composite timber beams with elastic shear connections are increasingly adopted due to their efficient use of material and enhanced structural performance. However, existing design procedures, such as the γ-method specified in EN 1995-1-1, are primarily based on linear assumptions and often neglect geometric nonlinearity. This simplification may lead to underestimation of deformation and stiffness reduction in slender composite members subjected to higher load levels. In this study, an enhanced numerical framework is developed in MATLAB to investigate the nonlinear behavior of two-layer composite timber beams. The proposed approach integrates a tangent-stiffness updating scheme with the von Kármán strain formulation, enabling consideration of geometric nonlinearity, including P–Δ effects and membrane forces. The numerical results demonstrate that geometric nonlinearity leads to a significant increase in mid-span deflection, ranging from 22% to 31% compared with linear analysis under advanced loading stages. In addition, the analysis identifies a critical connection stiffness threshold, beyond which the composite system approaches full interaction behavior. The proposed framework provides a practical and reliable computational tool for engineers, allowing more accurate assessment of deformation and stiffness characteristics and contributing to safer and more rational design of slender composite timber beams.

Keywords: Timber Structures; Composite Timber Beam; γ - Method; Elastic-Shear Connection; MATLAB; Geometric Nonlinearity.

1. INTRODUCTION

In recent years, composite timber members have attracted increasing attention in structural engineering due to their efficient material utilization, favourable strength-to-weight ratio, and applicability in sustainable construction systems [1,2]. Mechanically connected layered timber beams are increasingly used in floors, roofs, and medium-span structures because composite action between layers can significantly improve flexural stiffness and serviceability performance compared with conventional solid timber members [3,4]. Previous studies demonstrated that the global response of such systems is strongly governed by the degree of interaction between layers, which depends on the stiffness of the shear connection and the resulting interlayer slip behaviour [5,6]. In addition, mechanically laminated timber systems with semi-rigid interlayer connections have also been investigated as an effective alternative for increasing section capacity while maintaining relatively simple fabrication techniques [7].

The mechanical performance of partially composite timber beams is influenced by several parameters, including connection rigidity, fastener arrangement, layer configuration, and interlayer deformation compatibility [4–7]. Experimental investigations on mechanically jointed timber beams showed that the use of mechanical fasteners can provide substantial bending resistance while still introducing noticeable stiffness reduction due to partial interaction effects [4]. Similarly, studies on inclined screw connections demonstrated that the slip modulus depends not only on timber density and fastener geometry, but also on withdrawal stiffness and local deformation mechanisms within the connection zone [6]. Such findings highlight the importance of accurately representing interlayer slip behaviour when analysing composite timber members.

To evaluate partial composite action, EN 1995-1-1 introduces a simplified analytical procedure for mechanically jointed beams in Annex B, commonly referred to in technical literature as the γ-method [8]. The method employs an interaction coefficient γ to account for the influence of finite slip stiffness on the effective flexural rigidity of the composite section. Owing to its relatively simple formulation, transparent mechanical interpretation, and consistency with practical design procedures, the γ-method has become one of the most widely adopted approaches for estimating stiffness and serviceability deflection of composite timber beams [5,8]. Analytical studies based on partial interaction theory further demonstrated that equivalent stiffness formulations can effectively represent the deformation behaviour of layered composite members while maintaining relatively low computational complexity [5].

Alongside simplified analytical procedures, finite-element modelling has become an important tool for investigating the structural behaviour of timber composite systems. Numerical studies on timber–concrete composite beams demonstrated that finite-element methods can effectively reproduce load–deflection response, interface slip, stiffness degradation, and stress redistribution under increasing loading [1,9,10]. Different modelling strategies, including continuum-based finite elements, frame-based formulations, and nonlinear spring representations, have been adopted to simulate partial interaction between connected layers [9,10]. Comparative investigations further showed that simplified finite-element approaches can provide satisfactory agreement with experimental measurements while maintaining significantly lower computational effort than fully detailed three-dimensional simulations [11].

Recent numerical investigations also demonstrated the applicability of finite-element analysis for evaluating deformation behaviour and structural efficiency in laminated timber and bamboo-based composite beam systems [2,12]. These studies highlighted the increasing importance of computational analysis in assessing deformation mechanisms and serviceability performance in sustainable structural systems. Furthermore, recent investigations on deterioration modelling and long-term performance assessment have emphasized the importance of stiffness evolution and deformation prediction in timber structural systems subjected to sustained actions and environmental effects [13].

