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FINITE DIFFERENCE APPROXIMATION FOR SOLVING TRANSIENT HEAT CONDUCCTION EQUATION OF THE BRICK

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ABSTRACT: In this paper, we studied the numerical approximation of transient heat conduction equation of the brick .Brick is a natural building material used in construction, usually shaped in parallel, made of clay, sand and water, with a small amount of hay (crushed and soft straw) added to the clay block before cutting bricks to dry it under the sun. The bricks are strongly resistant and are not easily affected by weather. We applied the Finite differences method to the thermal conductivity equation for bricks. The technique is described and illustrated with a numerical example. The obtained solutions are compared with the available exact solutions and the solutions obtained by Finite difference method Results showed that Finite difference method is a very promising method for obtaining approximate solutions to transient heat conduction equation of the brick. As evidenced by numerical results obtained from brick in resistance to external factors such as rain, air, sounds and others.

Keywords: Finite Difference Method, Transient Heat Conduction Equation, Brick, Matlab.

1. INTRODUCTION

Finite difference method is one of several techniques for obtaining numerical solution of partial differential equation. Finite difference method describes functions as discrete values across a grid and approximate their derivatives as differences between points on the grid. Knowing a little about how difference methods are formulated and in what regimes they are stable can help save a lot of time, both in the design of finite differencing algorithms, and in the time that they take to run, the finite difference approximations for derivatives are one of the simplest and of the oldest methods to solve differential equations.

We need a fast, realistic and reliable method to solve the heat conduction equation of the Brick.

Advantages of using clay brick:

• very lite:
Its weights are 40-55% less than other building materials, which reduces loads on concrete foundations for construction.

• Heatproof:
It consists of thermal insulating materials more than any other materials, which saves 40% of the electrical energy.

• High pressure:
It has a high-pressure equivalent to three times the other building materials. It is used to build the bearing walls without columns.

• Fire resistance:
It is incinerated in ovens with a temperature of more than 1000 ° C, which gives immunity against fire and reduce its spread in buildings and facilities.

Resists moisture leakage:
It has a low water absorption rate of not more than 12% which prevents moisture leakage.

• Soundproof:
Features a high rate of sound absorption.

• Ease of work:
Helps in the ease of construction and electrical wiring and plumbing for light weight and the presence of spaces.

• Low maintenance costs:
Free of salt, which increases its age, prevents corrosion and maintains its color and shape and does not require maintenance.

2. FINITE DIFFRENCE METHOD FOR HEAT CONDUCTION EQUATION

The linear second order partial differential equation

$$Au_{xx} + 2Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu + G = 0$$

as a parabolic equation if $B^2 - AC = 0$. A parabolic equation holds in an open domain or in a semi-open domain.

The partial differential equation governing the flow of heat in the rod is given by the parabolic equation

$$\boldsymbol{u_t = c^2 u_{xx}, \quad 0 \le x \le l, \qquad t > 0.}$$

Where $\boldsymbol{c^2}$ is a constant and depends on the material properties of the rod.

The heat conduction equation of the Brick

$$\boldsymbol{u_t = (0.0038)u_{xx}}$$
$$\boldsymbol{u(0,t) = u(1,t) = 0, \forall t \in (0, t_F)}$$
$$\boldsymbol{u(x,0) = u_0(x), \quad \forall x \in [0,1],}$$

Where $\boldsymbol{t_F}$ Denotes the terminal time for the model. Here without loss of generality, we assume that the spatial domain is $[\boldsymbol{0}, \boldsymbol{1}]$.
At first divide the physical domain $(\boldsymbol{0}, \boldsymbol{t_F}) \times (\boldsymbol{0}, \boldsymbol{1})$ by $\boldsymbol{N} \times \boldsymbol{J}$ uniform grid points

$$\boldsymbol{t_n = n\Delta t, \Delta t = \frac{t_F}{N}, n = 0, 1, \cdots, N,}$$
$$\boldsymbol{x_j = j\Delta x, \Delta x = \frac{1}{J}, j = 0, 1, \cdots, J.}$$

Then, we denote the approximate solution $\boldsymbol{u_j^n} \approx \boldsymbol{u(x_j, t_n)}$ .At an arbitrary point $(\boldsymbol{x_j}, \boldsymbol{t_n})$. To obtain afinite difference scheme, we need to approximate the derivatives in (1) by some finite differences.
(Explicit scheme)
Substituting

$$\boldsymbol{u_t(x_j, t_n) \approx (u_j^n - u_j^{n-1})/\Delta t,}$$
$$\boldsymbol{u_{xx}(x_j, t_n) \approx (u_{j+1}^n - 2u_j^n + u_{j-1}^n)/(\Delta x)^2,}$$

