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THE MODIFIED DECOMPOSITION METHOD FOR SOLVING VOLTERRA INTEGRAL EQUATION OF THE SECOND KIND USING MAPLE

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ABSTRACT: In this paper, Volterra Integral equations of the second kind arise in several applications. These include applications in biology, physics, chemistry and engineering. In recent years, much work has been carried out by researchers in mathematics and engineering in applying and analyzing novel numerical and semi-analytical methods for obtaining solutions of integral equations of the second kind. The Modified Decomposition Method for solving Volterra integral equations of the second kind by using Maple program, By the Taylor expansion of components apart from the zeroth term of the Adomian series solution. Some numerical examples are provided to demonstrate the validity of the method, so all calculations can be easily using Maple to find that exact solution. In the current paper, the principle of the Modified decomposition method is described and its advantages are Some illustrative examples are presented and the results show that the solutions obtained by using this technique have a close agreement with series solutions obtained with the help of the Modified Decomposition Method using Maple17.

Keywords: Volterra integral equation of the second kind, Modified Decomposition method, Maple17.

1. INTRODUCTION

The modified decomposition method [1]-[3] has been efficiently used to solve linear and nonlinear problems such as differential equations and integral equations.

The modified decomposition needs only a slight variation from the standard decomposition method may provide the exact solution by using iterations only and sometimes without using the so-called adomain polynomials, its effectiveness is based on the assumption that the function f can be divided into two parts and the paper choice of f_1 and f_2 .

2. THE MODIFIED DECOMPOSITION METHOD

Adomian decomposition method the use of the recurrence relation:

$$\boldsymbol{u_0(x) = f(x)},$$
$$\boldsymbol{u_{k+1}(x) = \lambda \int_0^x K(x,t) u_k(t)\, dt,\ k \geq 0}, \quad (1)$$

where the solution $\boldsymbol{u(x)}$ is expressed by an infinite sum of components defined before by

$$\boldsymbol{u(x) = \sum_{n=0}^{\infty} u_n(x)} \quad (2)$$

In view of (1), the components $\boldsymbol{u_n(x), n \geq 0}$ can be easily evaluated. The modified decomposition method [1]-[4] introduces a slight variation to the recurrence relation (1) that will lead to the determination of the components of $\boldsymbol{u(x)}$ in an easier and faster manner. For many cases, the function $\boldsymbol{f(x)}$ can be set as the sum of two partial functions, namely $\boldsymbol{f_1(x)}$ and $\boldsymbol{f_2(x)}$. In other words, we can set

$$\boldsymbol{f(x) = f_1(x) + f_2(x)}. \quad (3)$$

In view of (3), we introduce a change in the formation of the recurrence relation (1). To minimize the size of calculations, we identify the zeroth component $\boldsymbol{u_0(x)}$ by one part of $\boldsymbol{f(x)}$, namely $\boldsymbol{f_1(x)}$or$\boldsymbol{f_2(x)}$. The other part of $f(x)$ can be added to the component $u1(x)$ among other terms.
In other words, the modified decomposition method introduces the modified recurrence relation:

$$\boldsymbol{u_0(x) = f_1(x)},$$
$$\boldsymbol{u_1(x) = f_2(x) + \lambda \int_0^x K(x,t) u_0(t)\, dt},$$
$$\boldsymbol{u_{k+1}(x) = \lambda \int_0^x K(x,t) u_k(t)\, dt,\ k \geq 1}. \quad (4)$$

This shows that the difference between the standard recurrence relation (1) and the modified recurrence relation (4) rests only in the formation of the first two components $u0(x)$ and $\boldsymbol{u_1(x)}$ only. The other components $\boldsymbol{u_j, j \geq 2}$remain the same in the two recurrence relations. Although this variation in the formation of $\boldsymbol{u_0(x)}$ and $\boldsymbol{u_1(x)}$ is slight, however it plays a m,ajor role in accelerating the convergence of the solution and in minimizing the size of computational work. Moreover, reducing the number of terms in $f_1(x)$ affects not only the component $u_1(x)$,but also the other components as well. This result was confirmed by several research works.

