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3-D CRITICAL SLIP SURFACE BASED ON ARTIFICIAL INTELLIGENT

* Nesrin Ali Morsey[1], Samir Gad [2] and M.Abo Bakr [3]

[1]Faculty of engineer, Mechanics and Foundation Engineering, Al-Azhar University, Cairo, Egypt; [2,3]

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ABSTRACT: The problem of slope stability is an important issue for human and structural safety. To prevent landslides, the stability analysis of the three-dimensional (3D) slope is essential, but the most important and difficult problem is how to determine the 3D critical slip surface with the minimum safety factor in the earth slope. The critical slip surface of two-dimensional 2D soil slope stability was calculated by several researchers using optimization techniques. The Grey Wolf optimization technique (GWO) was applied to evaluate the critical slip surface (CSS) of soil slopes. This is one of the optimization techniques used to solve several engineering problems to determine the critical slip surface and the corresponding 3D soil slope safety factor. As the fundamental particle, a general rotating spherical shape was introduced for 3D slope stability research. The results showed the efficiency and effectiveness of GWO in evaluating the (CSS) of 3D soil slope compared with other optimization methods, as the safety factor is lower than the other results, and the failure form is much more close to the natural slope shape. By using GWO the critical slip surface of a slope can be located in a larger scope and more rapidly than the previously presented methods.

Keywords: Slope stability, Limit equilibrium, Optimization

1. INTRODUCTION

Slope stability analysis is a method for determining the stability of earth and rock-fill dams, embankments, excavated slopes, and natural slopes in soil and rock using an analytical or empirical approach. Analyses are primarily focused on determining the reasons for a slope failure, or the elements that could potentially trigger a slope movement, resulting in a landslide, as well as preventing, slowing, or stopping such movement by mitigation techniques. The stability of a slope is essentially controlled by the ratio between the available shear strength and the acting shear stress, which is called a safety factor. A previously stable slope can be influenced by a variety of predisposing variables or processes that reduce the safety factor - either by increasing shear stress or decreasing shear strength - and ultimately lead to slope failure. The slope stability benefits are not only for an essential environment for human survival but also an important aspect of engineering construction. The determination of the slope stability of the critical slip surface is very relevant for regional landslide forecasting and assessment and provides a scientific basis for the management of the regional geological environment and geological hazard assessment. The two-dimensional slope stability analysis remains the most common method of analysis in slope engineering practice for its simplicity Mishra, Ramana, and Damodar [1] presented teaching–learning-based optimization (TLBO). Mishra [2] presented the application multiverse optimization algorithm (MVO) which applies the method on soil slopes containing a band of weak layers sandwiched between two strong layers, to determine if the proposed algorithm can capture the presence of the weak soil layer. S. H. Li, Wu, and Luo [3] applied the improved whale optimization algorithm (IWOA). Gao, [4] applied the meeting ant colony. Bai et al. [5] used the simulated annealing algorithm (SAA).Y. X. Li and Yang [6] used a new method called smoothed particle hydrodynamic (SPH), which is a mesh-less, particle-based numerical approach in which the problem domain is represented or discretized by nodes or particles. Despite the popularity of 2D slope stability methods, these plain-strain problems imagined slopes in the third dimension to be symmetric and indefinitely long. However, most engineering problems have three-dimensional characteristics that cannot be taken into account by a conventional two-dimensional plane. In essence, a three-dimensional (3D) problem is a slope stability problem and its analysis involves sound geotechnical expertise as well as the efficient use of a sophisticated design computer code. Several methods are used to evaluate the stability state of a slope, the most common of which is the limit equilibrium method (LEM) Kalatehjari and N. Ali [7]. Several researchers have researched (3D) LEM analysis, including Hovland [8], Hunger, Zhang [9], Z. Xing [10], Lam and Fredlund [11], Cheng [12], etc was used the slicing method in 2D analyses has been extended into 3D analysis with columns. The static behavior of the slope on the verge of failure is performed by this technique and the equilibrium of the soil body in the static condition is established. Consequently, no stress-strain relationship is considered and Fredlund [13] is not studied for the corresponding deformation within the soil body. As a consequence, it is necessary to assume the shape of any possible slip surface that determines the lower boundary of the sliding body. To evaluate the critical slip surface (CSS) as the least stable slip surface of all possibilities, a numerical ratio as a safety factor (FOS) is used. At present, in evaluating the critical slip surface and the corresponding safety factor, different computational methods have different accuracy was used Troncone and Shklyaev [14,15], As a practical way of using NURBS surface and ellipsoidal surface to model a three-dimensional sliding surface by Cheng, Liu[16]. Ahangar-Asr [17] used a three-dimensional approach with a genetic algorithm (GA) to calculate the minimum safety factor of slope and the shape and direction of the corresponding failure surface. Using an optimization method to obtain the nonspherical critical slip surface, Hajiazizi and Tavana[18] calculated the three-dimensional nonspherical critical slip surface In slopes, which is more consistent with the actual slip surface in nature by using the three-dimensional alternating variable local gradient. Duncan [19], Akhtar [20], and Kalatehjari & Ali [7] made extensive reviews of existing 3D LE procedures available since 1969. However, after more than four decades, 3D procedures have not yet been approved by geotechnical practitioners compared to the many 2D procedures used today.Extending 2D processes to 3D requires more assumptions for statically determining the problem and constructing the 3D geometry of a real slope and determining its critical failure surface are problems that are still not well understood Akthar [20]. According to all previous works, the safety factor using 3D analysis methods is always higher than that from the corresponding 2D analysis. In the last decades, there are many optimizations techniques proposed to solve the very complicated engineering problems so far, many of them inspired by hunting and search behaviors. Of particular interest is that the gray wolves have a very strict social dominant hierarchy this led to the proposal of a new optimizations techniques algorithm inspired by grey wolves, and investigate its abilities in solving benchmark and real problems. as to the 3D critical slip, the surface is one of these very complicated problems so many optimization techniques were applied to try to reach the best solution, to be too Near to the natural soil behavior. The presented method is simple, flexible, and derivation-free. This allows an accurate simulation to computer scientists and assists other scientists to learn it quickly and apply it to their problems. This motivated us to apply the new technique Based on what I have provided from good results on others engineering applications.

