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LIMIT ANALYSIS OF A STRIP LOAD ON THE HALF-PLANE USING A NOVEL EFFECTIVE STRESS FIELD AND NONLINEAR PROGRAMMING

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ABSTRACT: This paper presents a method to determine a strip load's actual failure load on the half-plane. Solving the problem of calculating natural stresses and deformations in the soil is a practically impossible task. An alternative may be formed by limit analysis based on plasticity theory. The fundamental theorems of the idea of plasticity (the upper and lower- bound theorems) aim to give a possible upper or lower limit of the stresses the deformations. For materials with friction, such as soil, for which the yield condition is the Mohr-Coulomb criterion, the limit theorems of plasticity are not good, except for $\phi = 0$ (i.e., purely cohesive materials). For such a material, the theory predicts that the volume is constant during plastic deformations, which agrees with experimental evidence. Thus, the author proposes a novel effective stress field based on the shear potential having volume remains constant during plastic deformations, to properly apply the limit theorem. In this paper, the limit analysis is formulated in the form of nonlinear programming. Several numerical examples show that the novel effective stress field achieves high reliability compared with existing results.

Keywords: Limit analysis, Strip load, Novel effective stress field, Shear potential, Nonlinear programming

1. INTRODUCTION

Soil is a porous material consisting of particles, water, and air. The particles constitute the grain skeleton. In the pore structure of all soils, the pores are connected. The water fills the space and constitutes a single continuous body. In this water body, pressure may be transmitted and the water may also flow through the pores. The pressure in the pore water is denoted as the pore pressure [1-7]. On an element of soil, normal stresses, as well as shear stresses, may act. The simplest case is isotropic normal stress (Fig.1).

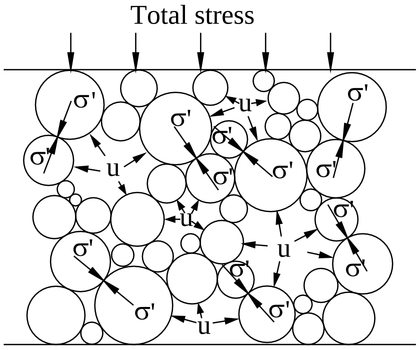

Fig.1 Modelling of effective stress in soils

According to Arnold Verruijt [3], in the interior of the soil (for instance, at a cross-section in the center), this stress is transmitted by a pore pressure u in the water and by stresses in the particles. The stresses in the particles are partly generated by the concentrated forces acting on the contact points between the particles, and partly by the pressure in the water, which almost surrounds the particles. It can be expected that the deformations of the particle skeleton are almost completely determined by the concentrated forces on the contact points because the structure can only deform by sliding and rolling at these contact points. The pressure in the water results in an equal pressure on all the grains. It follows that this pressure acts on the entire surface of a cross-section and that, by subtracting u from the total stress, a measure for the contact forces is obtained. Effective stress is a measure of the concentrated forces acting on the contact points of a granular material. Thus, the author proposes that the effective stress field in the soil is determined based on the shear potential [8,9].

In considerations of limit analysis, not all of the details of the constitutive relations are taken into account but one aspect is given priority, namely the failure criterion of the material [3,10-14]. For soils, a suitable yield condition is the Mohr-Coulomb criterion, described by a friction angle ϕ and a cohesion c. Also, not all the conditions of equilibrium and compatibility equations are taken into account, only a subset of these equations. The purpose of limit analysis is not to determine the complete field of actual stresses and strains, but only to determine certain limiting values. The problem may be to determine a lower bound for the maximum allowable load on a soil body or to determine an upper bound for this maximum load. If a lower bound for the failure load can be found, no failure will certainly occur as long as the real load remains below this lower bound. If an upper bound can be found, failure will certainly occur if the real load is greater than this upper bound. In its simplest form, the theory of plasticity uses a single constant failure condition, which is a function of the stresses only. This condition expresses that for certain combinations of stresses at a point in the material, the deformations increase without bounds (this is called plastic yielding) and that, for smaller stresses, no plastic deformations will occur. A material with such a simple yield condition is called a perfectly plastic material. For soils, a suitable yield condition is the Mohr-Coulomb criterion, although more complex yield conditions have also been studied.