Two important remarks related to the modified method [1]-[4] can be made here. First, by proper selection of the functions $f_1(x)$ and $f_2(x)$, the exact solution $u(x)$ may be obtained by using very few iterations, and sometimes by evaluating only two components. The success of this modification depends only on the proper choice of $f_1(x)$ and $f_2(x)$, and this can be made through trials only. A rule that may help for the proper choice of $f_1(x)$ and $f_2(x)$ could not be found yet. Second, if $f(x)$ consists of one term only, the standard decomposition method can be used in this case.

3. EXAMPLE

Example1. Consider the Volterra integral equation of the second kind

$$u(x) = 1 + 2x + \sin x + x^2 - \cos x - \int_0^x u(t)dt .$$

Applying the Modified Decomposition Method using Maple we find

Table 1 Numerical results and exact solution of Volterra integral equation for example 1

x	$u(x)$	$Exact = 2x + \sin x$	$Error$
0.10000	0.2998334	0.2998334	0.0000000
0.20000	0.5986693	0.5986693	0.0000000
0.30000	0.8955202	0.8955202	0.0000000
0.40000	1.1894183	1.1894184	0.0000000
0.50000	1.4794255	1.4794256	0.0000001
0.60000	1.7646425	1.7646429	0.0000004
0.70000	2.0442177	2.0442191	0.0000014
0.80000	2.3173561	2.3173603	0.0000042
0.90000	2.5833269	2.5833376	0.0000107
1.00000	2.8414710	2.8414958	0.0000248

It is obvious that each component of $u_{i,j}$ is nearly by exact solution. This in turn gives the exact solution by

$$Exact = 2x + \sin x$$

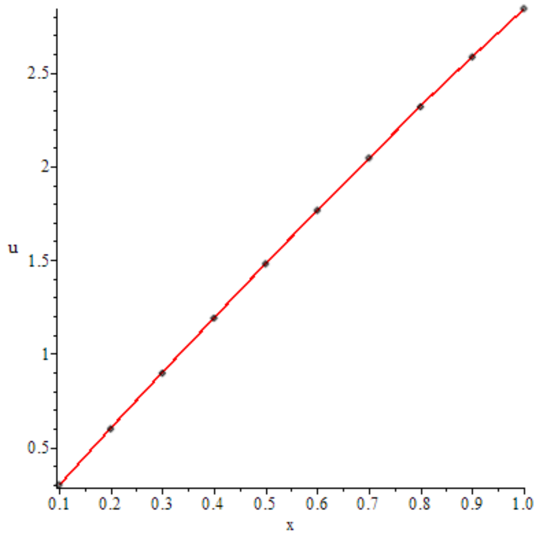

Fig.1 Plot 2D of the exact solutions result Of Volterra integral equation for example 1.

Example2. Consider the Volterra integral equation of the second kind

$$u(x) = 1 + x + x^2 + \frac{1}{2}x^3 + \cosh x + xsinhx - \int_0^x xu(t)dt .$$

Applying the Modified Decomposition Method using Maple we find

Table 2 Numerical results and exact solution of Volterra integral equation for example 2

x	$u(x)$	$Exact = 1 + x + \cosh x$	$Error$
0.10000	2.1050042	2.1050042	0.0000000
0.20000	2.2200668	2.2200667	0.0000000
0.30000	2.3453385	2.3453385	0.0000000
0.40000	2.4810724	2.4810723	0.0000000
0.50000	2.6276260	2.6276259	0.0000001
0.60000	2.7854652	2.7854652	0.0000000
0.70000	2.9551690	2.9551674	0.0000016
0.80000	3.1374349	3.1374288	0.0000061
0.90000	3.3330864	3.3330613	0.0000251
1.00000	3.5430806	3.5429900	0.0000900

It is clear that each component of $u_{i,j}$ is almost an accurate by exact solution. This in turn gives the exact solution by

$$Exact = 1 + x + \cosh x$$

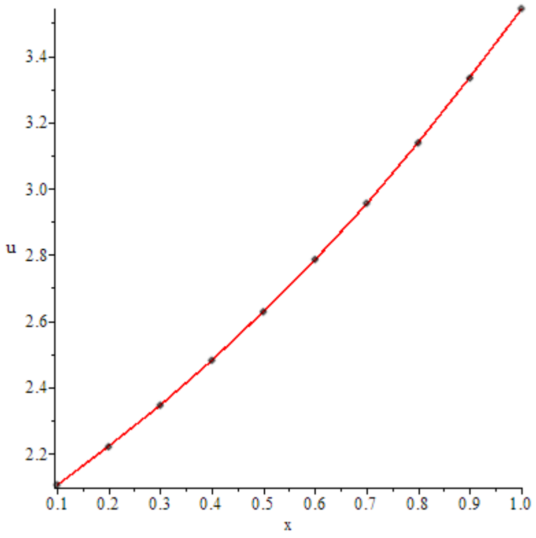

Fig. 2 Plot 2D of the exact solutions result Of Volterra integral equation for example 2.