Into (1), another difference scheme for (1) can be constructed as:

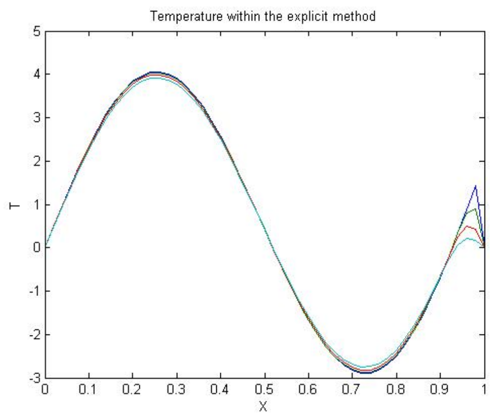
$$\boldsymbol{\frac{u_j^n - u_j^{n-1}}{\Delta t} = \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{(\Delta x)^2}, \quad 1 \le j \le J - 1, 1 \le n \le N.}$$

3. SEVERALS EXAMPLES

(i) Example1. Find the solution of the heat conduction equation of the Brick

$$\boldsymbol{u_t = (0.0038)u_{xx} \quad 0 < x < 1, \quad t > 0;}$$
$$\boldsymbol{u(0,t) = u(1,t) = 0, \ t > 0;}$$
$$\boldsymbol{u(x,0) = 2\sin(\pi x/2) - \sin \pi x + 4\sin 2\pi x, \quad 0 \le x \le 1.}$$

Applying the finite difference method using Matlab, then the result show as follows.



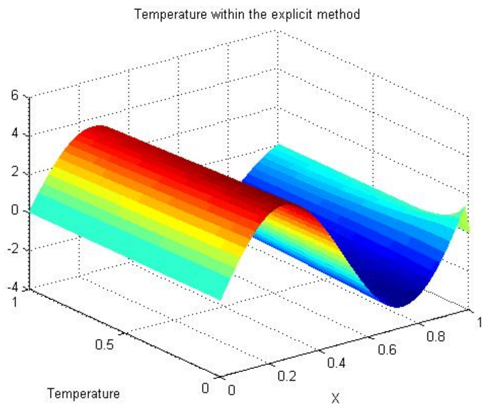
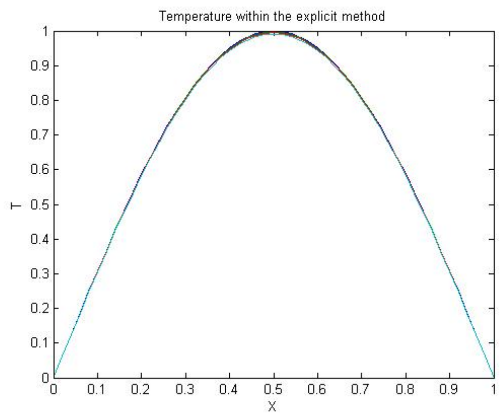


Fig.1 Temperature distributions at several times for the heat conduction of the Brick for example 1.

(ii) Example2. Find the solution of the heat conduction equation of the Brick

$$\boldsymbol{u_t = (0.0038)u_{xx} \quad 0 < x < 1, \quad t > 0;}$$
$$\boldsymbol{u(0,t) = u(1,t) = 0, \ t > 0;}$$
$$\boldsymbol{u(x,0) = \sin \pi x, \quad 0 \le x \le 1.}$$

Applying the finite difference method using Matlab, then the result show as follows.



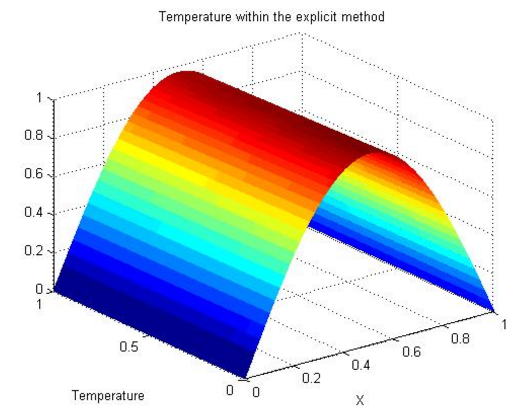


Fig.2 Temperature distributions at several times for the heat conduction of the Brick for example 2.

(iii) Example3. Find the solution of the heat conduction equation of the Brick

$$u_t = (0.0038)u_{xx} \quad 0 < x < 1, \quad t > 0;$$
$$u(0,t) = u(1,t) = 0, \ \ t > 0;$$
$$u(x,0) = \sin 2\pi x - \sin 5\pi x, \quad 0 \le x \le 1.$$

Applying the finite difference method using Matlab, then the result show as follows.