When studying these proofs, it appears that they have only limited validity. The most important restriction is that for a material with friction, such as soil (for which the yield condition is the Mohr-Coulomb criterion, with a friction angle ϕ and a cohesion *c)*, the theorems are only valid if, during plastic deformation, a continuing volume expansion occurs, of magnitude sinϕ times the rate of shear deformation [3,10-13,15]. This seems to be an unrealistic behavior, as it can be expected that in the case of continuing plastic deformations the volume will remain practically constant [3]. This has often been confirmed in experimental studies. An ever-continuing plastic volume expansion would mean that the material expands without bounds, which seems to be improbable. This means that the basic theorems of plasticity are not valid for soils, except when $\phi = 0$, i.e. for purely cohesive materials. For such a material, the theory predicts that the volume is constant during plastic deformation and this is in agreement with experimental evidence [3].

In this study, the novel effective stress field based on the shear potential having a constant volume during plastic deformation is used to apply the limit theorem properly. The limit analysis is formulated in the form of nonlinear programming. The failure load of a strip load on the half-plane is determined in the case of purely cohesive soils and for soils with internal friction. The results of the calculation are compared with existing solutions to prove the reliability and suitability of combining the nonlinear programming method with the novel stress field.

2. RESEARCH SIGNIFICANCE

In this paper, the author proposes a novel effective stress field based on the shear potential having a constant volume during plastic deformation, to properly apply the limit theorem. The nonlinear programming method combined with the novel stress field can be a viable and valuable tool for limit analyses of a strip load on the half-plane (no assumption is required regarding the slip-line or the stress state). It can be extended to consider the bearing capacity of general shallow foundations.

3. A NOVEL EFFECTIVE STRESS FIELD IN SOIL

To distinguish the effective stress field in soil, based on the shear potential and elastic theory, we first need to study the elastic stress field in the soil.

If the soil is considered to be elastic material, the elastic stress field in the soil can be determined by its displacement field and strain. In the plane strain problem, stress is unknown but the stress field can be determined by solving the optimization problem [16]:

$$\begin{cases} Z = \int_V \dfrac{1}{E}\left[\dfrac{\sigma_x^2+\sigma_y^2}{2} - \nu.\sigma_x.\sigma_y \right. \\ \qquad\qquad \left. + (1+\nu)\dfrac{\tau_{xy}^2+\tau_{yx}^2}{2}\right] dV \to \min \\ \dfrac{\partial \sigma_x}{\partial x} + \dfrac{\partial \tau_{yx}}{\partial y} = 0 \\ \dfrac{\partial \sigma_y}{\partial y} + \dfrac{\partial \tau_{xy}}{\partial x} = 0 \end{cases} \tag{1}$$

where:

Z is the elastic deformation potential in the plane strain problem;

E, ν is the elastic modulus and Poisson's ratio of the soil, respectively, and;

σ_x, σ_y, τ_{xy}, τ_{yx} is the stress state at a point in the soil, see Fig. 2.

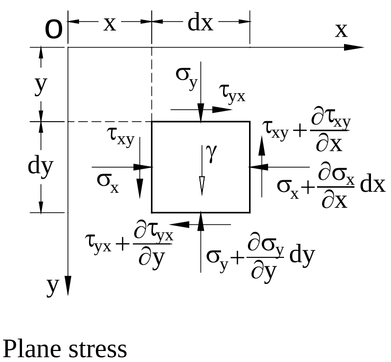

Fig.2 Plane stress

In Eq. (1), we do not consider the self-weight. V is the area of the consideration domain. By variational calculus [17], we can prove that the problem (1) gives us sufficient equations to determine the stress state in the soil. It should be noted that stresses are functions of the coordinates $\sigma_x(x,y)$, $\sigma_y(x,y)$, $\tau_{xy}(x,y)$ that need to be determined.