2. RESEARCH SIGNIFICANCE

A conventional issue in geotechnical engineering that is still challenging for researchers is assessing the critical slip surface of a soil slope. This issue involves a huge process of searching. Plus the growing numbers of possible solutions, their processes are becoming slow. By knowing that the safety factor determined by using artificial intelligence is always less than the one calculated by traditional methods, which means that it is more efficient to solve any engineering problem by similarity to any natural problem. This algorithm has higher computational efficiency than the other optimization techniques. In addition, it is fast, accurate, and very simple for the user. The framework of this study was helping engineers assess slope performance, which is meaningful in slope engineering design.

3. METHODOLOGY

In this section, using the Grey wolf optimization techniques, a 3D slope stability analysis method will be determined and the limit equilibrium method is proposed to determine the critical slip surface and the corresponding safety factor of 3D soil slopes. With this paper, assumed a spherical slip surface shape, and to get the center points and the radius of the best sphere a MATLAB program will be used. Two main problems should be identified: (a) the position of the critical slip surface and (b) the required safety factor for the critical slip surface

3.1 Limit Equilibrium Method

Spherical shapes or ellipsoidal shapes are widely used in 3D slope stability slip surfaces. Here, for the 3D slope stability analysis, the spherical shapes of slip surfaces are considered. The following "Eq. (1)"can be used to define the spherical slip surface.