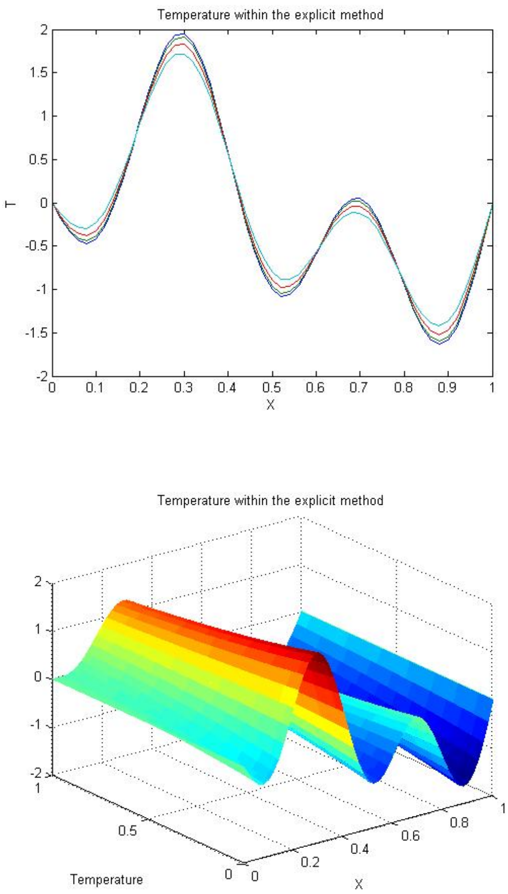


Fig.3 Temperature distributions at several times for the heat conduction of the Brick for example 3.

(iv) Example 4. Find the solution of the heat conduction equation of the Brick

$$u_t = (0.0038)u_{xx} \quad 0 < x < 1, \quad t > 0;$$
$$u(0,t) = u(1,t) = 0, \ \ t > 0;$$
$$u(x,0) = \sin(x) - 3\cos 4x, \quad 0 \le x \le 1.$$

Applying the finite difference method using Matlab, then the result show as follows.

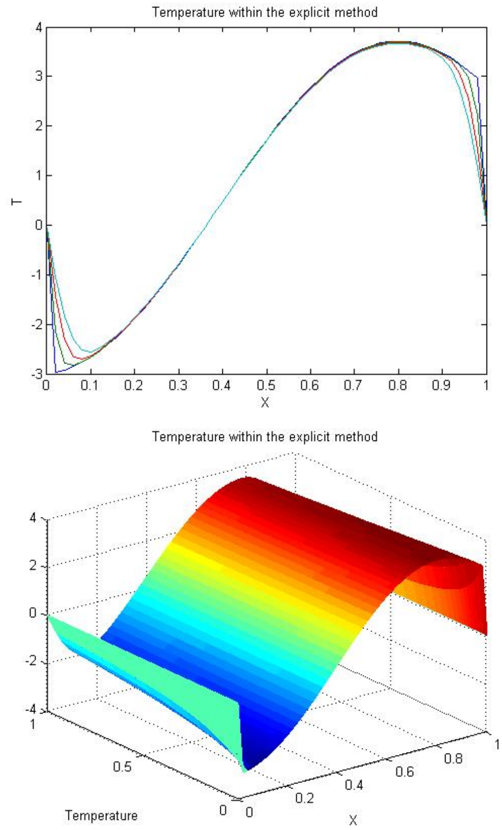
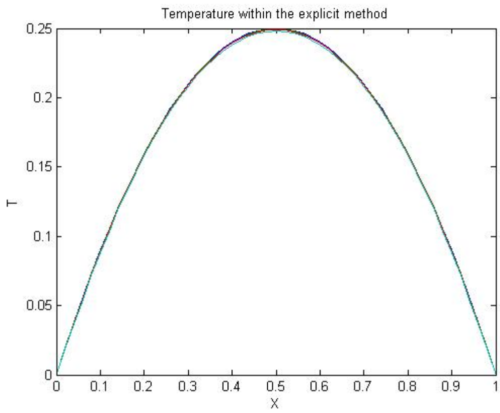


Fig.4 Temperature distributions at several times for the heat conduction of the Brick for example 4.

(v) Example 5. Find the solution of the heat conduction equation of the Brick

$$u_t = (0.0038)u_{xx} \quad 0 < x < 1, \quad t > 0;$$
$$u(0,t) = u(1,t) = 0, \ \ t > 0;$$
$$u(x,0) = x(1-x), \quad 0 \le x \le 1.$$

Applying the finite difference method using Matlab, then the result show as follows.