Example 3. Consider the Volterra integral equation of the second kind

$$\boldsymbol{u(x) = e^x - xe^x + \sin x + x\cos x - \int_0^x xu(t)dt\,.}$$

Applying the Modified Decomposition Method using Maple we find

Table 3 Numerical results and exact solution of Volterra integral equation for example 3.

x	$u(x)$	$Exact = e^x + \sin x$	$Error$
0.10000	1.2050043	1.2050000	0.0000043
0.20000	1.4200721	1.4200000	0.0000721
0.30000	1.6453790	1.6450000	0.0003790
0.40000	1.8812430	1.8820000	0.0007570
0.50000	2.1281468	2.1290000	0.0008532
0.60000	2.3867613	2.3890000	0.0022387
0.70000	2.6579704	2.6580000	0.0000296
0.80000	2.9428970	2.9450000	0.0021030
0.90000	3.2429300	3.2430000	0.0000700
1.00000	3.5597528	3.5610000	0.0000000

It is interesting to point that each component of $u_{i,j}$ is nearly by exact solution. This in turn gives the exact solution by

$$\boldsymbol{Exact = e^x + \sin x}$$

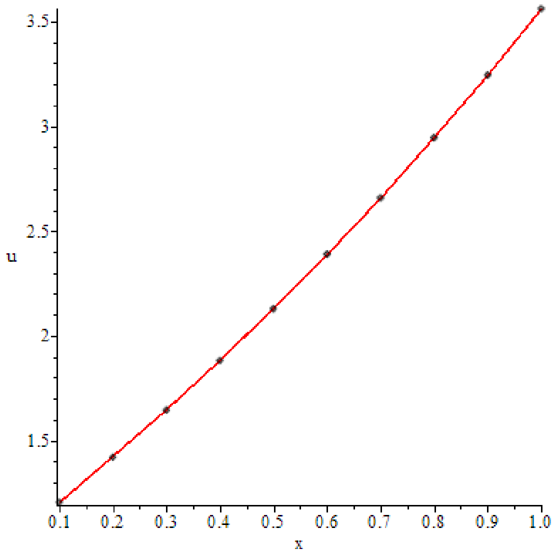

Fig. 3 Plot 2D of the exact solutions result Of Volterra integral equation for example 3.

Example 4. Consider the Volterra integral equation of the second kind

$$\boldsymbol{u(x) = \sinh x + \cosh x - 1 - \int_0^x u(t)dt\,.}$$

Applying the Modified Decomposition Method using Maple we find

Table 4 Numerical results and exact solution of Volterra integral equation for example 4.

x	$u(x)$	$Exact = \sinh x$	$Error$
0.10000	0.1001668	0.1001668	0.0000000
0.20000	0.2013360	0.2013360	0.0000000
0.30000	0.3045203	0.3045203	0.0000000
0.40000	0.4107523	0.4107523	0.0000000
0.50000	0.5210953	0.5210954	0.0000001
0.60000	0.6366536	0.6366540	0.0000004
0.70000	0.7585837	0.7585851	0.0000014
0.80000	0.8881060	0.8881102	0.0000042
0.90000	1.0265167	1.0265275	0.0000108
1.00000	1.1752012	1.1752263	0.0000251