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2 \quad (1)$$

Where (x_0, y_0, z_0) are the coordinates of the center point (O) of a sphere, and r is the radius of the sphere. Different numerical methods can simulate the stress field of a slope under natural conditions. "The main stresses σ_1,σ_2 and σ_3 of the representative element can be determined by numerical simulation, as shown in Fig.1 assuming that the slip surface area S is dA and the total

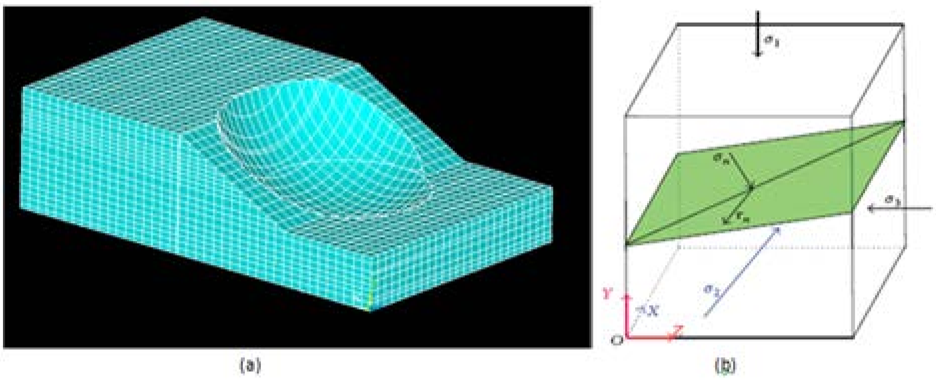

Fig.1a Assumptions for the slip surface of 3D soil slopes, and Fig.1b Free body diagram of a slice

stress on the slip surface S is P_i "Eq.(2)":

$$P_i = \sqrt{\sigma_i^2 + \tau_i^2} \quad (2)$$

Where: The normal stresses, and the shear stress on the slip surface, respectively are σi and τi. According to force equilibrium analysis, the total stress, Pi, "Eq. (3), (4), (5)" can be divided into three parts along with the axis directions of X, Y, and Z:

$$\mathbf{P_i(x) = \sigma_3 \cos\alpha} \quad (3)$$

$$\mathbf{P_i(y) = \sigma_2 \cos\beta} \quad (4)$$

$$\mathbf{P_i(z) = \sigma_1 \cos\gamma} \quad (5)$$

Where: the angles between the normal direction of the slip surface S and the axis directions of X, Y, and Z, respectively are α, β, and γ. the relationship between these there angles is given by "Eq. (6)":

$$\mathbf{(\cos\alpha)^2 + (\cos\beta)^2 + (\cos\gamma)^2 = 1} \quad (6)$$

The normal stress and total stress can be determined as follows by "Eq. (7), (8)":

$$\mathbf{\sigma_i = \sigma_3(\cos\alpha)^2 + \sigma_2(\cos\beta)^2 + \sigma_1(\cos\gamma)^2} \quad (7)$$

$$\mathbf{P_i = \sqrt{\sigma_3^2(\cos\alpha)^2 + \sigma_2^2(\cos\beta)^2 + \sigma_1^2(\cos\gamma)^2}} \quad \mathbf{(8)}$$

The shear stress can be computed in "Eq. (9):

$$\mathbf{\tau_i^f = C_i + \sigma_i \tan\emptyset_i} \quad (9)$$

Where: Ci and φi are the cohesion and friction angle of the soil, respectively. The safety factor In terms of moment equilibrium F_m, the safety factor can be obtained by summing up the moment of all forces over the failed mass around a rotation axis as "Eq. (10), similarly, by summing forces in the X direction over the failed mass, the safety factor for force equilibrium F_f can be derived as "Eq. (11)

$$F_m = \frac{M_R}{M_S} = \frac{\sum_{i=1}^{n}(C_i L_i + \sigma_i L_i \tan\emptyset_i)}{\sum_{i=1}^{n}\tau_i L_i} \quad (10)$$

$$\mathbf{F_f = \frac{\tau^f}{\tau} = \frac{\sum_{i=1}^{n}\tau_i^f}{\sum_{i=1}^{n}\tau_i} = \frac{\sum_{i=1}^{n}(C_i + \sigma_i \tan\emptyset_i)}{\sum_{i=1}^{n}\tau_i}} \quad (11)$$