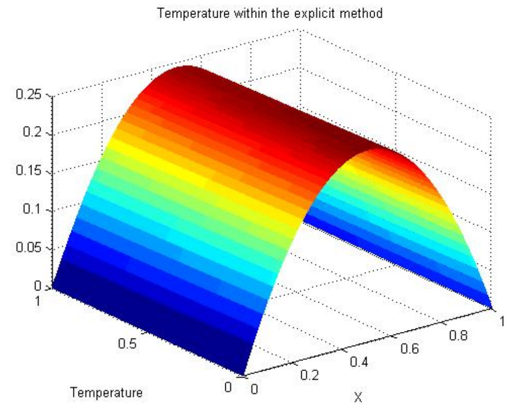


Fig.5 Temperature distributions at several times for the heat conduction of the Brick for example5.

(vi) Example 6. Find the solution of the heat conduction equation of the Brick

$$u_t = (0.0038)u_{xx} \quad 0 < x < 1, \quad t > 0;$$
$$u(0,t) = u(1,t) = 0, \quad t > 0;$$
$$u(x,0) = -\sin(3\pi x) + \frac{1}{4}\sin(6\pi x), \quad 0 \le x \le 1.$$

Applying the finite difference method using Matlab, then the result show as follows.

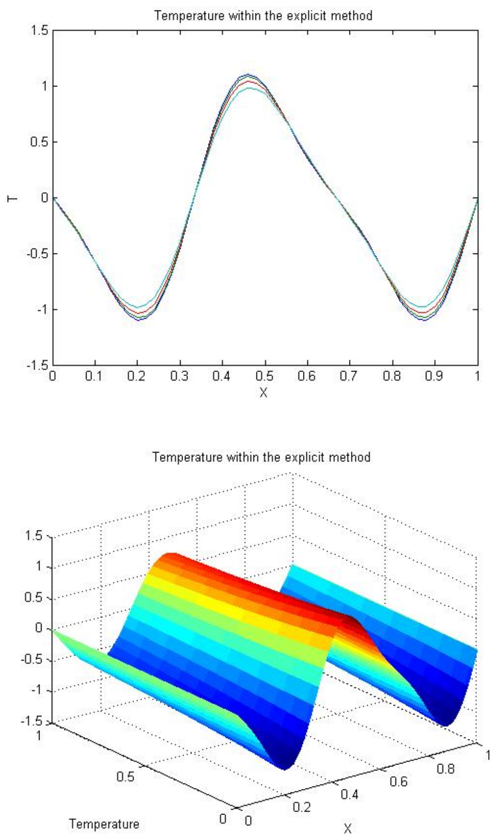


Fig.6 Temperature distributions at several times for the heat conduction of the Brick for example6.

(vii) Example 7. Find the solution of the heat conduction equation of the Brick

$$u_t = (0.0038)u_{xx} \quad 0 < x < 1, \quad t > 0;$$
$$u(0,t) = u(1,t) = 0, \quad t > 0;$$
$$u(x,0) = 1 + \cos(2\pi x), \quad 0 \le x \le 1.$$

Applying the finite difference method using Matlab, then the result show as follows.

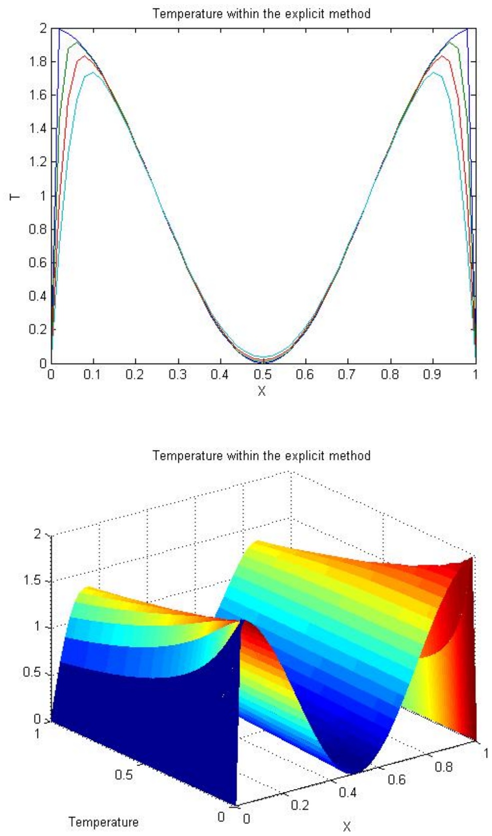
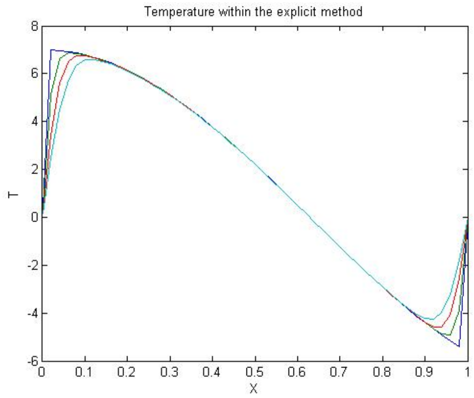