Despite these developments, most practical applications of the γ-method remain limited to first-order linear elastic analysis. For slender composite beams subjected to moderate or large transverse deflections, geometric nonlinearity may become increasingly important. Under such conditions, lateral deformation generates additional axial stretching and membrane-force effects that alter the tangent stiffness and modify the equilibrium path of the structure [14]. Classical nonlinear finite-element formulations demonstrated that second-order effects may substantially influence deformation behaviour even when the material response remains elastic [14]. Consequently, linear formulations may underestimate deflection and overestimate stiffness under advanced loading stages.

In practical timber construction, perfectly straight members are rarely achieved after fabrication and installation. Initial geometric deviations may arise from manufacturing tolerances, moisture-induced deformation, assembly imperfections, and long-term environmental effects. Although these imperfections are often relatively small, they may amplify second-order effects and increase the sensitivity of slender members to geometric nonlinearity. Existing studies generally evaluate such effects through load–deflection comparisons, while the evolution of tangent stiffness and membrane-force development in composite timber beams remains insufficiently discussed.

Advanced continuum-based finite-element simulations can accurately represent nonlinear structural behaviour; however, they often require detailed constitutive relationships, extensive material characterization, and relatively high computational effort [9,10]. Such approaches may therefore be impractical for rapid parametric studies and preliminary engineering assessment. In contrast, simplified computational procedures based on equivalent stiffness concepts provide a more transparent and computationally efficient framework for practical nonlinear evaluation.

The primary mechanical motivation for considering geometric nonlinearity in the present study lies in the membrane effect generated by large transverse deformation. As the beam deflects, axial stretching develops along the longitudinal direction, producing additional internal membrane forces that influence the tangent stiffness and overall structural response. This second-order behaviour becomes increasingly significant in slender composite systems with partial interaction and may lead to noticeable stiffness degradation even when the material response remains elastic [14,15].

In this study, a MATLAB-based nonlinear analytical–numerical framework is developed for two-layer composite timber beams with elastic shear connections. The formulation combines the γ-method with a tangent-stiffness updating procedure and the von Kármán strain-displacement relationship in order to account for geometric nonlinearity, including P–Δ effects and membrane-force development. The composite beam is discretised using Euler–Bernoulli beam elements, while partial interaction between layers is represented through elastic shear connections with finite slip stiffness. The proposed procedure enables the evaluation of load–deflection response, tangent stiffness evolution, membrane-force development, and nonlinear response amplification under varying geometric and connection parameters.

The aim of this study is to demonstrate that a relatively compact γ-based computational framework can reproduce the principal nonlinear mechanical characteristics of composite timber beams within the serviceability range while maintaining significantly lower computational complexity than detailed continuum-based finite-element models. The proposed approach provides a practical numerical tool for comparing connection alternatives, evaluating stiffness evolution, and supporting the rational design of slender composite timber members subjected to geometric nonlinear effects.

2. RESEARCH SIGNIFICANCE

This study addresses a practical limitation in the design of two-layer composite timber beams with elastic shear connections, where geometric nonlinearity is commonly neglected in standard applications of the γ-method in EN 1995-1-1. An enhanced MATLAB-based framework is proposed by integrating von Kármán geometric nonlinearity into a tangent-stiffness updating scheme. The approach enables efficient consideration of P–Δ effects and membrane forces while retaining the simplicity of design-oriented calculations. The results provide engineers with a practical tool for more accurate deformation assessment and rational design of slender composite timber beams.

3. THEORETICAL BACKGROUND AND ANALYSIS METHOD

3.1. Mechanical Model and Assumptions

The γ-method presented in Annex B of Eurocode 5 provides a simplified approach to transform a two-layer composite timber system connected by an elastic shear connection into an equivalent single-material beam with an effective bending stiffness EI_{eff}.

In practical structures, the connection between the two layers is rarely perfectly rigid and can transmit only part of the shear stress, resulting in relative slip and partial interaction between the layers. The main assumptions are as follows:

- The timber material behaves elastically with a modulus of elasticity E.

- Material: The composite timber beam consists of two layers, each having dimensions b × t and span length L. The timber is assumed to be orthotropic, working in the serviceability range under linear elastic behavior with a longitudinal modulus of elasticity E. The cross-sectional area of each layer is A = b·t, the second moment of area is $I = bt^3/12$, and the distance from the centroid of each layer to the neutral axis is a= t/2.

- Connection between layers: The layers are connected through an elastic shear connection characterized by a shear stiffness per unit length K (N/mm). The stiffness K is derived from the serviceability shear modulus of the fasteners K_{ser} (N/mm) considering the fastener spacing s (mm) along the beam axis and the number of fastener rows n_r as shown in Fig. 1.

$$K \approx \frac{n_r \cdot K_{ser}}{s} \tag{1}$$

- Bending assumption: The beam is modeled as an Euler–Bernoulli beam, neglecting shear deformation within each layer (valid for sufficiently large L/t ratios), with partial interaction between layers due to the finite shear stiffness K.