The functional Lagrange expansion of Eq. (1) is written as:

$$F=\int_V \frac{1}{E}\left[\frac{\sigma_x^2+\sigma_y^2}{2}-\nu.\sigma_x.\sigma_y+(1+\nu)\frac{\tau_{xy}^2+\tau_{yx}^2}{2}\right]dV$$
$$+\int_V \lambda_1(x,y)\left[\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}\right]dV+$$
$$+\int_V \lambda_2(x,y)\left[\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}\right]dV\rightarrow\min \tag{2}$$

The constrained optimization problem (1) is converted into the unconstrained optimization problem (2). λ_1, λ_2 are Lagrange factors and they are unknown in the problem. By variational calculus, we get six equations:

$$\begin{cases}\frac{1}{E}(\sigma_x-\nu.\sigma_y)=\frac{\partial\lambda_1}{\partial x}\\ \frac{1}{E}(\sigma_y-\nu.\sigma_x)=\frac{\partial\lambda_2}{\partial y}\\ \frac{1+\nu}{E}\tau_{yx}=\frac{\partial\lambda_1}{\partial y}\\ \frac{1+\nu}{E}\tau_{xy}=\frac{\partial\lambda_2}{\partial y}\\ \frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}=0\\ \frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}=0\end{cases} \tag{3}$$

Eq. (3) is the system of six equations containing the unknowns: σ_x, σ_y, τ_{yx}, τ_{xy}, λ_1 and λ_2.

In this problem, the shear stress τ_{xy} = τ_{yx}. Taking the sum of the third and fourth equations in Eq. (3) gives:

$$\tau_{xy}=\tau_{yx}=\frac{E}{2(1+\nu)}\left[\frac{\partial\lambda_1}{\partial y}+\frac{\partial\lambda_2}{\partial x}\right]=G\left[\frac{\partial\lambda_1}{\partial y}+\frac{\partial\lambda_2}{\partial x}\right] \tag{4}$$

where G is the shear modulus of soil.

By balancing the dimensions, we see that λ_1 and λ_2 give the dimensions of the displacement.

Moreover, λ_1 is the displacement in the x-direction and λ_2 is the displacement in the y-direction. Eq. (4) is the relationship between the shear stress and the displacement of the elastic medium. The problem can be solved by removing two implicit functions λ_1 and λ_2 in Eq. (3).

The first four equations of Eq. (3) can be converted into an equation of stress as follows.

Taking the derivative of the first equation in Eq. (3) with respect to y and combining it with the third equation, gives:

$$\frac{\partial}{\partial y}\left(\sigma_x-\nu\sigma_y\right)=E\frac{\partial}{\partial y}\frac{\partial\lambda_1}{\partial x}=E\frac{\partial}{\partial x}\frac{\partial\lambda_1}{\partial y}=$$
$$=E\frac{1+\nu}{E}\frac{\partial\tau_{yx}}{\partial x}=(1+\nu)\frac{\partial\tau_{xy}}{\partial x} \tag{4a}$$

Taking the derivative of the second equation in Eq. (3) with respect to *x* and combining it with the fourth equation, gives:

$$\frac{\partial}{\partial x}\left(\sigma_y-\nu\sigma_x\right)=E\frac{\partial}{\partial x}\frac{\partial\lambda_2}{\partial y}=E\frac{\partial}{\partial y}\frac{\partial\lambda_2}{\partial x}$$
$$=E\frac{1+\nu}{E}\frac{\partial\tau_{xy}}{\partial y}=(1+\nu)\frac{\partial\tau_{xy}}{\partial y} \tag{4b}$$

Derivation of Eq. (4*a*) with respect to *y* gives:

$$\frac{\partial^2}{\partial y^2}\left(\sigma_x-\nu\sigma_y\right)=(1+\nu)\frac{\partial^2\tau_{xy}}{\partial x\partial y} \tag{4c}$$