Fig.7 Temperature distributions at several times for the heat conduction of the Brick for example7.

(viii) Example 8. Find the solution of the heat conduction equation of the Brick

$$u_t = (0.0038)u_{xx} \quad 0 < x < 1, \quad t > 0;$$
$$u(0,t) = u(1,t) = 0, \quad t > 0;$$
$$u(x,0) = 7\cos(\frac{5}{2}x), \quad 0 \le x \le 1.$$

Applying the finite difference method using Matlab, then the result show as follows.



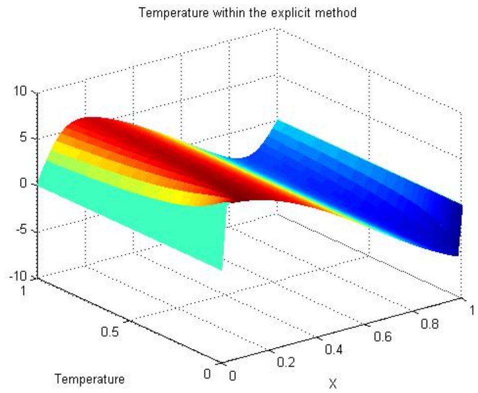
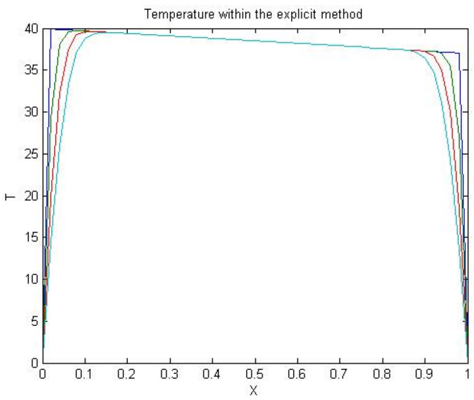


Fig.8 Temperature distributions at several times for the heat conduction of the Brick for example8.

(ix) Example 9. Find the solution of the heat conduction equation of the Brick

$$\boldsymbol{u_t = (0.0038)u_{xx} \quad 0 < x < 1, \quad t > 0;}$$
$$\boldsymbol{u(0,t) = u(1,t) = 0, \ \ t > 0;}$$
$$\boldsymbol{u(x,0) = 40 - 3x, \quad 0 \le x \le 1.}$$

Applying the finite difference method using Matlab, then the result show as follows.



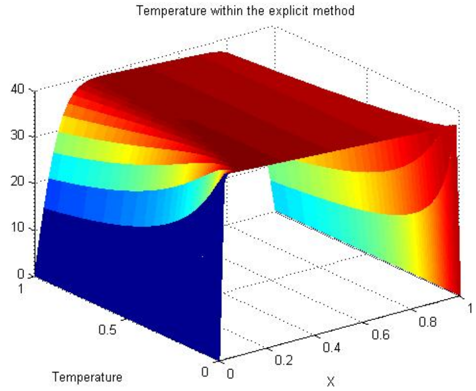


Fig.9 Temperature distributions at several times for the heat conduction of the Brick for example9.

4. CONCLUSION

In this paper, the calculations were performed by the MATLAB program for the programming and the corresponding evolutionary laws on the basis of the one-dimensional mathematical model of one-dimensional thermal conductivity using the Finite differences method of solving the heat-conduction equation for the bricks. All the examples show that the finite difference method is a powerful mathematical tool to solve the heat conduction equation for bricks. The graphic shapes we have provided to determine the higher resolution and simplicity of the proposed method. In addition, it should be noted that the method described can be easily circulated to build more heat for different materials. This work focuses on bricks because it is resistant to high temperatures, friction and various chemical effects. It is used in the construction of heaters and lining furnaces used in the iron and steel industry in the iron and steel industry and the non-ferrous industries such as copper, zinc and lead in the manufacture of cement, lime and glass.

5. ACKNOWLEDGMENTS

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