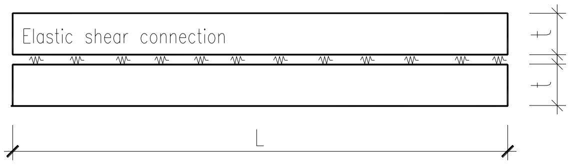

Fig. 1 Two-Layer Timber Composite Beam

- Nonlinearity: Both the material and the connection behave elastically; geometric nonlinearity of the von Kármán type (P–Δ) is considered.

- Boundary conditions and loading: The beam is simply supported at both ends, with ω(0) = ω(L) = 0 and free rotation at the supports. To activate the P–Δ effect, the overall axial shortening of the system is constrained (global longitudinal restraint). The bending load is applied through a four-point loading configuration, with two equal loads positioned at L/3 and 2L/3 along the span as shown in Fig.2.

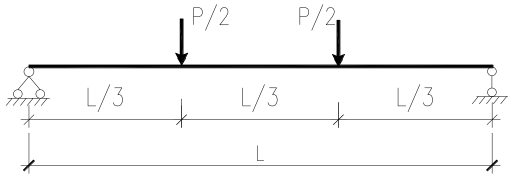

Fig. 2 Analysis scheme and four-point-loading configuration

3.2. Gamma Method and effective bending stiffness

3.2.1. Limiting cases and effective bending stiffness

- The bending stiffness of a two-layer system lies between two limiting cases:

$-EI_{nc}=E \cdot 2I$ (soft connection)

$-EI_{fc} = E \cdot \left[2 \cdot (I + A \cdot a^2)\right]$ (fully rigid connection)

The actual behavior of the composite beam lies between these limits and is characterized by the coefficient γ (0 < γ < 1), which reduces the coupling term EAa^2 according to the degree of connection [7]:

$$\gamma = \frac{1}{1+\pi^2 \dfrac{EA}{KL^2}} \tag{2}$$

The coefficient γ depends on the ratio between the axial stiffness EA/L^2 and the shear stiffness K. When K is large or L is short, γ → 1 (fully composite); conversely, when K is small or L is long, γ → 0 (non-composite).

Accordingly, (3) give the effective bending stiffness [5]:

$$EI_{eff} = E \cdot 2I + 2 \cdot \gamma EAa^2 \tag{3}$$

To compare the degree of composite action among different connection configurations, a normalized connection efficiency index can be used:

$$\eta = \frac{EI_{eff} - EI_{nc}}{EI_{fc} - EI_{nc}} \in [0,1], \qquad (4)$$

where η = 1 corresponds to a fully rigid connection, and η = 0 corresponds to a soft connection. The origin of the expression defining the γ coefficient can be explained from the differential equation describing the relative slip between the two layers [7]. Consider two layers with longitudinal displacements $u_1(x)$ and $u_2(x)$, and deflection ω(x). The relative slip is:

$$\delta(x) = u_1(x) - u_2(x) - 2a\frac{d\omega}{dx} \qquad (5)$$

The distributed shear force in the connection is q(x) = K·δ(x). From the equilibrium of axial forces in each layer:

$$EA\frac{d^2u_1}{dx^2} = q(x), \quad EA\frac{d^2u_2}{dx^2} = -q(x) \qquad (6)$$

one obtains the governing equation for δ(x):

$$EA\frac{d^2\delta}{dx^2} - 2K\cdot\delta = 0 \qquad (7)$$

Solving this equation with boundary conditions δ(0)=δ(L)=0 and a sinusoidal bending moment distribution $M(x)=M_0\cdot\sin(\pi x/L)$ yields the slip function δ(x). Substituting it into the equation leads to the dimensionless parameter $\pi^2EA/(KL^2)$, which represents the ratio between axial stiffness and shear stiffness.

The constant π appears due to the sinusoidal distribution of slip in a simply supported beam (first vibration mode). When formulating the global moment–curvature relationship, the coupling term EAa^2 is reduced by a factor $\left(1+\pi^2\frac{EA}{KL^2}\right)^{-1}$, it means that γ as defined in Annex B of [7].

Therefore, the parameter $\pi^2EA/(KL^2)$ has the following physical interpretation:

+ When the parameter is large → soft connection, large slip, small γ;

+ When the parameter is small → stiff connection, small slip, γ approaches 1.

3.2.2. Application in bending analysis

In all linear bending equations (Euler–Bernoulli theory), the bending stiffness EI can be replaced by the effective stiffness EI_{eff} to determine deflection, rotation, or stress.