Where τf and τ are the total shear resistance and shear stress on the slip surface, respectively the safety factor will be determined as the same as two-dimensional (2D) in the case of a three-dimensional (3D) slip surface for moment and force. In this case, by using the limit equilibrium method as shown in Fig.1.a, the slip surface will be divided into numbers of slices to decide the slope elements are cut by a spherical surface with many different elements interacting with the cutting surface, and the slip surface consists of a group of elements. The stress distribution of a slip surface element in one representative component is shown in Fig.1.b.

4. GREY WOLFS OPTIMIZATION (GWO)

This work proposes a new Grey Wolf Optimizer (GWO) to simulate the leadership hierarchy; four types of grey wolves are used, such as alpha, beta, delta, and omega. In addition, Mirjalili [21] implements the three main stages of hunting, looking for prey, encircling prey, and attacking prey.

4.1. Concept of Grey Wolves

The Grey Wolf (Canis lupus) is part of the family Canidae. Of particular interest, they have a very strict hierarchy of social superiority, as shown in Fig.2. The leaders are called alpha it is a male or female. The alpha wolf is often referred to as the dominant wolf because the pack should obey

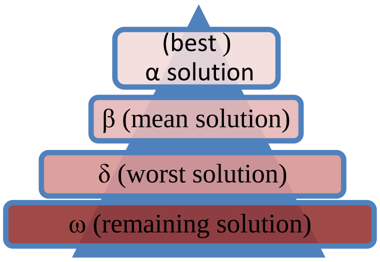

Fig.2 Grey wolf Hierarchy

his/her commands Mech [22]. The alpha is responsible for making hunting, sleeping place, time to wake, and soon decisions, these decisions are dictated to the other wolves, which follows alpha. However, alphas have some democratic behavior. By keeping their tails down, the other wolves obey alpha, finally, the alpha wolves must mate on the pack only. Alpha must be the best member of the package management, not the strongest member of the pack. This implies that a team's structure and discipline are more significant than its strength. In grey wolves, the second level is beta; it is the subordinate wolves that help the alpha in decision-making or other pack activities. The beta wolf may also be male or female, and if one of the alpha wolves passes away or becomes very old, it might be the best candidate to be the alpha. The beta wolf must respect the alpha wolf, and he must be respected by the other wolves. In the grey wolf, the lowest level is the omega, the most suffering wolves that often submit to all other wolves. They are the last wolves allowed to eat. The babysitters in the pack could also be the omega. Scouts, sentinels, elders, hunters, and caretakers are all members of the Delta category. Scouts are in charge of keeping an eye on the territory's limits and alerting the pack if there is any danger. Sentinels protect and ensure the pack's safety. Elders are the experienced wolves who have been alpha or beta in the past. Finally, the caretakers are responsible for caring for the weak, ill, and wounded wolves in the pack. Group hunting is another fascinating social activity of grey wolves, in addition to the social hierarchy of wolves. The key stages of grey wolf hunting are as follows, according to Muro et al. [23]

1) Tracking, chasing and approaching the prey
2) Pursuing, encircling, and harassing the prey until it stops moving
3) Attack towards the prey

To develop GWO and perform optimization, the hunting strategy and the social hierarchy of grey wolves are mathematically modeled.