Derivation of Eq. (4*b*) with respect to *x* gives:

$$\frac{\partial^2}{\partial x^2}\left(\sigma_y-\nu\sigma_x\right)=(1+\nu)\frac{\partial^2\tau_{xy}}{\partial x\partial y} \tag{4d}$$

Taking the sum of Eq. (4*c*) and Eq. (4*d*) gives:

$$\frac{\partial^2}{\partial y^2}\left(\sigma_x-\nu\sigma_y\right)+\frac{\partial^2}{\partial x^2}\left(\sigma_y-\nu\sigma_x\right)=2(1+\nu)\frac{\partial^2\tau_{xy}}{\partial x\partial y} \tag{4e}$$

If we take the derivative of the fifth equation with respect to x and the sixth equation concerning y in Eq. (3), then the sum of them gives:

$$2\frac{\partial^2\tau_{xy}}{\partial x\partial y}=-\frac{\partial^2\sigma_x}{\partial x^2}-\frac{\partial^2\sigma_y}{\partial y^2} \tag{4f}$$

Substituting Eq. (4*f*) into Eq. (4*e*) and reducing it gives:

$$\frac{\partial^2}{\partial x^2}\left(\sigma_x+\sigma_y\right)+\frac{\partial^2}{\partial y^2}\left(\sigma_x+\sigma_y\right)=0 \quad (5)$$

Eq. (5) is a compatibility equation in the form of normal stress. Thus, in the plane strain problem, the set of equations convert to three basic equations of the elastic theory:

$$\begin{cases}\nabla^2\left(\sigma_x+\sigma_y\right)=0\\ \dfrac{\partial\sigma_x}{\partial x}+\dfrac{\partial\tau_{yx}}{\partial y}=0\\ \dfrac{\partial\sigma_y}{\partial y}+\dfrac{\partial\tau_{xy}}{\partial x}=0\end{cases} \quad (6)$$

where ∇^2 is the symbol for the Laplace operator,

$$\nabla^2=\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2}$$

Eq. (1) shows the problem of determining elastic stress field in soil. When the stress is unknown, the stress field can be determined according to the minimum potential energy (Eq. (6)).

The shear potential is equal to the total potential energy (elastic deformation potential *Z*) minus volume deformation potential [16]. In the plane strain problem, the shear potential is:

$$Z_s=Z-Z_0=\frac{1}{4G}\left[\left(\sigma_x-\sigma_m\right)^2+\left(\sigma_y-\sigma_m\right)^2+2\tau_{xy}^2\right] \quad (7)$$

where Z, Zs, and Z_0 are the elastic deformation potential Z, shear potential, and volume deformation potential, respectively; G is the shear modulus of the soil; and σ_x, σ_y, τ_{xy} is the stress state at a point in the soil

$$\sigma_m=\frac{\sigma_x+\sigma_y}{2}$$

By substituting σ_m into Eq. (7), we have:

$$\begin{aligned}Z_s&=\frac{1}{4G}\left[\left(\sigma_x-\frac{\sigma_x+\sigma_y}{2}\right)^2\right.\\&\qquad\left.+\left(\sigma_y-\frac{\sigma_x+\sigma_y}{2}\right)^2+2\tau_{xy}^2\right]=\\&=\frac{1}{2G}\left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2\right]\end{aligned} \quad (8)$$

Thus, the plane strain problem of determining the effective stress field in soil, based on the shear potential is:

$$\left.\begin{aligned}&Z_s=\int_V\frac{1}{2G}\left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2\right]dV\to\min\\&\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}=0\\&\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}-\gamma=0\end{aligned}\right\} \quad (9)$$

where γ is the volume weight of soil.

To demonstrate that problem (9) is determinacy, using the variational calculus, we can prove that Eq. (9) gives us enough equations to determine the stress state in the soil.

The functional Lagrange expansion of Eq. (9) is written as:

$$\begin{aligned}F&=\int_V\frac{1}{2G}\left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\left(\frac{\tau_{xy}+\tau_{yx}}{2}\right)^2\right]dV\\&+\int_V\lambda_1(x,y)\left[\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}\right]dV+\\&+\int_V\lambda_2(x,y)\left[\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}-\gamma\right]dV\to\min\end{aligned} \quad (10)$$

where λ_1, λ_2 are unknown Lagrange factors.