For example, for a simply supported beam under a uniformly distributed load q:

$$\delta_{max} = \frac{5qL^4}{384(EI)_{eff}} \qquad (8)$$

By using the γ-method, the influence of the shear connection stiffness K on serviceability deflection and the degree of composite action can be quickly evaluated without solving complex differential equations.

Notes: It can thus be observed that the γ-method results from the analytical solution of the bending problem of a composite beam with an elastic shear connection, which has been standardized in [7] as a set of simplified expressions (Annex B) for the design of composite timber beams. This method accurately represents the partial interaction behavior of elastic connections and serves as a foundation for the development of more advanced analytical approaches.

3.3. Geometric Nonlinearity According to von Kármán (P–Δ)

3.3.1. Scope and assumptions

The geometric nonlinear analysis considered here applies to cases of small strains but moderate rotations or deflections. The von Kármán approximation is adopted for the Euler–Bernoulli beam with the following assumptions:

- Shear deformation within each layer is neglected.

- Both the material and the interlayer connection behave elastically (only geometric nonlinearity is considered).

3.3.2 von Kármán strain for the Euler–Bernoulli beam

For the lateral displacement ω(x) (deflection) and the axial displacement u(x) of the neutral axis, the generalized axial strain (in the reduced Green–Lagrange form) at a fiber located a distance z from the neutral axis is expressed as:

$$\varepsilon(x,z) \approx u'(x) + \frac{1}{2}(\omega'(x))^2 - z\omega''(x) \qquad (9)$$

where k(x) = ω''(x) is the curvature of the neutral axis (sign convention according to bending), θ(x) = ω'(x) is the rotation of the cross-section. The nonlinear geometric component lies in the quadratic term $\frac{1}{2}(\omega')^2$, which induces axial membrane forces in the neutral axis even in the absence of external axial loading.

3.3.3. Axial membrane force and boundary constraint

The average axial force over the cross-section is given by [3]:

$$N(x) = \int_A E\cdot\varepsilon(x,z)dA \approx EA_{eff}\left(u'(x) + \frac{1}{2}\cdot(\omega'(x))^2\right) \qquad (10)$$

since ∫Az·dA = 0 along the neutral axis.

Under moderate and large transverse deflections, the von Kármán strain-displacement relationship introduces an additional nonlinear axial strain component associated with beam curvature [10,11]. In the present study, the support-loading configuration corresponds to a simply supported beam subjected to overall longitudinal restraint. Consequently, the total axial elongation along the span is assumed to be zero. The average membrane strain $\bar{\varepsilon}_0$ can therefore be approximated as:

$$\frac{1}{L}\int_0^L u'(x)dx \approx 0 \rightarrow \ \bar{\varepsilon}_0 \equiv \frac{1}{L}\int_0^L \left(u'(x)\right) + \frac{1}{2}\theta^2(x))dx \approx \frac{1}{2L}\int_0^L \theta^2(x)dx \tag{11}$$

As a result, a self-induced membrane force N_{mem} arises due to geometry [9-11]:

$$N_{mem} = EA_{eff}\,\bar{\varepsilon}_0 = \frac{EA_{eff}}{2L}\int_0^L \left(\theta(x)\right)^2 dx \geq 0 \tag{12}$$

where:

-u′(x) is the axial strain component,

-θ(x) is the beam rotation,

- $\bar{\varepsilon}_0$ denotes the average membrane strain generated by geometric nonlinearity.

It is observed that as deflection increases, the rotation θ(x) also increases, leading to accumulated geometric elongation. Consequently, a positive membrane force develops, increasing the tangent stiffness of the system, a "stiffening" type P–Δ effect within the serviceability range. In cases with initial compression or external axial force, the sign and magnitude of N may differ; however, in this study, only the case without external axial loading is considered.

3.3.4. Equilibrium equation and "geometric stiffness"

In the Euler–Bernoulli beam model, the bending moment obeys a linear elastic law. The response is characterized by an effective bending stiffness.

$$M(x) = EI_{eff}\cdot k(x) = EI_{eff}\cdot \omega''(x) \tag{13}$$

When a membrane force N_{mem} exists, the tangent linearized transverse equilibrium equation becomes:

$$EI_{eff}\cdot \omega''''(x) + N_{mem}\cdot \omega''(x) = q(x) \tag{14}$$

where q(x) is the equivalent distributed or converted concentrated load [11].

Eq. (14) represents the tangent linearization of the geometrically nonlinear beam problem according to von Kármán [11]. The term $N_{mem}\cdot\omega''(x)$ is the origin of the geometric stiffness matrix, which modifies the tangent stiffness depending on the current deflection state.