4.2. Mathematical Model and Algorithm

4.2.1 Social hierarchy

The social hierarchy of wolves in the mathematical model when developing GWO is considered the best approach to alpha (α), so, the second and third-best solutions will be beta (β) and delta (δ), so the remaining candidate solutions are omega (ω). Hunting (optimization) is guided by α, β, and δ. The ω wolves follow these three wolves

4.2.2 Encircling prey

Grey wolves encircle prey during the hunt, to mathematically model encircling behavior the following "Eq. (12), (13) are proposed:

$$\vec{\mathbf{D}} = \left|\vec{\mathbf{C}}.\vec{\mathbf{X}_{\mathbf{p}}}(\mathbf{t}) - \vec{\mathbf{X}}(\mathbf{t})\right| \tag{12}$$

$$\vec{\mathbf{X}}(\mathbf{t}+\mathbf{1}) = \vec{\mathbf{X}_{\mathbf{p}}}(\mathbf{t}) - \vec{\mathbf{A}}.\vec{\mathbf{D}} \tag{13}$$

Where t indicates the current iteration, A and C are coefficient vectors, Xp is the position vector of the prey, and indicates the position vector of a grey wolf. The vectors A and C are calculated by "Eq. (14), (15):

$$\vec{\mathbf{A}} = \mathbf{2}\vec{\mathbf{a}}\,.\vec{\mathbf{r}_{\mathbf{1}}} - \vec{\mathbf{a}} \tag{14}$$

$$\vec{\mathbf{C}} = \mathbf{2}.\vec{\mathbf{r}_{\mathbf{2}}} \tag{15}$$

Where components of a are reduced linearly from 2 to 0 throughout iterations and r1, r2 are random vectors in [0,1]. To see the effects of equations (1) and (2), A two and three-dimensional position vector and some of the possible neighbors are shown in Fig.3a,b.

4.2.3 Hunting

Generally, the hunt is led by the alpha. The beta and delta could often also engage in hunting. However, there is no idea about the optimal position of the prey in a search area. To mathematically mimic the hunting behavior of grey wolves, the alpha was thought to be the best solution. Beta and delta are then supposed to provide a better understanding of the location of the prey. Thus, other studies such as omega will be saved and forced to change their positions according to the best search location.

4.2.4 Attacking prey (exploitation)

The grey wolves attacking the prey when it stops moving, to reach the prey in the mathematically model the value of $\vec{a}$ must be decreased. Note that the fluctuation range of$\vec{A}$ is also decreased by $\vec{a}$. Where $\vec{A}$is a random value in the interval [-a, a], and (a) is decreased from (2 to 0) on iterations. When random values of A $^{\rightarrow}$are in [-1,1], the next position of a search agent can be in any position between its current position and the position of the prey.

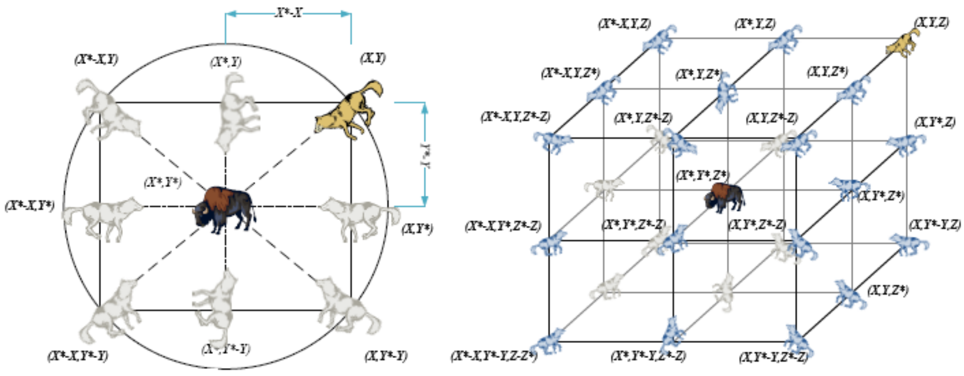

Fig.3 2D and 3D position vectors and their possible next locations

Fig.4.a shows that if |A|<1 that mean the wolves forces to attack the prey.