By variational calculus, we get six equations:

$$\begin{cases}\dfrac{1}{4G}\left(\sigma_x-\sigma_y\right)=\dfrac{\partial\lambda_1}{\partial x}\\ \dfrac{1}{4G}\left(\sigma_y-\sigma_x\right)=\dfrac{\partial\lambda_2}{\partial y}\\ \dfrac{1}{4G}\left(\tau_{xy}+\tau_{yx}\right)=\dfrac{\partial\lambda_1}{\partial y}\\ \dfrac{1}{4G}\left(\tau_{xy}+\tau_{yx}\right)=\dfrac{\partial\lambda_2}{\partial x}\\ \dfrac{\partial\sigma_x}{\partial x}+\dfrac{\partial\tau_{yx}}{\partial y}=0\\ \dfrac{\partial\sigma_y}{\partial y}+\dfrac{\partial\tau_{xy}}{\partial x}-\gamma=0\end{cases} \quad (11)$$

By balancing the dimensions, we see that λ_1 and λ_2 have the dimensions of the displacement. Moreover, λ_1 is the displacement in the *x*-direction and λ_2 is the displacement in the y-direction. Taking the sum of the first equation and second equation in Eq. (11), we get:

$$\frac{\partial \lambda_1}{\partial x}+\frac{\partial \lambda_2}{\partial y}=\varepsilon_x+\varepsilon_y=0 \qquad (12)$$

From Eq. (12) we can see that the volume deformation is zero. This is an important factor to apply in the limit analysis method for which the yield condition is the Mohr-Coulomb criterion.

Taking the derivative of the first equation in Eq. (11) with respect to y and combining it with the third equation, we get:

$$\frac{\partial}{\partial y}\left(\sigma_x-\sigma_y\right)=4G\frac{\partial}{\partial y}\frac{\partial \lambda_1}{\partial x}$$
$$=4G\frac{\partial}{\partial x}\frac{\partial \lambda_1}{\partial y}=\frac{\partial}{\partial x}\left(\tau_{xy}+\tau_{yx}\right) \qquad (13a)$$

and by taking the derivative of the second equation in Eq. (11) with respect to x and combining it with the fourth equation, we get:

$$\frac{\partial}{\partial x}\left(\sigma_y-\sigma_x\right)=4G\frac{\partial}{\partial x}\frac{\partial \lambda_2}{\partial y}$$
$$=4G\frac{\partial}{\partial y}\frac{\partial \lambda_2}{\partial x}=\frac{\partial}{\partial z}\left(\tau_{xy}+\tau_{yx}\right) \qquad (13b)$$

Derivation of the first equation in Eq. (13*a*) with respect to *y* gives:

$$\frac{\partial^2}{\partial y^2}\left(\sigma_x-\sigma_y\right)=\frac{\partial^2}{\partial x\partial y}\left(\tau_{xy}+\tau_{yx}\right) \qquad (13c)$$

Derivation of Eq. (13*b*) with respect to *x* gives:

$$\frac{\partial^2}{\partial x^2}\left(\sigma_y-\sigma_x\right)=\frac{\partial^2}{\partial y\partial x}\left(\tau_{xy}+\tau_{yx}\right) \qquad (13d)$$

Eq. (13*c*) minus Eq. (13*d*) yields:

$$\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2}\right)\sigma_x-\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2}\right)\sigma_y=0 \qquad (14)$$

Eq. (11) has only three equations:

$$\begin{cases}\nabla^2\left(\sigma_x-\sigma_y\right)=0\\ \dfrac{\partial \sigma_x}{\partial x}+\dfrac{\partial \tau_{yx}}{\partial y}=0\\ \dfrac{\partial \sigma_y}{\partial y}+\dfrac{\partial \tau_{xy}}{\partial x}-\gamma=0\end{cases} \qquad (15)$$

where ∇^2 is the symbol for the Laplace operator:

$$\nabla^2=\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2}$$

Eq. (15) have three equations for finding unknowns such as σ_x, σ_y and τ_{xy}. Thus, the problem of determining the effective stress field in soil (9) can be solved.