For an element of length l_e, assuming linear interpolation of θ. Under this assumption, the conventional form of the geometric stiffness matrix $\mathbf{K}_g$ can be expressed as [11]:

$$K_g = \frac{N_{mem}}{30\cdot l_e}\begin{bmatrix} 36 & 3l_e & -36 & 3l_e \\ 3l_e & 4l^2{}_e & -3l_e & -l^2{}_e \\ -36 & -3l_e & 36 & -3l_e \\ 3l_e & -l^2{}_e & -3l_e & 4l^2{}_e \end{bmatrix} \tag{15}$$

In the two-node Euler–Bernoulli beam element (four degrees of freedom: ω_1, θ_1, ω_2, θ_2), the linear bending stiffness matrix is [11]:

$$K_b = \frac{EI_{eff}}{l^3{}_e}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l^2{}_e & -6l_e & -2l^2{}_e \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & -2l^2{}_e & -6l_e & 4l^2{}_e \end{bmatrix} \tag{16}$$

where l_e is the element length and EI_{eff} is the effective bending stiffness determined by the γ-method.

When considering geometric nonlinearity (P–Δ), the total tangent stiffness matrix is obtained by summing the two matrices:

$$K_t = K_b + K_g \tag{17}$$

3.4. Analysis procedure using MATLAB

Based on the theoretical background presented in Sections 2.1–2.3, the authors developed a computational model in MATLAB to simulate the bending behavior of a two-layer glued laminated timber beam with elastic shear connection, taking into account geometric nonlinearity according to the von Kármán theory. The model was formulated using the finite element method, with Euler–Bernoulli beam elements that include two degrees of freedom at each node as shown in Fig. 3.

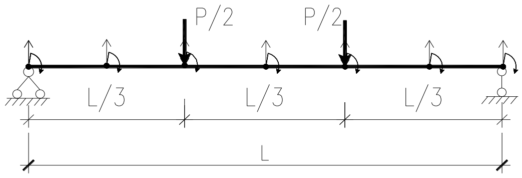

Fig. 3. Discretization of the beam and associated degrees of freedom

a. Input Data Definition

The geometric parameters b, t, L, the material modulus E, and the connection stiffness K are defined as input data.

The beam is divided into n_e finite elements (two-node Euler–Bernoulli elements with four degrees of freedom). Boundary conditions are specified as simply supported with no overall axial deformation. The load is applied incrementally in n_{step}, distributed at two positions, L/3 and 2L/3.

b. Calculation of Effective Parameters

From the material and connection properties, the composite interaction coefficient γ and the effective bending stiffness EI_{eff} are determined using the γ-method. These values are then assigned to the linear bending stiffness matrix $\mathbf{K}_b$ and used in the nonlinear computation.

c. Assembly of the Stiffness Matrix

The element stiffness matrices are formulated and assembled into the global bending stiffness matrix [$\mathbf{K}_b$]. The displacement vector {$\mathbf{U}$}=0 and the load vector {$\mathbf{F}$}are initialized.

At this stage, nonlinearity is not yet considered, and the system is in a purely elastic state.

d. Incremental Solution and Update of Axial Forces

The load is gradually increased from 0 to P_{max}. At each load step, the current displacements and rotations are used to compute the axial membrane force N_{mem} according to the expression (10) given in Section 2.3. This value is incorporated into the geometric stiffness matrix $\mathbf{K}_g$ to update the tangent stiffness matrix as $\mathbf{K}_t=\mathbf{K}_b+\mathbf{K}_g$.

The incremental equilibrium equation $\mathbf{K}_t \cdot \Delta\mathbf{U}=\Delta\mathbf{F}$ is solved for the system's degrees of freedom, and the displacements are accumulated for the next load step.

e. Updating and Convergence Check

After each iteration, the displacement vector $\{\mathbf{U}\}$ is updated, and the displacement error $\|\Delta\mathbf{U}\|$ is compared with the convergence tolerance. If the error exceeds the limit, the load increment is reduced or the number of Newton–Raphson iterations is increased for the current load step.

The procedure continues until the target load or deflection limit is reached.

f. Output and Post-Processing

Upon completion of the analysis, the program automatically generates plots, including the load–deflection (P-δ) curves for both linear and nonlinear cases, the distribution of axial forces N_{mem}, the tangent stiffness $K_t(P)$, and the nonlinearity index $\varphi=\delta_{non}/\delta_{lin}$. The results are normalized and represented as parameter maps of δ_{non}/L versus K, L, and for parametric evaluation.

To ensure the numerical stability and accuracy of the proposed procedure, a mesh convergence study was conducted. The number of elements (n_{el}) was varied from 10 to 80. It was observed that the mid-span deflection converged at n_{el} = 40, with a relative difference of less than 0.5% compared to finer meshes. Consequently, a mesh of 40 elements was adopted for all subsequent simulations to balance accuracy and computational cost.