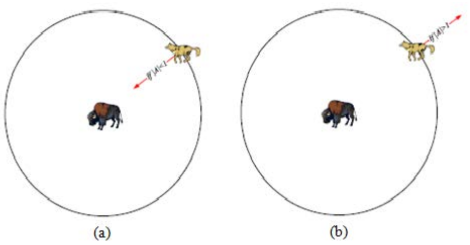

Fig.4 Attacking prey versus searching for prey

4.2.5 Search for prey (exploration)

Alpha, beta, and delta diverge from each other in real life to hunt for prey and unite to attack prey. So, a random value greater than 1 or less than -1 was used by the mathematical model to force the search agent to move far from the prey. Also, Fig.4.b shows that in the case of |A|>1 this forces the grey wolves to diverge from the prey to hopefully find better prey. Another component that must take into consideration in GWO is $\vec{C}$. Which seen in "Eq. (15) the $\vec{C}$ vector contains random values from [0, 2]. To randomly affirm (C>1) or decrease the concentration of the effect of prey in the measurement of distance (C<1) in the "Eq. (14), this part provides random weights for prey. This factor C is not reduced, like A, linearly. We forced C not only at initial iterations but also at final iterations to be random values at all times. The C vector can also be seen as the impact of obstacles in nature. Finally, the search process starts with the GWO algorithm selecting a random number of grey wolves. Alpha, beta, and delta wolves estimate the likely prey location after several iterations. Each candidate solution updates its distance from the prey. To emphasize discovery and exploitation, parameter (a) is modified from 2 to 0 respectively. Candidate solutions are used when |A|>1 to move further from the prey and when |A|<1 to move towards the prey.

5. CASE STUDY

to ensure that GWO's new optimization techniques are relevant for solving the 3D soil slope stability problems. In this example: Chen, J.Wang [24] will be used in this comparison, as shown in Fig.5 the soil slope consists of three different types of soil three layers: layer (1) is 11.9 m height, layer (2) is 0.3 m and layer (3) is 11 m height from top to bottom, the length is 80 m, width is 50 m, slope inclination is 27°, and the coordinate Z is a constant is equal to (25). Also, Table 1 will present the soil parameters. The complete set of input parameters includes E is the elastic modulus; υ is Poisson's ratio; γ is the density; c is the cohesion, and Φ is the friction angle.

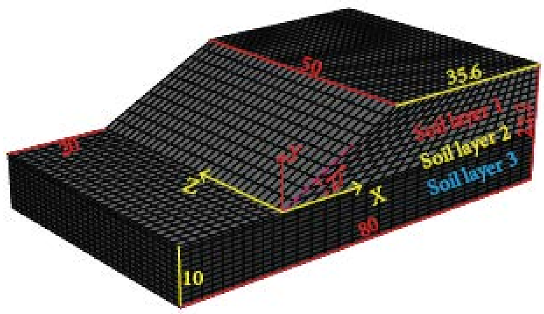

Fig.5 Geometric condition of the 3D slop

Table 1 Mechanical parameters of the soil slope.

soil	Soil type	E MPa	υ	γ g/cm^3	C KPa	Φ°
L1	Clay	500	0.3	1.92	29	20
L2	Weak sand clay	500	0.3	1.92	10	0
L3	clay	500	0.3	1.92	29	20

6. DISCUSSION AND RESULTS

An old problem in geotechnical engineering is determining the critical slip surface of a soil slope, and the 3D critical slip surface is still a very complicated problem for researchers. Since this problem has a large number of potential solutions, GWO was used in slope stability analysis to get rid of these limitations, based on its promising results in advanced engineering problems. After using geo studio and Matlab programs to apply the Gray wolf optimization techniques GWO in the previous soil slope stability, the results obtained are The soil collapse as part of the spherical shape as shown in Fig.6, the spherical that caused this form of collapse is the best solution of the solving program that depends on solving the gray wolf GWO mathematics equations. This is the best solution that minimizing soil slip surface factor of safety.