It is important to note that each component of $u_{i,j}$ is almost accurate by exact solution. This in turn gives the exact solution by

$$\boldsymbol{Exact = \sinh x}$$

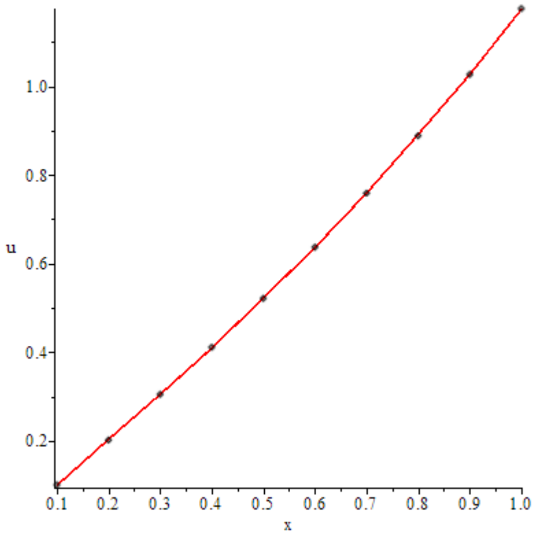

Fig. 4 Plot 2D of the exact solutions result Of Volterra integral equation for example 4.

Example 5. Consider the Volterra integral equation of the second kind

$$u(x) = e^x + xe^x - x - \int_0^x xu(t)dt\,.$$

Applying the Modified Decomposition Method using Maple we find

Table 5 Numerical results and exact solution of Volterra integral equation for example 5.

x	$u(x)$	$Exact = e^x$	$Error$
0.10000	1.1008212	1.1051709	0.0000000
0.20000	1.2064797	1.2214028	0.0000000
0.30000	1.3215859	1.3498588	0.0000000
0.40000	1.4505445	1.4918247	0.0000000
0.50000	1.5975986	1.6487213	0.0000001
0.60000	1.7668722	1.8221188	0.0000004
0.70000	1.9624114	2.0137527	0.0000014
0.80000	2.1882247	2.2255409	0.0000042
0.90000	2.4483235	2.4596031	0.0000108
1.00000	2.7467615	2.7182818	0.0000251

It is obvious that each component of $u_{i,j}$ is nearly by exact solution. This in turn gives the exact solution by

$$Exact = e^x$$

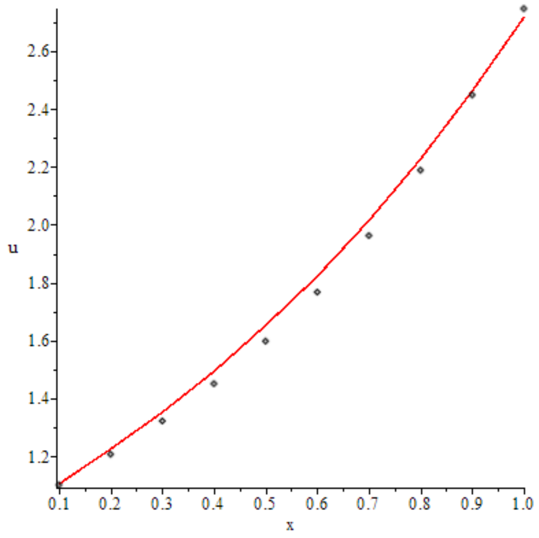

Fig. 5 Plot 2D of the exact solutions result Of Volterra integral equation for example 5.

Example 6. Consider the Volterra integral equation of the second kind

$$u(x) = x + x^2 - 2x^3 - x^4 + 12\int_0^x (x-t)u(t)dt\,.$$

Applying the Modified Decomposition Method using Maple we find

Table6 Numerical results and exact solution of Volterra integral equation for example 6.

x	$u(x)$	$Exact = x + x^2$	$Error$
0.10000	0.1100000	0.1100000	0.0000000
0.20000	0.2400000	0.2400000	0.0000000
0.30000	0.3900000	0.3900000	0.0000000
0.40000	0.5600000	0.5600000	0.0000000
0.50000	0.7500000	0.7500000	0.0000000
0.60000	0.9600000	0.9600000	0.0000000
0.70000	1.1900000	1.1900000	0.0000014
0.80000	1.7100000	1.4400000	0.0000042
0.90000	2.0000000	1.7100000	0.0000108
1.00000	2.7467615	2.0000000	0.0000251