4. RESULTS AND DISCUSSION

4.1 Input Parameters and Validation

To illustrate the computational procedure described in the previous sections, a two-layer timber composite beam subjected to symmetric four-point bending under simply supported conditions is considered. This loading configuration is commonly used in experiments to determine deflection and to validate bending models. The example is performed for a two-layer glued laminated timber beam with elastic–slip connection, evaluated using the γ-method of Eurocode 5, Annex B. The beam is simply supported and loaded at four symmetric points, and geometric nonlinearity (P–Δ effect) is activated by imposing zero overall axial deformation along the span (Fig. 4).

Geometric and material parameters: Span L=2.4m; width of each layer b=60 mm; thickness t=30 mm; elastic modulus E=11000 MPa; shear stiffness K=600 N/mm. The load consists of two concentrated forces P/2 applied at L/3 and 2L/3. The beam is discretized into 40 two-node Euler–Bernoulli elements (four degrees of freedom per element).

Analysis: The computations are performed in MATLAB using an in-house code, and two cases are compared:
- Linear elastic analysis (without P–Δ effects);
- Geometric nonlinear analysis including self-induced membrane forces.

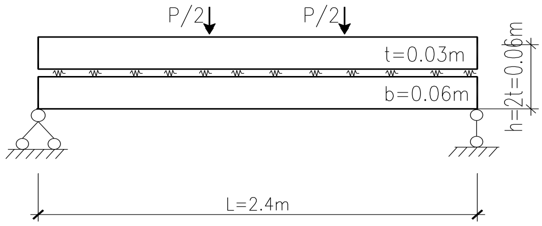

Fig. 4. Case Study: Bending Analysis of a Two-Layer Timber Composite Beam

The effective bending stiffness EI_{eff} is evaluated via the γ-method and assigned to the element stiffness matrices. The global tangent stiffness is updated iteratively using the Newton–Raphson procedure until convergence is achieved.

The linear results obtained from the MATLAB procedure show perfect agreement with the analytical γ-method specified in EN 1995-1-1 [7] for low load levels, verifying the baseline accuracy of the numerical model.

4.2 Nonlinear Behavior and Parametric Study

Using the developed MATLAB framework, the numerical results are reported and discussed in the following subsections.

4.2.1 Load–deflection relationship (P–δ)

The results indicate that the P–δ curve of the geometrically nonlinear beam. In the initial stage, the two curves nearly coincide, demonstrating that the initial elastic stiffness is captured accurately. As the load increases, second-order effects become apparent, reducing the stiffness and causing the nonlinear P–δ curve to bend more noticeably.

At higher load levels, approaching approximately 80–90% of the maximum applied load, the nonlinear deflection exceeds the linear prediction by about 20–30%, depending on the beam slenderness and connection stiffness. This deformation amplification is attributed to second-order (P–Δ) effects, in which self-induced axial membrane forces progressively reduce the tangent stiffness and amplify lateral deflections. These findings emphasize the necessity of considering geometric nonlinearity when analyzing slender beams or beams subjected to near-limit loading, in order to achieve more accurate predictions of deflection and structural stability.

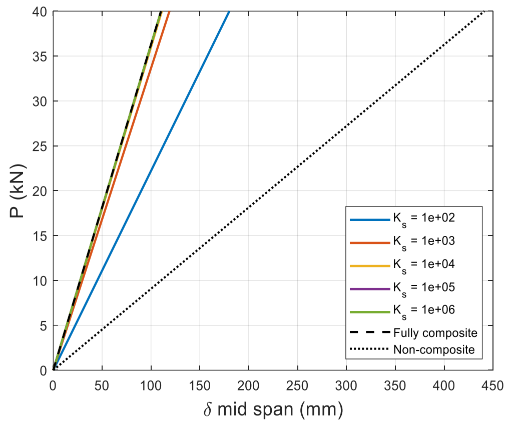

Fig. 5. Influence of shear-connection stiffness K_s on the load–deflection response

4.2.2 Influence of the shear connection stiffness K_s on the effectiveness of composite action

Fig. 6 illustrates the relationship between the shear-connection stiffness K_s and the composite-action efficiency η, which characterizes the degree of cooperative action between the layers in a multi-layer composite beam. Here, K_s represents the shear stiffness of the slip connection between the two layers.