By comparing the problem of determining the elastic stress field in the soil (6) with the problem of determining the effective stress field in the soil based on the shear potential (15), we see that the difference is the sign "±" in the first equation.

Now, we have identified the effective stress field in the soil as being the deterministic static, we have enough equations to be able to solve the soil mechanics problems (e.g. external loads).

4. LIMIT ANALYSIS OF A STRIP LOAD ON THE HALF-PLANE

The problem of a strip load on a half-plane is shown in Figure 3.

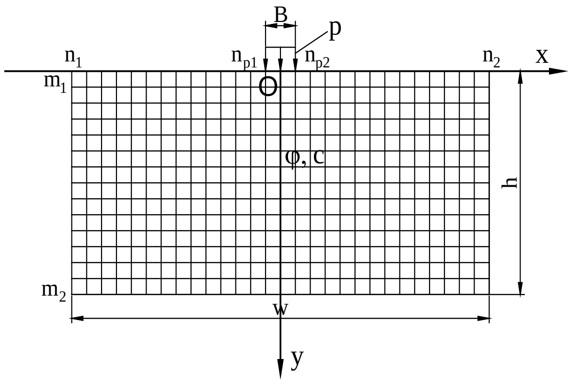

Fig.3 Strip load on a half-plane

The weight of the soil will be disregarded in this problem.

4.1 Methodology

The plastic limit theorems of Drucker et al. (1952) can be employed to obtain upper and lower bounds of the failure load for stability problems, such as the critical heights of vertical cuts, or the bearing capacity of soils. The conditions required to establish a lower bound or upper bound are essential and are described below.

4.1.1 Lower bound theorem

The loads, determined from the stress distribution, must satisfy: (a) the equilibrium equations; (b) the stress boundary conditions; and (c) not violate the yield condition by not being greater than the true failure load. The stress distribution that satisfies items (a), (b), and (c) has been termed a statically admissible stress field, for the problem under consideration. Hence, the lower bound theorem may be rewritten as follows: the true failure load is larger than the load corresponding to an equilibrium system.

4.1.2 Upper bound theorem

The loads, determined by equating the external rate of work to the internal rate of dissipation in an assumed deformation mode (or velocity field) must satisfy: (a) the velocity boundary conditions and (b) the strain and velocity compatibility conditions, and not be less than the true failure load. A velocity field that satisfies the above conditions has been termed a kinematically admissible velocity field. The upper bound theorem considers only velocity or failure modes and energy dissipations. The stress distribution need not be in equilibrium and is only defined in the deforming regions of the mode.

4.2 Numerical Method

Solving this problem by the analytical method is very difficult. Therefore, the authors solved the problem by the finite difference method. The soil mass is divided into the differential grid as shown in Figure 3. At each node, there are three unknowns: σ_x, σ_y, τ_{xy}.

As the applied load p is increased, the stress intensity reaches the yield value and the yield zone increases (the stress field should also satisfy the condition that the yield condition is never violated). Eventually, the load reaches the value that triggers the failure mechanism in the soil, then we know the failure load p_u. Hence, the load p is unknown in the problem. The objective function of a strip load on half-plane problems is written as follows:

$$Z_1 = \int_V \frac{1}{2G}\left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right] dV - p \rightarrow \min \quad (16)$$

The objective function (16) must satisfy two equilibrium equations and the constraints as follows:

- soils can only transfer compressive normal stresses and no tensile stresses:

$$\sigma_x \geq 0 \ \& \ \sigma_y \geq 0 \quad (17)$$

- the Mohr-Coulomb yield criterion is not violated:

$$\tau_{\max} - \frac{\sigma_x + \sigma_y}{2}\sin\phi - c.\cos\phi \leq 0 \quad (18)$$