It is important to note that each component of $u_{i,j}$ is almost accurate by exact solution. This in turn gives the exact solution by

$$Exact = x + x^2$$

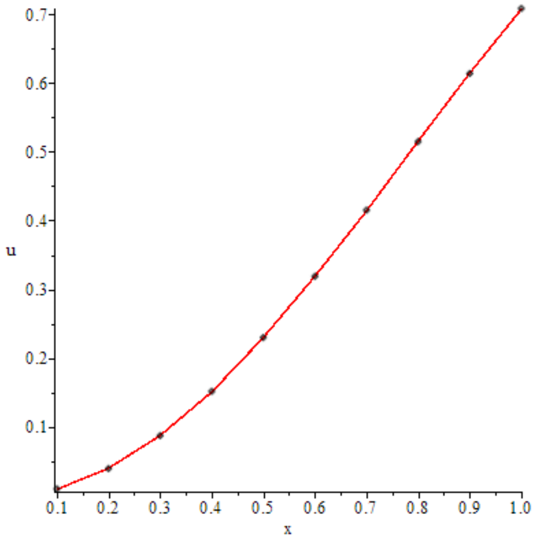

Fig. 6 Plot 2D of the exact solutions result Of Volterra integral equation for example 6.

Example 7. Consider the Volterra integral equation of the second kind

$$u(x) = -\frac{1}{2}x + \frac{1}{4}\sin(2x) + sin^2(x) + \int_0^x u(t)dt .$$

Applying the Modified Decomposition Method using Maple we find

Table 7 Numerical results and exact solution of Volterra integral equation for example 7.

x	$u(x)$	$Exact = sin^2(x)$	$Error$
0.10000	0.0099667	0.0099667	0.0000000
0.20000	0.0394695	0.0394695	0.0000000
0.30000	0.0873322	0.0873322	0.0000000
0.40000	0.1516466	0.1516466	0.0000000
0.50000	0.2298488	0.2298488	0.0000000
0.60000	0.3188211	0.3188211	0.0000000
0.70000	0.4150164	0.4150164	0.0000000
0.80000	0.5145997	0.5145998	0.0000001
0.90000	0.6136008	0.6136010	0.0000002
1.00000	0.7080734	0.7080728	0.0000006

It is clear that each component of $u_{i,j}$ is almost an accurate by exact solution. This in turn gives the exact solution by

$$Exact = sin^2(x)$$

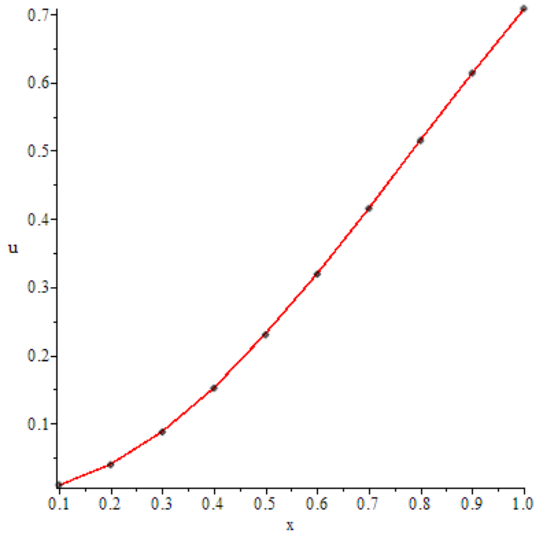

Fig. 7 Plot 2D of the exact solutions result Of Volterra integral equation for example 7.

Example 8. Consider the Volterra integral equation of second kind

$$u(x) = \frac{1}{2}x - \frac{1}{4}\sinh(2x) + sinh^2(x) + \int_0^x u(t)dt .$$

Applying the Modified Decomposition Method using Maple we find

Table 8 Numerical results and exact solution of Volterra integral equation for example 8.

x	$u(x)$	$Exact = sinh^2(x)$	$Error$
0.10000	0.0100334	0.0100334	0.0000000
0.20000	0.0405362	0.0405362	0.0000000
0.30000	0.0927326	0.0927326	0.0000000
0.40000	0.1687175	0.1687175	0.0000000
0.50000	0.2715403	0.2715403	0.0000000
0.60000	0.4053278	0.4053278	0.0000000
0.70000	0.5754492	0.5754492	0.0000000
0.80000	0.7887322	0.7887322	0.0000001
0.90000	1.0537364	1.0537366	0.0000002
1.00000	1.3810972	1.3810978	0.0000006