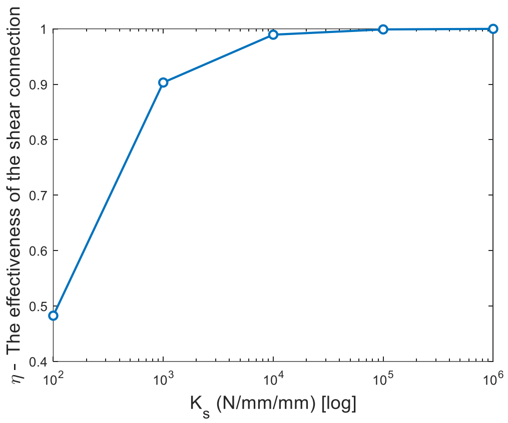

Fig. 6. Influence of shear-connection stiffness $K_s$on the load–deflection response

As K_s increases from 10^2 to 10^4 N/mm^2, the efficiency η rises rapidly from approximately 0.45 to nearly 1.0, indicating a significant improvement in the interlayer shear interaction. When $K_s \geq 10^5$N/mm^2, η approaches its maximum value, corresponding to a fully composite state in which the two layers behave as a single structural unit.

These results show that the shear-connection stiffness plays a decisive role in the overall composite efficiency: when K_s is low, the beam exhibits only partial composite action, resulting in a reduced bending stiffness; in contrast, sufficiently large K_s provides stiffness levels close to those of a fully composite member. Therefore, in the design of multi-layer composite beams, a minimum shear-connection stiffness of approximately 10^4 N/mm^2 is recommended to ensure the desired level of composite action.

4.2.3 Influence of the shear connection stiffness K_s on the effectiveness of composite action

Fig. 7 shows that the P–δ curve of the geometrically nonlinear beam exhibits a pronounced downward curvature compared with the linear reference response, clearly reflecting the influence of second-order (P–Δ) effects. In the low-load range, the two curves nearly coincide, indicating that the initial elastic stiffness is accurately captured. However, as the load increases, the tangent stiffness gradually decreases, causing the nonlinear curve to bend more sharply and the deflection to grow rapidly.

At higher load levels, the discrepancy between the two models becomes significant: the nonlinear deflection may exceed the linear prediction by 20–30%, indicating the gradual loss of stiffness and stability due to geometric effects. These results confirm that geometric nonlinearity must be considered when analyzing slender beams or beams subjected to near-limit loading, in order to achieve accurate predictions of displacement and overall stability.

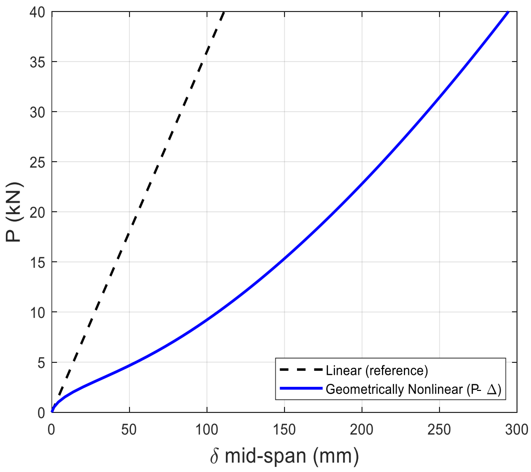

Fig. 7. Load–deflection P–δ curves for linear and geometrically nonlinear analyses

4.2.4 Development of second-order membrane force and corresponding tangent stiffness of the beam

Fig. 8 presents the evolution of the membrane force N_{mem} with respect to the applied load P (left), and the variation of the tangent stiffness k_t=dP/dδ with midspan deflection δ (right). The results show that the membrane force increases nonlinearly with load, demonstrating the pronounced influence of the

P–Δ effect in the large-deflection range. The increasing curvature of the N_{mem}–P curve indicates that the internal force develops faster than the nominal load, a characteristic feature of geometric-induced secondary effects.

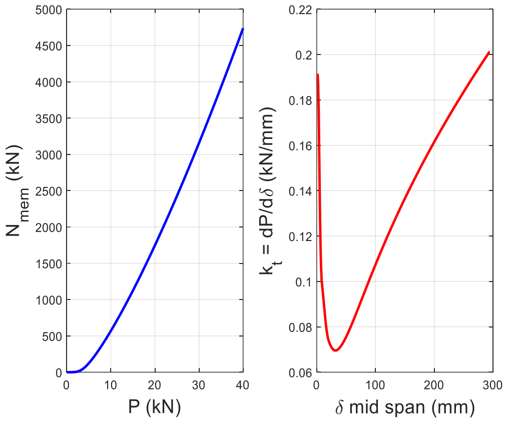

Fig. 8. Membrane-force development and tangent-stiffness reduction due to geometric nonlinearity

The k_t-δ plot shows that the tangent stiffness decreases sharply at the initial stage from approximately 0.20 to 0.07 kN/mm when δ<100 mm indicating a rapid loss of initial stiffness. Beyond this point, the curve gradually rises and stabilizes around 0.20 kN/mm, reflecting a redistribution of internal forces as the structure approaches a new geometrically compatible equilibrium. This behavior is consistent with the mechanics of geometric nonlinearity, in which the stiffness initially softens due to large deformations and subsequently increases slightly as the deformed shape becomes "stretched" under higher loads.