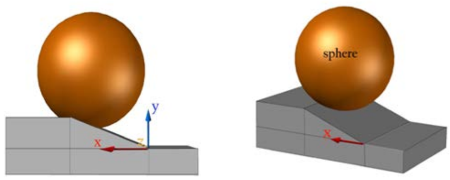

Fig.6 The soil geometry and the spherical shape in 3D soil slope

The program starts with circle center (5.1, 19.6, 25) and radius (R=24.4m). The results are the spherical center is (6.33, 19.24, and 25) with radius R=24.7m the results tell us that the displacement in X-direction was increased, and the displacement in Y- the direction was decreased it means that the sphere moved closer to the sliding block and cutting a large part of, this change was very obvious in the values of resisting and sliding moments MR=396.8KN.m and the MS=188.7KN.m as shown in Fig.7 which presented also the minimum safety factor value F.O.S =2.103 which calculated by divided the previous values of the resisting moment by sliding moment. After running the program they are several solutions of a factor of safety appeared one of these results to forces and the other results are to moment as shown in Fig.8 the intersection between the two curved was the best solution of the gray wolves. Fig.9 presented the slip surface and the spherical shape which causes the minimum factor of safety. The validity and effectiveness of the proposed GWO algorithm are investigated by comparing the results by several previous diverse examples as shown in Yu-Chuan Yang [25] the researcher applied the finite element method at the same problem then compared the results by previous methods such as Zhang' result [26], Zhou' result [27] and Chen' result [28], Then the comparison was tabulated in Table 2. As shown in the previous table the results calculated by using the GWO are better than the compared results, first: the factor of safety is less than the others it found to be (2.103) but the calculated radius is bigger than the others (r =24.7) at the same time the Y value of the sphere center is small it means the cut weight is big and have a minimum factor of

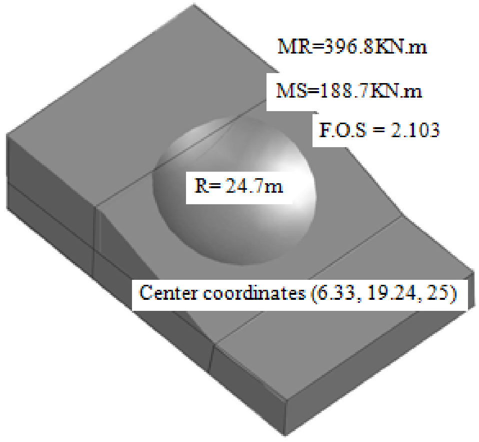

Fig.7 The circular coordinates its radius and the minimum factor of safety

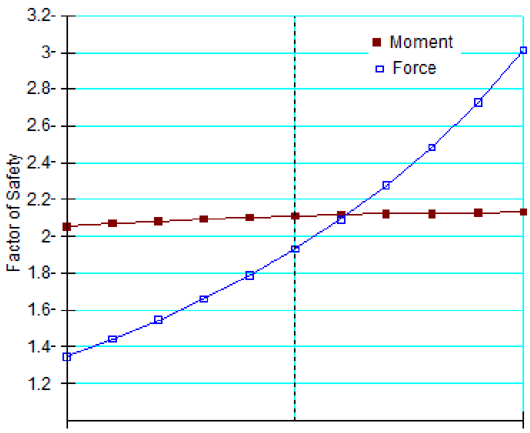

Fig.8 The best solution of GWO factor of safety

Table 2 Comparison between GWO method results and previous work results

method	Center(m)	Radius	F.O.S
Zhou[27]	(5.1, 19.6, 25)	24.4	2.192
Chen[28]	(5.1, 19.6, 25)	24.4	2.262
Zhang[26]	(5.1, 19.6, 25)	24.3	2.122
Yang [25]	(6.1, 19.3, 25)	24.5	2.135
GWO	(6.33, 19.24, 25)	24.7	2.103

safety as the same time. The factor of safety calculated to force and moment as shown in fig.8, the factor of safety rang was narrow in case of the moment it was between 2.05 to 2.15, but the factor of safety calculated by force has a wide range it was between 1.35 to reach 3. As shown in Fig.10 presented the normal stress results which are in the range of 5.7 kPa to 140 kPa. The normal stress on the slip surface is related to the stress field of the soil slope and can be used to compute the shear resistance of the slip surface. Although there is little difference in the shape of the spherical slip