It is interesting to point that each component of $u_{i,j}$ is nearly by exact solution. This in turn gives the exact solution by

$$Exact = sinh^2(x)$$

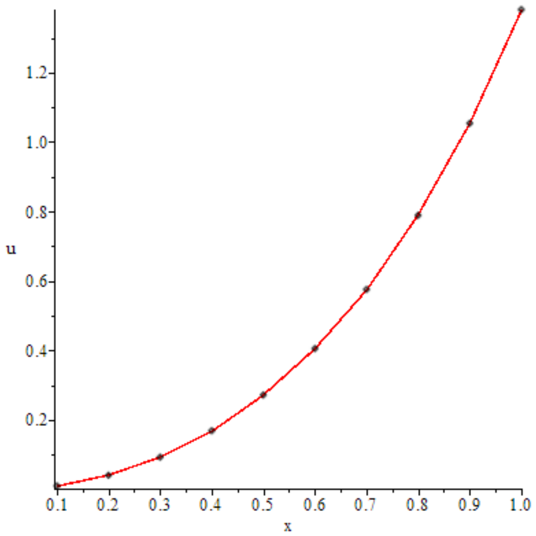

Fig. 8 Plot 2D of the exact solutions result Of Volterra integral equation for example 8.

4. CONCLUSIONS

In this paper, the Modified Decomposition Method to the solution of Volterra integral equation numerical results demonstrates that our method is an accurate and reliable numerical technique for solving Volterra integral equation. Finally, The Modified Decomposition Method using Maple can be easily extended and applied to linear or nonlinear Fredholm and Volterra integral equations of the first or second kind.

5. ACKNOWLEDGMENTS

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6. REFERENCES

[1] Abdul-Majid. W., A reliable Modification of Adomain Decomposition Method,. Applied Mathematics and Computation, Vol. 102, No.1, 1999, pp.77-86.

[2] Abdul-Majid. W., and Said. A. El-S., Anew Modification of the Adomain Decomposition Method for Linear and Nonlinear Operators,. Applied Mathematics and Computation, Vol. 122, No.3, 2001, pp.393-405.

[3] Abdul-Majid. W., Linear and Nonlinear Integral Equations, Higher Education Press, Beijing, China, 2011.

[4] Peter. L., Analytical and Numerical Methods for Volterra Equations, Studies in Applied Mathematics 7, SIAM, Philadelphia, 1985.

[5] Dalal A.M, Amani..Z.B.l, Badreeh..M. G., Numerical Solution of Volterra Integral Equation of Second Kind Using Implicit Trapezoidal, Journal of Advances In Mathematics, Vol8,No.2, pp.1540-1553,2014.

[6] Dalal .A. M., Adomian Decomposition Method of Fredholm Integral Equation of the Second Kind Using Maple, Journal of Advances In Mathematics, Vol9, No.1,pp.1868-1875,2014.

[7] Dalal. A.M., Application of Adomian Decomposition Method for Solving of Fredholm Integral Equation of the Second Kind, European Journal of Science and Engineering, Vol9, No.2,pp.1-9,2014.

[8] Dalal .A.M., Adomian Decomposition Method for Solving of Fredholm Integral Equation of the Second Kind Using Matlab, International Journal of GEOMATE, Dec. 2016, Vol. 11, Issue 28, pp.2830-2833.Special Issue on Science, Engineering and Environment, ISSN: 2186-2990, Japan.

[9] Dalal A.M. and Honida M.M., Numerical Solution of System of Three Nonlinear Volterra Integral Equation Using Implicit Trapezoidal, Journal of Mathematics Research, Vol.10, No. 1, February 2018, ISSN 1916-9795 E-ISSN 1916-9809.

[10] Mohammad .A, and Ahmad .I, Optimal Homotopy Asymptotic Method for Solving the Linear Fredholm Integral Equations of the First Kind, Abstract and Applied Analysis, vol.2013, Article ID 278097,6 pages.

[11] Hashmi M.S., N. Khan, and S. Iqbal, "Optimal Homotopy Asymptotic Method For Solving Nonlinear Fredholm Integral Equations of Second Kind, Applied Mathematics and Computation, vol.218, no22, pp.10982-10989, 2012.

[12] Islam, S., Khan, Y., Faraz, N., Austin, F., Numerical Solution of logistic differential equations by using the Laplace decomposition method, World Appl. Sci. J. 8(9), 1100-1105, 2010.

[13] Khan, Yasir, An Effective Modification of the Laplace Decomposition Method for Nonlinear Equations. Int. J. Nonlin. Sci. Num.Simul. 10(11-12), 1373-1376, 2009.

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