- boundary conditions exist on the soil surface and to infinity.

$$y = 0, \ \ n_{p1} \div n_{p2}: \ \ \sigma_y \geq 0, \ \ \tau_{xy} = 0 \quad (19)$$

$$y = 0, \ \ n_1 - n_{p1} \ \& \ n_{p2} - n_{n2}: \ \ \sigma_y = 0, \ \ \tau_{xy} = 0 \quad (20)$$

$$y = m_2: \ \ (\sigma_y^{(m_2,n)} - \sigma_y^{(m_2-1,n)})^2 \rightarrow \min \quad (21)$$

$$x = n_1: \ \ (\sigma_x^{(m,n_1)} - \sigma_x^{(m,n_1+1)})^2 \rightarrow \min \quad (22)$$

$$x = n_2: \ \ (\sigma_x^{(m,n_2)} - \sigma_x^{(m,n_2-1)})^2 \rightarrow \min \quad (23)$$

The boundary conditions (21), (22), and (23) are developed according to the extreme Gauss's principle method to ensure that the problem is considered in the infinite half-plane [18].

The limit analysis becomes an optimization problem. The objective function (16) is a nonlinear (quadratic) function for stress. The constraints are two linear equilibrium equations, linear inequalities (17), and nonlinear inequalities (18). Such a problem is the nonlinear programming problem. The author wrote a program in Matlab to solve the problem.

4.3 Numerical Examples

Example 1: Determining the true failure load of a strip load on a half-plane. The width of strip load B = 2 m. The soil is considered to be weightless (γ = 0) and frictionless (ϕ = 0), the only relevant property is the cohesive strength c. The difference grid size is Δx = Δy = 1m.

The problem is realized by a Matlab code [8].

The results of the calculation are shown in Fig. 4.

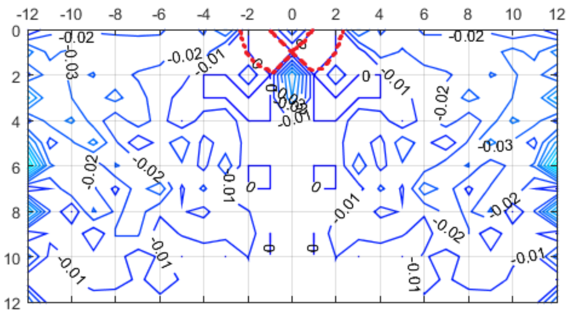

Fig. 4 Chart of the ability for yield

The number on the contour is the value of the expression:

$$\tau_{\max} - \frac{\sigma_x + \sigma_y}{2}\sin\phi - c.\cos\phi \quad (24)$$

The zero contours are the lines connecting the points that reach the yield value. The remaining contours are less than zero, i.e. the zones have not reached the yield value. Nowhere violates the yield condition (satisfying the lower bound theorem).

In Fig. 4, at the value of the load p=5.14c, the yield points develop and connect as a slip-line extending to the surface (bold dashed lines and zero value). At that time, it can be seen that the mass soil has formed a failure mechanism. Therefore, the failure load in the case of a strip load is p_u=5.14c. This value agrees with the well-known 'exact' Prandtl's solution (upper and lower bounds of the failure load are equal to 5.14c) [19]. The shape of the failure mechanism is the same as the Prandtl mechanism.

This confirms the validity of the novel effective stress field based on the shear potential and the nonlinear programming method.

Example 2: Determining the true failure load for soil with the angle of internal friction ϕ and the cohesion c.

The problem is realized by a Matlab code [8].

The result of the calculation is shown in Fig. 5.

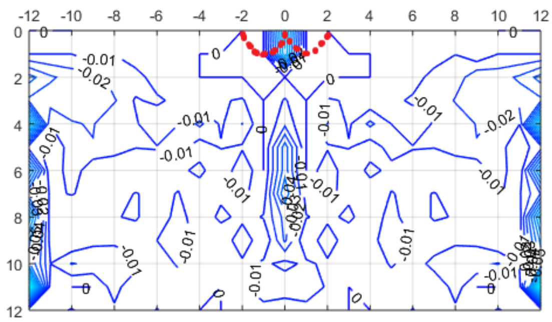

Fig. 5 Chart of the ability for yield

For ϕ ranging from 1^0 to 40^0, the corresponding failure loads are compared with the Prandtl's solution shown in Table 1 and Fig. 6.