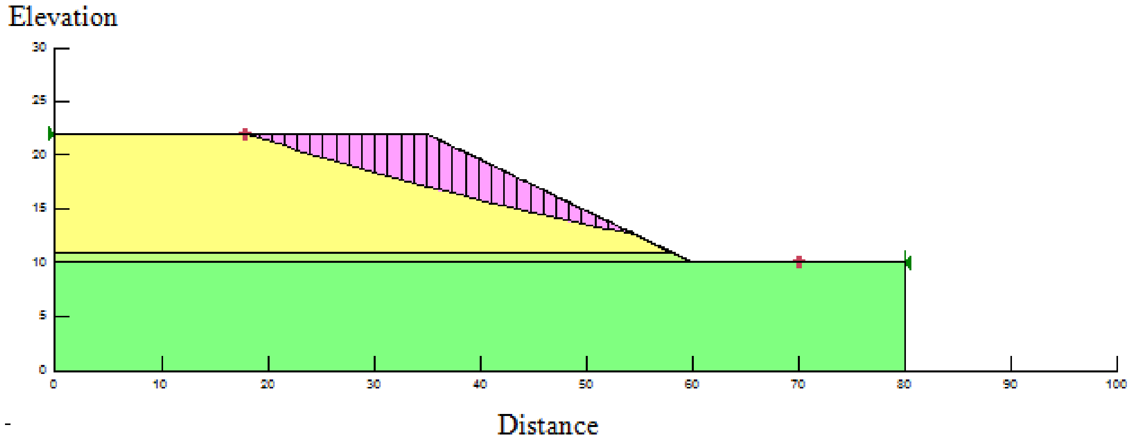

Fig.9 The slip surface and the spherical shape

surface, the safety factor of benchmark soil slope using the presented method is close to those of other methods. The simulated results indicate that the proposed method can be used for the spherical stability analysis of a 3D soil slope.

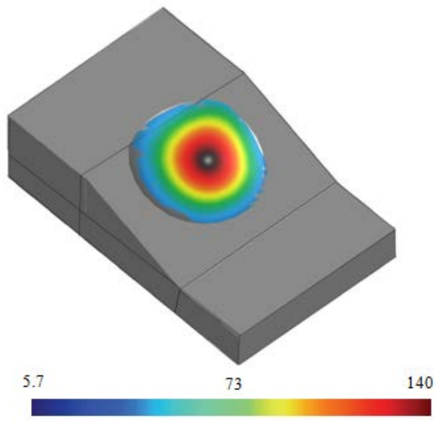

Fig.10 The normal stress results range

7. CONCLUSION

This paper demonstrates the prediction of the soil slip surface and its factor of safety by using optimization techniques. So the after simulating this problem with the Gray wolf behavior and running the program many times the following conclusion was obtained:

1) The MatLab software was developed and validated as a generalized three-dimensional stability analysis model.

2) The factor of safety was calculated two times one was respected to moment and the other was respected to force, in the two cases the was respected to force, in the two cases the program was running many times, then the results were started from 1.35 to reach 3, but in the case of using moment the numbers were larger, they were started by 2.05 until 2.15.

3) The factor of safety chosen and compare with the others researchers' work was the intersection point between the solving by using both the moment and force methods and it was found to be 2.103.

4) A safety factor extracted from this software was satisfactorily compared with analytical solutions and other published examples of issues after calculated the percentage between this work results and the results of the previous works it found that the safety factor calculated by Chen is the biggest one which increased by 7.56% than the Grey wolves one. In case that the smallest factor of safety was calculated by Zhang and it is considered the nearest one to this work but still larger than it by 0.9%. The percentage results indicated that the safety factor of Grey Wolfs is still lower than any previous study.

5) The GWO has higher computational efficiency, faster and easier than the other optimization techniques.

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