Overall, the results confirm that the beam is significantly affected by geometric nonlinearity and second-order effects, highlighting the need for analysis models that account for stiffness variation with displacement to accurately capture structural stability and realistic deflection behavior.

4.2.5 Combined influence of shear-connection stiffness and beam length

The two figures illustrate the combined influence of the shear-connection stiffness K_s and the beam length L on the key response quantities of the composite beam. In Fig.9, the color variation changes only mildly along both axes, indicating that the investigated quantity (e.g., equivalent bending stiffness or composite-action efficiency η) increases gradually with increasing K_s and L, but with relatively low sensitivity.

When K_s is sufficiently large (≥ 10^4N/mm^2), the results approach a saturated value, implying that the slip connection becomes stiff enough to ensure nearly full composite action, with negligible dependence on the beam length.

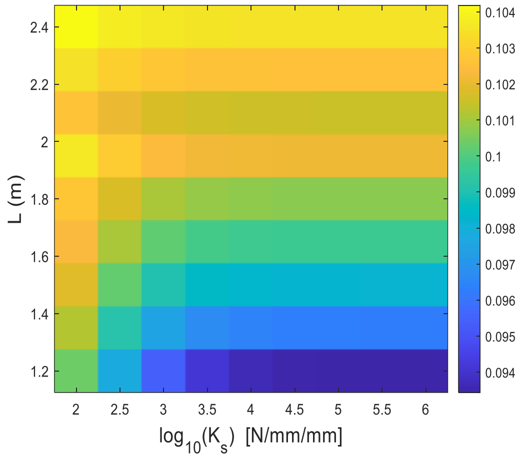

Fig. 9. Effective bending stiffness distribution with respect to L and K_s

In contrast, Fig.10 shows a much more pronounced variation: the response quantity increases significantly when L decreases or K_s increases, indicating that deflection and geometric sensitivity are substantially reduced for stiffer connections or shorter beams.

This confirms that, in practical design, both parameters L and K_s play important roles in governing the overall behavior: Longer beams→ stronger geometric nonlinearity; Softer connections→ reduced composite efficiency and increased deflection.

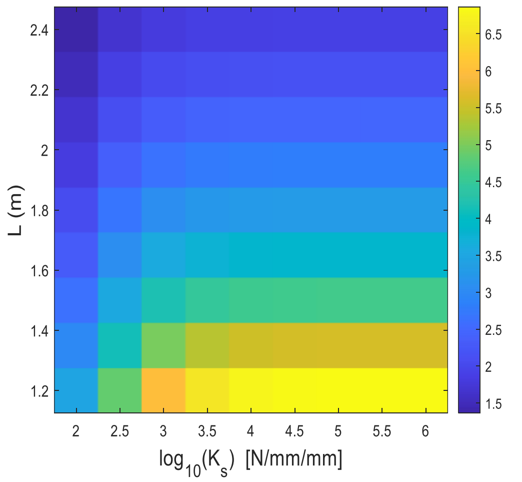

Fig. 10 Heatmap of composite-action coefficient η versus beam length L and shear stiffness K_s

5. CONCLUSIONS

The analysis results show that the behavior of the composite beam is strongly influenced by geometric nonlinearity and the shear-connection stiffness between layers. The load–deflection (P–δ) response reveals that the P–Δ effect significantly reduces stiffness as the load increases, causing the actual deflection to exceed the linear prediction by 20–30%.

The composite-action coefficient η and the equivalent bending stiffness EI_{eff} increase rapidly with the shear-connection stiffness K_s; when $K_s \geq 10^4$ N/mm^2, the beam approaches a fully composite state, whereas for $K_s < 10^3$ N/mm^2, interlayer slip leads to a substantial reduction in the global stiffness. At the same time, the beam length L has a clear influence: longer beams exhibit stronger geometric nonlinear effects and greater sensitivity to K_s.

The results for bending moment and tangent stiffness indicate that the system stiffness decreases rapidly at the initial loading stage and then gradually stabilizes as a new geometric equilibrium is reached. The L–K_s heatmaps further confirm these trends, showing that short beams with stiff shear connections provide the highest mechanical efficiency.

Overall, the study demonstrates that accounting for both geometric nonlinearity and slip-connection characteristics is essential for accurately predicting the behavior of composite beams. The findings also provide practical guidance for determining the minimum required shear-connection stiffness and permissible beam lengths in structural design.

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