Table 1. Failure load (p_u/c)

ϕ (^{0})	Proposed	Prandtl	% difference
0	5.14	5.14	0.00
1	5.37	5.38	-0.17
5	6.29	6.49	-3.06
10	7.72	8.34	-7.49
15	9.74	10.98	-11.27
20	12.62	14.83	-14.93
25	17.33	20.72	-16.38
30	24.92	30.14	-17.33
35	37.12	46.12	-19.51
40	59.08	75.31	-21.56

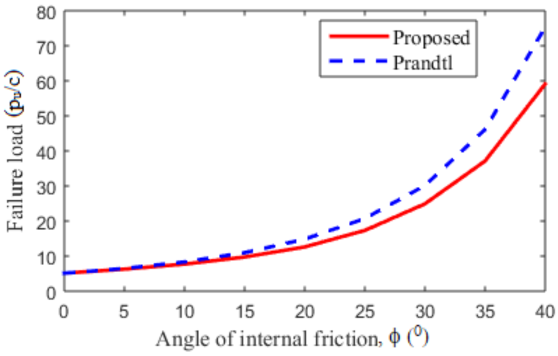

Fig. 6 Failure load of a strip load on a half-plane

The value of (p_u/c) is the cohesion bearing capacity factor N_c in the formula for determining the bearing capacity of a strip foundation. Prandtl's formula has been extended by Keverling Buisman (1940), Von Terzaghi (1943), Caquot and Kérisel (1953; 1966), Meyerhof (1951; 1953; 1963; 1965), Brinch Hansen (1970), Vesic (1973; 1975) and Chen (1975) to become the complete formula for calculating the bearing capacity [1-7,11]. The factor N_c is still used according to Prandtl's solution.

It can be observed that, for low friction angles the failure load is almost the same as Prandtl's solution, and the failure mechanism looks like the Prandtl mechanism. For soil with high friction angles, the failure load is smaller than Prandtl's solution (by about 15-22%) and the failure mechanism looks like the circular (bold dashed lines and zero values), and not like the Prandtl mechanism, see Fig. 5.

As in the previous example, the limit theorems have been applied to determine failure loads for a purely cohesive soil (φ = 0). For soils with internal friction, the basic theorems of the theory of plasticity (the upper and lower-bound theorems) are not valid when the volume changes during failure. This means that the Prandtl mechanism leads to the conclusion that the failure load obtained is an upper bound to the correct value. The novel effective stress field in the soil is based on the shear potential having a constant volume during plastic deformation, and so the limit theorems are valid. The failure loads are smaller, especially for higher friction angles, and therefore safer and more accurate than the existing results [1-7,11,12,19,20]. This change in failure mechanism is a sign of redistribution of the stresses.

5. CONCLUSION

The novel effective stress field in the soil, based on the shear potential is the deterministic static, and we have enough equations to solve the problems in soil mechanics. The nonlinear programming method, combined with the novel stress field, allows the application of the limit theorems to directly determine the failure load of a strip load on the half-plane (no assumption is required regarding the slip-line or the stress state).

For purely cohesive soils, the results of the calculation of the failure load of a strip load on the half-plane agree with Prandtl's solution. This confirms the validity of the novel stress field and the nonlinear programming method's ability to solve the problem.

For soils with internal friction, the results of the calculation of the failure load are smaller than Prandtl's solution and are, therefore, safer and more accurate than the existing results.

The method can be a viable and valuable tool for limit analyses of a strip load on the half-plane. It can be extended to consider the bearing capacity of the general type of shallow foundation on a soil characterized by its cohesion c, friction angle ϕ, and volume weight γ.

6. REFERENCES

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